A Syntactic Approach to Satisfaction

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Abstract. Most of the research on propositional logic satisfiability follows the Davis-Putnam approach, which is based on a semantic view that all the possible assignments of true values to propositional symbols should be tested. This paper proposes an algorithm that is based on a syntactic view, that explores the properties of the normal forms of a given theory to verify its satisfiability. Any propositional theory can be represented either by its conjunctive normal form (CNF) or by its disjunctive normal form (DNF). The proposed algorithm, given a propositional theory represented by a CNF, calculates, using a specially designed representation, the minimal DNF, where minimal is defined as the smallest set of non contradictory, non subsumed dual clauses. Each one of the minimal dual clauses represents (minimally) a set of semantic assignments that satisfy the theory. Therefore, if we generate all minimal dual clauses, we have a syntactic description of all possible assignments. The main idea is that the number of minimal dual clauses is always less (or in the worst case equal) than the number of assignments and this is especially true for difficult theories. The paper also presents some preliminary experimental results, obtained with a Common Lisp implementation.

1 Introduction

The importance of the propositional logic satisfiability problem (SAT) can be hardly overemphasized: it is the first (and the prototype) NP-complete problem [5], it presents very interesting properties with respect to its complexity behavior [14], analogous, in mathematical terms, to phase transitions in physical systems [11], and it has a wide range of applications, e.g., computer aided design of integrated circuits, logic verification, timing analysis. One of the first Artificial Intelligence problems [22], it has deserved increasing interest in recent years, from science magazines [10] to the most important scientific journals [21].

Most of the research on SAT solving algorithms follows the Davis-Putnam [6] approach, which is based on a *semantic* view that all the possible *assignments* of truth values to propositional symbols should be tested. In this paper, on the other hand, an algorithm is proposed which is based on a *syntactic* view that explores the properties of the normal forms of a given theory to verify its satisfiability. Any propositional theory can be represented either by its *conjunctive normal form (CNF)* or by its *disjunctive normal form (DNF)*. Given an *ordinary formula*

W, i.e., a well-formed expression of the full propositional logic syntax, there are algorithms for converting it into a formula W_c, in CNF, and into a formula W_d, in DNF, such that $W \Leftrightarrow W_c \Leftrightarrow W_d$ (e.g., [25], [26], [27]). To transform a formula from one clause form to the other, only the distributivity of the logical operators $\vee$ and $\wedge$ is needed.

The proposed algorithm calculates, given a propositional theory represented by a CNF W_c, the *minimal* representation of its DNF W_d, where minimal is defined as the smallest set of non contradictory, non subsumed dual clauses. In the literature, the non subsumed set is sometimes called *condensed* [9] and, when inference is also taken into account, *prime implicants* [12, 13]. Each one of the minimal dual clauses represents (minimally) a set of semantic assignments that satisfy the theory. Therefore, if we generate all minimal dual clauses, we have a syntactic description of all possible assignments. The main idea is that the number of minimal dual clauses is always less (or in the worst case equal) than the number of assignments and this is specially true for *difficult* theories, i.e., those near the complexity edge.

In particular, the proposed algorithm can be used to solve the satisfiability problem, if it is terminated when the first minimal dual clause is found, but we are also interested in the complete set of minimal dual clauses for knowledge representation purposes [2]. The goal of this paper is to present the algorithm and to analyze its performance properties. The knowledge representation applications are just sketched and will be the subject of a future paper.

The paper is organized as follows. In Section 2, we introduce some notation for normal forms that explicitly represents the relations between them. In Sections 3 and 4, we describe the proposed algorithm and give some examples. In Section 5, we present some preliminary experimental results obtained with a proof-of-concept Common Lisp implementation of the algorithm. In Section 6, the application of the algorithm as a knowledge representation tool in the field of autonomous agents is sketched. Finally, in Section 7, we conclude and comment upon some ongoing and future work.

2 Theory Representation

Let $P = \{P_1, \ldots, P_n\}$ be a set of propositional symbols and $L = \{\phi_1, \ldots, \phi_{2n}\}$ the set of their associated literals, where $\phi_i = P_j$ or $\phi_i = \neg P_j$. A *clause* C is a *generalized disjunction* [8] of literals: $C = [\phi_1, \ldots, \phi_{k_C}] \equiv \phi_1 \vee \ldots \vee \phi_{k_C}$ and a *dual clause* is a *generalized conjunction* of literals: $D = \langle \phi_1, \ldots, \phi_{k_D} \rangle \equiv \phi_1 \wedge \ldots \wedge \phi_{k_D}$.

