Increasing VCG Revenue by Decreasing the Quality of Items

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2^{nd} price auction

- There is one item for sale.
- All bidders submit their bids.
- The bidder with the highest bid takes the item.
- The winner pays the second highest bid to the auctioneer.

Example

Bidder	Value
b_1	6
b_2	100
b_3	3

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Bidder	Value
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b_3	3

Bidder b_2 takes the item and pays 6 to the auctioneer.

What if there more than one items?

VCG mechanism

Setting

- Many (possibly different) items for sale.
- Bidders bid a value for every possible subset of items.

VCG mechanism

- Chooses an allocation $\mathcal{A}$ for the items that maximize the social welfare $\mathcal{W}$ for the bidders.
- $u_i(\mathcal{A})$ is the utility of bidder i under allocation $\mathcal{A}$.
- $\mathcal{W}_{-i}$ is the optimal welfare when bidder i is excluded from the allocation.
- Bidder i pays: $p_i = \mathcal{W}_{-i} - (\mathcal{W} - u_i(\mathcal{A}))$

Example of VCG

2 items, 3 bidders

	Bidder 1	Bidder 2	Bidder 3
	100	0	50
	0	100	50
	100	100	120

Example of VCG

	Bidder 1	Bidder 2	Bidder 3
	100	0	50
	0	100	50
	100	100	120

- $\mathcal{A}$: {Bidder 1 gets the rod, Bidder 2 gets the reel}
- $u_1(\mathcal{A}) = 100, u_2(\mathcal{A}) = 100, u_3(\mathcal{A}) = 0$
- $\mathcal{W} = 200, \mathcal{W}_{-1} = 150, \mathcal{W}_{-2} = 150, \mathcal{W}_{-3} = 200$
- $p_1 = \mathcal{W}_{-1} - (\mathcal{W} - u_1(\mathcal{A})) = 150 - (200 - 100) = 50$
- $p_2 = 150 - (200 - 100) = 50$
- $p_3 = 200 - (200 - 0) = 0$

Problem. . .

VCG is not revenue monotone with respect to number of items.

2 identical items, 3 unit demand bidders*

Bidder	Value
b_1	100
b_2	100
b_3	10

* Each bidder has value v if he get at least one item and zero if he does not get any items at all.

Problem. . .

VCG is not revenue monotone with respect to number of items.

2 identical items, 3 unit demand bidders

Bidder	Value
b_1	100
b_2	100
b_3	10

- Each one of Bidder 1 and Bidder 2 gets one item.
- VCG payment is 10 for each one of them.
- Bidder 3 does not win any item.
- His payment is zero.

Total revenue of VCG: **20**

. . . and a solution.

VCG is not revenue monotone with respect to number of items.

Burn one of the items!

Bidder	Value
b_1	100
b_2	100
b_3	10

Total revenue of VCG: **100**

- Bidder 1 wins the item.
- VCG payment is 100 for him.
- Bidders 2 and 3 do not win any item.
- Their payment is zero.

Goal 1: Maximize the revenue collected by VCG by burning items.

Our Model

Items

- Items are characterized by k attributes $(q_1, \ldots, q_k)$.
- q_i represents the quality for $i - th$ attribute.
- Higher attribute value corresponds to higher quality.
- Quality for attribute i is given with a non-negative integer.
- Given two items $(q_1, q_2, \ldots, q_k)$ and $(q'_1, q'_2, \ldots, q'_k)$, if $q_i \geq q'_i$ for all i and $q_i > q'_i$ for some i, then we say the first item is "better".

Our Model - Example

PC attributes

- Attributes: CPU, RAM, HD
  - CPU: $\{i-3, i-5, i-7\}$
  - RAM: $\{2GB, 4GB, 8GB\}$
  - HD: $\{500GB, 1TB, 2TB\}$

- CPU: $i-3 \prec i-5 \prec i-7$
- RAM: $2GB \prec 4GB \prec 8GB$
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$$(i-3, 2GB, 500GB) \prec (i-5, 2GB, 2TB)$$

Our Model

Bidders' valuation functions

- Bidders valuation is given by a multistep function increasing with items' quality.
- Simple unit demand bidder: $(q_1, q_2, \ldots, q_k)$ and a value $c > 0$ such that

$$v_i(q'_1, q'_2, \ldots, q'_k) = \begin{cases} c & \forall j, q'_j \geq q_j \\ 0 & \text{Otherwise} \end{cases}$$

- He has value c if he receives at least one item of quality $(q_1, q_2, \ldots, q_k)$ and zero otherwise.

New option: Mark down attributes

Example

- Items have one attribute: $\{L, H\} \rightarrow L \prec H$.
- Bidders are simple and unit demand.
- We have two items of quality H.

Bidder	Limit	Value
b_1	H	100
b_2	H	100
b_3	L	40
b_4	L	40

New option: Mark down attributes

Original VCG

Bidder	Limit	Value
b_1	H	100
b_2	H	100
b_3	L	40
b_4	L	40

Original VCG

- Bidders 1 and 2 win one item each.
- VCG payment is 40 for each.
- Bidders 3 and 4 do not win any item.
- Their payment is zero.

Revenue: 80

New option: Mark down attributes

Item burning

Bidder	Limit	Value
b_1	H	100
b_2	H	100
b_3	L	40
b_4	L	40

Revenue: 100

Optimal Burning

Burn one of the items.

- Bidder 1 wins the item.
- VCG payment is 100 for him.
- Bidders 2, 3 and 4 do not win any item.
- Their payment is zero.

New option: Mark down attributes

Marking down

Bidder	Limit	Value
b_1	H	100
b_2	H	100
b_3	L	40
b_4	L	40

Revenue: 140

Optimal Marking Down

Present one item as it has quality *L*.

- Bidder 1 wins the item with quality *H*.
- VCG payment is 100 for him.
- Bidder 3 wins the item with quality *L*.
- VCG payment is 40 for him.
- Bidders 2and 4 do not win any item.
- Their payment is zero.

Research questions

1. **Optimal item burning**: Find the set of items to burn in order to maximize VCG revenue.
2. **Optimal marking down**: Find the optimal way to mark down items' attributes in order to maximize VCG revenue.
   - We assume that, all bidders are unit demand.

Results

Valuations	No of Attributes	Item burning	Marking Down
General	1	NP-hard	NP-hard
Simple	4	NP-hard	NP-hard
Simple	n-binary	NP-hard	NP-hard
Simple	1	P	P
Simple	2, 3	**?**	**?**