A propositional theory $\mathcal{L}(P)$ can be represented by its *conjunctive normal form (CNF)*: $W_c = \langle C_1, \ldots, C_m \rangle$ defined as a generalized conjunction of clauses, or by its *disjunctive normal form (DNF)*: $W_d = [D_1, \ldots, D_w]$ defined as a generalized disjunction of dual clauses.

The fundamental element in the algorithm is called a *quantum* and is defined as a pair (ϕ, F), where ϕ is a literal and $F \subseteq W_c$ is its set of *coordinates* that contains the subset of clauses in W_c to which the literal ϕ belongs. A quantum is noted ϕ^F, to remind us that F can be seen as a function $F : L \to 2^L$.

During the presentation of the algorithm, it is frequently necessary to refer to the set of negated literals, or quanta, of a given set of literals, or quanta[1]. To simplify the notation, we introduce the notion of *mirror*. The mirror of a quantum ϕ^F, noted $\overline{\phi}^{\overline{F}}$, is defined simply as the quantum associated with the negation of its literal: $\overline{\phi} = \neg\phi$. The quantum attribute mirror can also be seen as function: $\overline{\ \cdot\ } : L \to L$ and, from this point of view, $\overline{F}$ is the composition $F \circ \overline{\ \cdot\ } : L \to 2^L$.

This notation is extended to clauses and dual clauses, such that the mirror (dual) clause $\overline{C}$ of (dual) clause C is defined as the set of mirror literals associated with the literals in C[2].

Any dual clause in the DNF W_d is associated with a set of models, i.e., a set of assignments to the propositional symbols in P, that satisfy it. To each dual clause, we can associate a set of quanta: $\Phi = \langle \phi_1^{F_1}, \ldots, \phi_k^{F_k} \rangle$ such that $\cup_{i=1}^{k} F_i = W_c$, i.e., a dual clause is always associated with a set of literals $L_\Phi = \langle \phi_1, \ldots, \phi_k \rangle$ that contains at least one literal that belongs to each clause in W_c, spanning a path through W_c, and no pair of contradictory literals, i.e., if a literal belongs to L_Φ, its negation is excluded. To avoid the introduction of a new name, we call indistinctly the sets Φ and L_Φ dual clauses.

A set Φ represents a minimal dual clause, if the following condition is also satisfied: $\forall i \in \{1, \ldots, k\}, F_i \not\subseteq \cup_{j=1, j\neq i}^{k} F_j$. This condition states that each literal in L_Φ should represent *alone* at least one clause in W_c, otherwise it would be redundant and could be deleted. Given a theory, the set of all minimal Φ's is associated with the minimal representation of its DNF W_d.

Example 1. Consider the theory, whose CNF is given by:

$$
\begin{array}{ll}
0 : [\neg P_4, P_2, \neg P_3] & 5 : [\neg P_3, P_2, \neg P_1] \\
1 : [P_0, \neg P_2, \neg P_3] & 6 : [\neg P_2, \neg P_1, P_0] \\
2 : [P_0, \neg P_3, P_1] & 7 : [P_4, P_0, P_2] \\
3 : [\neg P_0, \neg P_2, P_3] & 8 : [\neg P_1, P_4, P_3] \\
4 : [\neg P_2, \neg P_3, \neg P_1] & 9 : [\neg P_3, \neg P_1, \neg P_4]
\end{array}
$$

Its minimal DNF has seven dual clauses that can be represented by the following sets of quanta, according to the definitions above:

$$
\begin{array}{l}
0 : \langle \neg P_4^{\{0,9\}}, P_0^{\{1,2,6,7\}}, \neg P_1^{\{4,5,6,8,9\}}, \neg P_2^{\{1,3,4,6\}} \rangle \\
1 : \langle \neg P_1^{\{4,5,6,8,9\}}, P_0^{\{1,2,6,7\}}, \neg P_3^{\{0,1,2,4,5,9\}}, \neg P_2^{\{1,3,4,6\}} \rangle \\
2 : \langle P_4^{\{7,8\}}, \neg P_3^{\{0,1,2,4,5,9\}}, \neg P_2^{\{1,3,4,6\}} \rangle \\
3 : \langle P_2^{\{0,5,7\}}, P_3^{\{3,8\}}, P_0^{\{1,2,6,7\}}, \neg P_1^{\{4,5,6,8,9\}} \rangle \\
4 : \langle \neg P_4^{\{0,9\}}, P_3^{\{3,8\}}, P_0^{\{1,2,6,7\}}, \neg P_1^{\{4,5,6,8,9\}} \rangle \\
5 : \langle P_4^{\{7,8\}}, \neg P_0^{\{3\}}, \neg P_1^{\{4,5,6,8,9\}}, \neg P_3^{\{0,1,2,4,5,9\}} \rangle \\
6 : \langle \neg P_0^{\{3\}}, P_2^{\{0,5,7\}}, \neg P_1^{\{4,5,6,8,9\}}, \neg P_3^{\{0,1,2,4,5,9\}} \rangle
\end{array}
$$

[1] Although clauses, dual clauses and sets of quanta are treated as sets, we note them using [ ] and ⟨ ⟩ according to their class.

[2] It should be noted that, differently from the literal case, the mirror of a clause is not the negation of this clause.

where each quantum is represented in the form: ϕ^F, with F its set of coordinates. For legibility reasons, the clauses in the sets F are represented by their numbers.

3 Simply a Search

The basic idea of the proposed algorithm is, given a propositional theory $\mathcal{L}$ represented by a CNF W_c, calculate the set of all Φ that represent the dual clauses in the minimal DNF W_d. If $\mathcal{L}$ is unsatisfiable then this set will be empty.

This problem can be seen as a search in a state space where each state is represented by an incomplete set Φ, associated with an incomplete dual clause in the minimal DNF W_d, and successor states are generated by adding a new quantum to the set, i.e., a new literal in the dual clause. Each incomplete set Φ has an associated *gap*, defined as the set of clauses to which none of its associated literals belong: $G_\Phi = W_c - \cup_{i=1}^{k} F_i$.

Any quantum, associated with literals that belong to the clauses in G_Φ, is, in principle, a relevant quantum to be added to Φ in order to generate a successor.

A space state search should begin in one or more initial states. A possible choice for these initial states is to select all quanta associated with the literals that belong to one specific clause $C_i \in W_c$. The choice of this clause is a first heuristic decision to be taken, e.g., for random theories choosing the clause that contains the most frequent literal in W_c or the one that contains the literal whose negated form is the most frequent literal in W_c, or some combination of both, seems to be sensible options. Once an initial clause is adopted, the problem reduces to a set of independent search problems, one for each literal in this clause, because any path through W_c must pass through exactly one literal in clause C_i.

Finally, the final states are defined as those that satisfies the condition to be a dual clause, i.e., a path through W_c: $\cup_{i=1}^{k} F_i = W_c$. To calculate the minimal set W_d, a complete search should be done but, if the goal is only to determine the satisfiability of $\mathcal{L}$, then when the first final state is found, the search stops.

3.1 Avoiding Redundancy

To keep disjoint the searches associated with each literal in the chosen initial clause C_i, it is necessary to restrict the simultaneous presence of two literals of C_i in some L_Φ to dual clauses Φ that originate from an initial state associated with only one of them. This means that each state Φ must remember its origins, in the form of a list of *forbidden* quanta X_Φ.

Example 2. Consider the theory of example 1, a possible best clause according to the heuristic discussed above is: $4 : [\neg P_3^{\{0,1,2,4,5,9\}}, \neg P_1^{\{4,5,6,8,9\}}, \neg P_2^{\{1,3,4,6\}}]$, where the literals are already sorted according to some quality criterion. States that originate from the best initial state $\langle \neg P_3^{\{0,1,2,4,5,9\}} \rangle$ can be extended to states that contain either $\neg P_1^{\{4,5,6,8,9\}}$ or $\neg P_2^{\{1,3,4,6\}}$ or both, but states that originate from the second best initial state $\langle \neg P_1^{\{4,5,6,8,9\}} \rangle$ cannot be extended to states that contain $\neg P_3^{\{0,1,2,4,5,9\}}$ and states that originate from $\langle \neg P_2^{\{1,3,4,6\}} \rangle$ can only be extended to states that do not contain neither $\neg P_3^{\{0,1,2,4,5,9\}}$ nor $\neg P_1^{\{4,5,6,8,9\}}$.

The same strategy can be used to avoid the generation of duplicated states in general. Usually, several quanta would qualify as possible extensions to some given dual clause. We propose to sort them according to the same quality criterion used to sort the quanta in the initial clause and to use the same method to restrict which quanta can be added to its successors. Given a dual clause Φ that can be extended by a set of different quanta, S_Φ, already sorted according to the adopted quality criterion, and two quanta, $\phi_i^{F_i}$ and $\phi_j^{F_j}$ in S_Φ, such that $\phi_i^{F_i}$ is better than $\phi_j^{F_j}$, we allow Φ to be extended by adding first $\phi_i^{F_i}$ and then $\phi_j^{F_j}$, or just by adding $\phi_j^{F_j}$. This implies adding new quanta to the forbidden list of each successor state, when it is generated.

The definition of the *quality criterion* used to sort the quanta is a second heuristic decision to be taken. Just to avoid duplicated states, any fixed arbitrary total order among the literals in L would be enough, because all possible combinations would be verified. But it is possible to find orders that are also complete, but avoid the generation of some combinations, that would, themselves or their successors, eventually be excluded by one of the pruning conditions (see Section 3.2). The information available to support the construction of such an order is: the gap of the dual clause, G_Φ, the coordinates of the quanta in S_Φ and the coordinates of their associated mirror quanta. Let $F_i^G = F_i \cap G_\Phi$ and $\overline{F}_i^G = \overline{F}_i \cap G_\Phi$ be the intersection of the quanta coordinates with the current gap, and $F_{ij} = F_i^G \cap F_j^G$, the intersection of the restricted coordinates of quanta i and j. A tentative set of rules that such an order would have to satisfy is:

- If $\mid F_i^G - F_{ij} \mid > \mid F_j^G - F_{ij} \mid$ then $\phi_i \succ \phi_j$ else $\phi_j \succ \phi_i$.
- If $\mid F_i^G - F_{ij} \mid = \mid F_j^G - F_{ij} \mid$ then, if $\mid \overline{F}_i^G - \overline{F}_{ij} \mid >$ $\mid \overline{F}_j^G - \overline{F}_{ij} \mid$ then $\phi_i \succ \phi_j$ else $\phi_j \succ \phi_i$.

The idea behind these rules is that a literal that covers alone more clauses in the current gap should be tried first and, in the case there are two that cover the same number of clauses, the one whose mirror literal covers the greater number of clauses should be preferred. This seems to be a sensible choice for random theories, but different or more elaborated conditions are surely possible.

The consequence of this redundancy avoiding mechanism is that each newly generated dual clause can be seen as the initial state of a new independent search, eliminating the necessity of backtracking.

3.2 Pruning the Search

Given a dual clause Φ, any new quantum to be included in it should satisfy the following *basic conditions*:

- The relevance condition: a new quantum ϕ_{ϕ}^{F} should only be included in Φ if $F_{\phi} \cap G_{\Phi} \neq \emptyset$. This condition restricts new quanta only to those that can decrease the gap associated with Φ.
- The non contradiction condition: if $\phi \in L_{\Phi}$ then $\neg\phi \notin L_{\Phi}$.
- The condensed condition: $\forall i \in \{1, \ldots, k\}, F_i^* = F_i - \cup_{j=1, i \neq j}^{k} F_j \neq \emptyset$. This condition restricts new quanta only to non-redundant ones. The clauses in the set F_i^* are called the *exclusive coordinates* associated with literal ϕ_i in dual clause Φ.

Example 3. Consider the theory of example 1 and a possible incomplete dual clause found during the search: $\Phi = \{\neg P_1^{\{4,5,6,8,9\}}, \neg P_2^{\{1,3,4,6\}}, \neg P_3^{\{0,1,2,4,5,9\}}\}$. The quantum $P_4^{\{7,8\}}$ qualify as a candidate to extend the dual clause Φ, because its coordinate set $F = \{7, 8\}$ intersects the gap of the dual clause, $G_{\Phi} = \{7\}$. But the exclusive coordinates associated with the quanta in Φ are: $\Phi = \{\neg P_1^{\{8\}^*}, \neg P_2^{\{3\}^*}, \neg P_3^{\{0,2\}^*}\}$
and the inclusion of $P_4^{\{7,8\}}$ would make $\neg P_1$ redundant. Therefore, because of the condensed condition, the dual clause Φ can not be extended by the quantum $P_4^{\{7,8\}}$.

The fact that including one literal in L_{Φ} imply the impossibility of including its negation leads to restrictions with respect to the clauses in G_{Φ}, i.e., those clauses that are not yet covered by Φ. These are the *gap conditions*:

- If there is a clause $C \in G_{\Phi}$ such that $C \subseteq \overline{L_{\Phi}}$, where $\overline{L_{\Phi}}$ is the set of the mirror quanta of L_{Φ}, then Φ contradicts one of the clauses in G_{Φ} and cannot represent a minimal dual clause.
- If there is a clause $C \in G_{\Phi}$ such that $\mid C - \overline{L_{\Phi}} \mid = 1$, i.e., L_{Φ} contradicts all literals in C except one, then the set L_{Φ} must contain this remaining literal, otherwise clause C would not be represented in Φ. Therefore, if this remaining literal does not qualify as a valid successor of Φ, according to the preceding conditions, then Φ cannot be extended to represent a minimal dual clause.
- Analogous considerations applies to the case in which there is a clause $C \in G_{\Phi}$ such that $\mid C \mid > \mid C - \overline{L_{\Phi}} \mid > 1$. In this case, at least one of the remaining literals in $C - \overline{L_{\Phi}}$ must qualify as a successor of Φ, according to the preceding conditions, otherwise Φ cannot be extended to represent a minimal dual clause.

The gap conditions can be described in a more principled way. Consider the set: $R_{\Phi} = \{C - \overline{L_{\Phi}} \mid C \in G_{\Phi} \text{ and } C \cap \overline{L_{\Phi}} \neq \emptyset\}$.

This *set of restrictions* represents, in the form of a logical theory in CNF, the gap conditions of the incomplete dual clause Φ. If the first gap condition is verified, then the empty clause belongs to R_{Φ}, which is, therefore, contradictory, and Φ can not be extended to represent a minimal dual clause. In the case of the second and/or third gap conditions, R_{Φ} must be coherent with respect to L_{Φ} and internally coherent, i.e., R_{Φ} should not contain a pair of contradictory unitary clauses. Some elements of R_Φ may also be redundant, i.e., if there are clauses $[\phi] \in R_\Phi$ and $C \in R_\Phi$, such that $\phi \in C$, then C is redundant. In order to better detect dual clauses that would become eventually contradictory, because of gap conditions, the minimal CNF of the theory R_Φ should be calculated for each newly generated dual clause Φ.

Example 4. Consider the search for dual clauses of the theory of example 1 at the moment in which the quantum $\neg P_0^{\{3\}}$ is considered as a possible extension of the incomplete dual clause $\Phi = \langle \neg P_1^{\{4,5,6,8,9\}} \rangle$, that has gap $\{0, 1, 2, 3, 7\}$ and list of forbidden quanta $\{\neg P_3^{\{0,1,2,4,5,9\}}\}$. The negation of the literals associated with the new set of quanta – $\langle \neg P_0^{\{3\}}, \neg P_1^{\{4,5,6,8,9\}} \rangle$ – appear in four clauses – 1, 2, 6 and 7 –, clause 6 is not in the gap, i.e., one or more of the literals in L_Φ occur in it. The remaining clauses are: $1 : [P_0, \neg P_2, \neg P_3]$, $2 : [P_0, \neg P_3, P_1]$ and $7 : [P_4, P_0, P_2]$.

In clause 1, $\neg P_0$ imply that the dual clause must include $\neg P_2$ or $\neg P_3$. In clause 2, $\neg P_0$ and $\neg P_1$ imply that the dual clause must include $\neg P_3$. In clause 7, $\neg P_0$ implies that the dual clause must include P_2 or P_4. The simplified clauses are: $1 : [\neg P_2, \neg P_3]$, $2 : [\neg P_3]$ and $7 : [P_4, P_2]$.

The new clause 1 is subsumed by the new clause 2. Therefore, the theory R_Φ is given by $R_\Phi = \langle [\neg P_3^{\{0,1,2,4,5,9\}}], [P_4^{\{7,8\}}, P_2^{\{0,5,7\}}] \rangle$.

The fact that $\neg P_3$ is in the forbidden list indicates that dual clause Φ can not be extended with $\neg P_0$, because of the gap conditions.

A third heuristic decision concerns the order in which these conditions should be tested. The order they are presented already proposes a possible priority, but the best order is clearly theory dependent. In the case of an implementation, the computational cost associated with testing each condition should also be taken into account.

3.3 Failure Propagation

The pruning conditions above are local conditions, in the sense that they depend only on information associated with one dual clause Φ. Quanta not satisfying the first two basic conditions can be easily avoided[3], but the third basic condition and the gap conditions are more complex. Failure that results from the condensed condition is a consequence of the specific composition of the set of quanta in the dual clause plus the new quantum and, according to section 3.1, this specific combination occurs only once.

This is not the case of the failures that result from the gap conditions. In this case, only a limited number of quanta (typically one or two) are responsible for the failure, and whenever this combination occurs it will cause a failure. In particular, all dual clauses that are generated from the same dual clause as the

[3] The non contradiction condition test can also be implemented using the list of forbidden quanta, X_Φ, it is only necessary to add to this list the mirror of each quantum included in the dual clause.

one in which the failure was detected share this fatal combination. It is possible to suitably update the forbidden list of this original dual clause to avoid testing these future failures in its successors.

A further way of propagating failure is the following: given a dual clause Φ and its set of possible extensions S_Φ, a set of new dual clauses is generated by including some of the quanta in S_Φ into Φ, call this set of used quanta S'_Φ. If the quanta in $S_\Phi - S'_\Phi$ were refused as successors of Φ, they will also be refused as successors of all its successors, therefore we can add them to the forbidden list of all successors of dual clauses of Φ.

3.4 The Algorithm

In the beginning of this section, the problem was defined as a search in a state space. This search is solved using a standard A* algorithm. To access the problem the search algorithm needs three interface functions: the initial state, the final state and the state successors. The initial state is defined as the best clause C_i in the theory $\mathcal{L}$, i.e., the clause whose literals coordinates cover the greatest number of clauses in W_c. To start the search, the literals in the best clause are sorted, i.e., the literals that represent more clauses in W_c are used first. A state Φ is final if $G_\Phi = \emptyset$, i.e., the coordinates of the associated literals cover the set W_c. The successor states are generated by the following algorithm:

Successors(Φ)
0. Initialize the successor list: $\Omega \leftarrow \emptyset$.
1. Determine the set of possible extensions:
   $\Theta \leftarrow \{\phi^F \mid \phi \in C \text{ and } C \in G_\Phi\} - X_\Phi$.
2. Sort Θ according to the *quality criterion* $\succ$ (see Section 3.1).
3. Verify the satisfiability of the clauses in the set of restrictions:
   if $\exists C \in R_\Phi, \Theta \cap C = \emptyset$ *then return* $\emptyset$.
4. Main loop: $\forall \phi^F \in \Theta$ do, let $\Phi^+ \leftarrow \Phi \cup \{\phi^F\}$
   if $\forall \phi_i^{F_i} \in \Phi, F_i^* \not\subset F$ — exclusive coordinates are compatible.
   and $\emptyset \notin R_{\Phi+}$ — new restrictions are not contradictory.
   and $\forall C \in R_{\Phi+}, C \not\subset X_\Phi$ — new restrictions are compatible with the forbidden list.
   then $\Omega \leftarrow \Omega \cup \{\Phi^+\}$ — create a new state.
5. *return* Ω.

Fig. 1. Successor Function

One important feature does not appear explicitly in the algorithm of figure 1: How the forbidden list to be associated with a new state − $X_{\Phi+}$ − is generated. The process is presented in Section 3.3. The algorithm is trivially correct and complete, because it is an implementation of the dual transformation with minimization, which is correct and complete by definition.

4 Failure Communication

Further improvement can be obtained if, besides propagating failure to successors, states "communicate" failure to all other active states in the search to which this specific failure is relevant. To accomplish this, a communication channel is necessary. We propose to add a new attribute to the quantum that contains, at each moment, the set of incomplete dual clauses to which the quantum belongs. Using this information the active dual clauses in the search that share the relevant quanta with the current dual clause can be identified.

Given a quantum ϕ^{F_ϕ}, we define the attribute F^d_ϕ as the set of incomplete dual clauses, not still processed by the search algorithm, to which the quantum belongs. When the search is completed, only minimal dual clauses remain and the attribute F^d_ϕ becomes the dual coordinates of the quantum, i.e., the set of all dual clauses to which the quantum belongs.

We are yet studying the heuristic potential of the use of the attribute F^d_ϕ, but, initially, we propose to use this information in two cases: when a failure occurs because of the gap conditions and when a new minimal dual clause is generated.

When a gap condition failure occurs in dual clause Φ, all the incomplete dual clauses associated with its quanta are selected. Those dual clauses that share the restriction associated with the gap condition, receive a communication that, if the quanta that fire this specific gap condition appear as candidates to extend the dual clause, a failure should occur. More formally, let $C \in W_c$ be the clause that fired the gap condition, this means that $C - \overline{L_\Phi}$ is not allowed in Φ. The dual clauses selected to receive the communication are those that share this restriction. The contents of the communication is the set of literals $\overline{C} \cap L_\Phi$ and its effect is that, whenever the last of these literals is considered to extend the dual clause, the search branch fails.

When a new minimal dual clause is generated, it may be the case that it has neighbors, i.e., other minimal dual clauses that differ from the one found by only one quanta. The exclusive coordinates of the quanta in the dual clause can be used to search for these neighbor solutions. The idea is, given one quantum in the minimal dual clause, to find another quantum that does not belong to the dual clause, is compatible with it and whose coordinates cover the exclusive coordinates of the given quantum.

Once the set of new solutions is constructed, all incomplete dual clauses that share quanta with them receive a communication that these specific combination of quanta was already found and that any search should stop before reproducing it.

5 Results

In order to test the relevance of the ideas discussed above, an experimental system was developed in the programming language Common Lisp [28]. This system includes a function that implements the proposed algorithm as defined in Section 3, without the failure communication mechanism (see Section 4). The algorithm was implemented with no optimization concerns, using Lisp structures and plain lists as data structures.

To give an idea of the absolute performance of the algorithm, we used the set of benchmark theories available at:
`http://www.intellektik.informatik.tu-darmstadt.de/SATLIB/`,
with 20, 50, 75 and 100 propositional symbols and 91, 218, 325 and 430 clauses, respectively.

The algorithm was tested both as decision procedure, i.e., halting at the first dual clause found, and as a generator of complete dual clause sets. Some of the obtained results are shown in tables 1, 2, 3 and 4, where *Time first* corresponds to the time until the first minimal dual clause is found, *Time all* the time needed to calculate the complete set of minimal dual clauses, *Calls first* and *Calls all* are the number of internal recursive calls for both the first and all clauses solutions and $\mid W_d \mid$ is the size of the minimal dual clause set. Each unit of time in these tables corresponds to 0.01 seconds. It is interesting to note that roughly theories with smaller dual clause sets are more difficult than theories with bigger ones, in the sense of it takes more time to find the first dual clause.

Problem	Time first	Calls first	Time all	Calls all	$\mid W_d \mid$
uf20-0110	9	327	31	1730	21
uf20-0111	8	301	19	819	5
uf20-0112	9	250	9	383	3
uf20-0113	7	260	8	330	1
uf20-0114	8	249	17	833	6
uf20-0115	8	229	16	693	4
uf20-0116	7	243	14	569	2
uf20-0117	7	218	7	255	1
uf20-0118	9	315	19	1342	14
uf20-0119	9	360	10	433	2

Table 1. Benchmark uf20-91

We also tested the program with some unsatisfiable theories of the benchmark uuf50-218. The results are shown in table 5.

The obtained results, although the limited range of parameters and low statistical significance, seems to indicate that the approach is promising. All the results were obtained with a compiled version of the system in the *CMU Common Lisp* [17] running on a *Celeron 500MHz, 256Mb*. The sources of the system as well as the data files will be made available on the Internet when appropriate.

6 Knowledge Representation

We intend to use propositional logic theories to represent world knowledge of autonomous robots. *Autonomous systems* should present some characteristics,

Problem	Time first	Calls first	Time all	Calls all	$\mid W_d \mid$
uf50-0110	231	1547	1433	17928	27
uf50-0111	218	1759	310	3889	1
uf50-0112	215	1521	338	5576	4
uf50-0113	234	1518	549	6503	9
uf50-0114	477	5307	5372	80159	244
uf50-0115	361	3503	1935	25558	40
uf50-0116	232	2248	747	8891	15
uf50-0117	213	1656	3560	55777	191
uf50-0118	254	2165	377	4778	7
uf50-0119	242	1732	1345	17953	24

Table 2. Benchmark uf50-218

Problem	Time first	Calls first	Time all	Calls all	$\mid W_d \mid$
uf75-010	1153	4141	122304	890578	1025
uf75-011	2377	10870	5940	39255	9
uf75-012	2111	16525	3029	28794	2
uf75-013	1915	8693	18028	113412	132
uf75-014	1154	6164	10131	96129	157
uf75-015	1673	12596	6301	49012	16
uf75-016	1648	7703	2596	20731	6
uf75-017	1032	3555	36273	256330	177
uf75-018	2154	12685	26104	258697	925
uf75-019	1712	10406	1714	10707	2

Table 3. Benchmark uf75-325

such as adaptation and self-organization, that allow them to deal with unexpected situations and to handle complex tasks, without human interference [7, 23]. To acquire and process information is one of the most important activities to support these characteristics. In recent works in the mobile robotics domain, it is possible to perceive a tendency towards hybrid approaches [1] that joins the best features of the planning [16] and sensor-based approaches [3]. The hybrid approach supplies an "intelligent" behavior to the robot, with human like capabilities, such as learning and adaptation. In this sense, Artificial Intelligence techniques have been used in mobile robotics to provide robots with intelligent behavior, mainly in navigation and map building problems [19, 29, 32]. There are also some applications in sensor fusion and control [30, 15].

We intend to use propositional logic to represent the world knowledge necessary to implement intelligent behavior in mobile robots [34, 33] in such a way that they can be considered cognitive autonomous agents. These cognitive agents should be able to learn characteristics of the world, to generalize their knowledge and to draw inferences upon this knowledge in order to accomplish complex tasks.

Problem	Time first	Calls first	Time all	Calls all	$\mid W_d \mid$
uf100-0110	9866	48883	29554	153574	18
uf100-0111	2700	5903	43735	223867	140
uf100-0112	3388	10667	28655	125415	46
uf100-0113	13149	100660	15131	118886	3
uf100-0114	3948	14042	8782	56146	2
uf100-0115	3401	6458	46507	279033	424
uf100-0116	4132	8909	35260	165054	66
uf100-0117	9694	48348	29048	181519	47
uf100-0118	2788	5760	79995	403377	433
uf100-0119	3923	12283	244049	1484480	1835

Table 4. Benchmark uf100-430

Problem	Time	Calls
uuf50-0110	139	2342
uuf50-0111	190	4270
uuf50-0112	109	2176
uuf50-0113	86	1393
uuf50-0114	105	2365
uuf50-0115	112	1947
uuf50-0116	129	2872
uuf50-0117	69	1359
uuf50-0118	141	3006
uuf50-0119	203	3431

Table 5. Benchmark uuf50-218

The adopted model is derived from the generic model for a cognitive agent presented in [2]. This model is based on three hypothesis: (i) Cognition is an emergent property of a cyclic dynamic self-organizing process [20, 31] based on the interaction of a large number of functionally independent units of a few types [4]. (ii) Any model of the cognitive activity should be epistemologically [18] compatible with the Theory of Evolution. That applies not only to the "hardware" components of this activity but also to its "psychological" aspects [35]. (iii) Learning and cognitive activities are closely related and, therefore, the cognitive modeling process should strongly depend on the cognitive agent's particular history [24].

The agents based on this model have three levels: reactive, instinctive and cognitive. The cognitive level could be defined as a set of non-contradictory propositional theories that represent the agent's knowledge about the world. The *states of the world*, relevant to a given theory, are defined as the possible truth assignments to a set of *primitive* propositional symbols that occur in this theory. We suppose that the world drifts along the possible states (i.e., assignments), but changing only one primitive propositional symbol assignment at each moment. The primitive propositional symbols can be *controllable* or *uncontrollable*.

Roughly, uncontrollable symbols correspond to *perceptions*, controllable ones to *actions*.

The agent is embodied in a mobile robot that wanders around the world and *perceives* the *primitive* propositional symbols, through the reactive and instinctive levels. It is important to note that the agent should recognize the situations and abstract similar information, grouping them into concepts, represented by propositional theories. Each concept is associated with a set of dual clauses that are satisfied in the corresponding situations. Using the proposed algorithm we can obtain the CNF associated with this set of dual clauses. This CNF is a rule-based representation of the concept and can be used to control the agent behavior.

Each theory can itself be interpreted as an *abstract* propositional symbol, that may occur in other theories. The idea is that each theory represents some *concept*, just to have a single word to mean either an *object* or a *situation* in the world. From the agent point of view, these concepts are characterized by some patterns of truth assignments, represented by its propositional theories. Therefore, the agent could act in the world through the primitive propositional symbols that it can *control* and update its internal states when the *uncontrollable* primitive symbols propositional change.

The fact that each theory is represented by both, CNF and DNF, provides the agent with a "holographic" representation of the world, where possible future situations and relevant behavior rules are available simultaneously. The goal is to demonstrate that this syntactical representation, it is suitable to implement the necessary cognitive capabilities of a simple autonomous agent.

7 Conclusion

The paper has presented an algorithm to calculate the minimal dual form of a theory and some preliminary results on its application to the random 3SAT problem. The following characteristics of the proposed algorithm make it different from most of the algorithms in the Davis-Putnam [6] thread: (i) The use of an explicit representation of the relations between CNF and DNF. (ii) The use of a syntactic property of the input theory, its set of minimal dual clauses, to guide the search, instead of the possible semantic assignments. (iii) The use of a redundancy avoiding mechanism that eliminates the need of backtracking. (iv) The propagation of failure information from one search point to others search points, in order to avoid useless search effort. Although some results were presented, the main goal of the paper was to present what we believe to be a different approach to the satisfiability problem and to motivate its application in the autonomous agent knowledge representation task.

On going work includes the failure communication implementation and the computational complexity analysis of the algorithm and a new implementation in the C++ programming language, to allow experiments with larger theories and comparisons with other systems. In the future, we also intend to develop a concurrent implementation of the algorithm to explore the fact that each new generated state of the search can be considered a new initial state of an independent search.

Future work also includes the extension of the algorithm to the first-order logic case and more investigation on the properties of the relation between the two minimal dual forms of a theory. Although the algorithm was defined to calculate the set of minimal dual clauses of a theory, it is absolutely symmetric and it is possible to obtain the minimal CNF, and its associated quanta, just executing the search in the other direction, beginning with the already calculated DNF. We believe that the explicit representation of these "holographic" relations, through the coordinates and exclusive coordinates of the quanta in both normal forms, has a high heuristic potential, specially in the first-order case.

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