Revenue Maximization via Hiding Item Attributes

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Auctions

Single item auction

We have an item for sale and there are several buyers
Seller → auctioneer
Buyers → bidders

1. Maximize the social welafare
2. Maximize auctioneer's revenue

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Buyers $\rightarrow$ bidders

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2^{nd} price auction

- All bidders submit their bids.
- The bidder with the highest bid takes the item.
- The winner pays the second highest bid to the auctioneer.

Example

Bidder	Value
b_1	6
b_2	100
b_3	3

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Bidder b_2 takes the item and pays 6 to the auctioneer.

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Probabilistic single item auctions

- The auctioneer will sell one item
- The item can take m different instantiations
- n bidders
- An instantiation is chosen randomly by nature, according to a common knowledge distribution $p \in \Delta(m)$
- Auctioneer sells the item via a 2^{nd} price auction

Information assymetry for the chosen instantiation

Bidders $\rightarrow$ probability distribution over instantiations
Auctioneer $\rightarrow$ actual realization

How the auctioneer can use this extra information in order to extract more revenue?

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How the auctioneer can use this extra information in order to extract more revenue?

An example

Suppose that the item has 4 possible instantiations, i_1, i_2, i_3, i_4 and each one is chosen with probability 1/4.

Bidders	i_1	i_2	i_3	i_4
b_1	1	0	0	0
b_2	0	1	0	0
b_3	0	0	1	0
b_4	0	0	0	1

The auctioneer always reveals the chosen instantiation.

Expected revenue

$r = \frac{1}{4} * r(i_1) + \frac{1}{4} * r(i_2) + \frac{1}{4} * r(i_3) + \frac{1}{4} * r(i_4) = 0$

- For i_1 bidder b_1 bids 1 and all other bidders bid 0, so $r(i_1) = 0$.
- For i_2 bidder b_2 bids 1 and all other bidders bid 0, so $r(i_2) = 0$.
- For i_3 bidder b_3 bids 1 and all other bidders bid 0, so $r(i_3) = 0$.
- For i_4 bidder b_4 bids 1 and all other bidders bid 0, so $r(i_4) = 0$.

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An example

No matter which item is chosen, the auctioneer does not reveal any information.

Item j is chosen.

- With probability $\frac{1}{4}$ is i_1, with $\frac{1}{4}$ is i_2, $\frac{1}{4}$ is i_3, with $\frac{1}{4}$ is i_4.
- Bidder b_1 has expected value for the item $\frac{1}{4}v(i_1) + \frac{1}{4}v(i_2) + \frac{1}{4}v(i_3) + \frac{1}{4}v(i_4) = \frac{1}{4}(1+0+0+0) = 1/4$, so he bids $\frac{1}{4}$.
- Bidder b_2 has expected value for the item $\frac{1}{4}v(i_1) + \frac{1}{4}v(i_2) + \frac{1}{4}v(i_3) + \frac{1}{4}v(i_4) = \frac{1}{4}(0+1+0+0) = 1/4$, so he bids $\frac{1}{4}$.
- $\dots$

Expected revenue

$$r = r(i_1) * \frac{1}{4} + r(i_2) * \frac{1}{4} + r(i_3) * \frac{1}{4} + r(i_4) * \frac{1}{4} = \frac{1}{4}(\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}) = 1/4$$

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Contents

Bundling

Bundling scheme

- The auctioneer a-priori declares a partition for the set of instantiations into bundles, and once an instantiation is randomly chosen, she reveals the cluster that contains it to the bidders.
- Then the bidders submit their bids.
- The winner and the payment for each cluster is determined according to the 2^{nd} price auction.

Optimal signaling problem

Find the partition that maximizes auctioneer's revenue.

An example - Optimal Bundling

	1	2	3	4
Bidder 1	1	0	0	0
Bidder 2	0	1	0	0
Bidder 3	0	0	1	0
Bidder 4	0	0	0	1

- With probability $\frac{1}{2}$ the signal is s_1
  - b_1 bids 1/2, b_2 bids 1/2, b_3 bids 0 and b_4 bids 0.
  - $r(s_1) = 1/2$
- With probability $\frac{1}{2}$ the signal is s_2
  - b_1 bids 0, b_2 bids 0, b_3 bids 1/2 and b_4 bids 1/2.
  - $r(s_2) = 1/2$

Expected revenue

$r = r(s_1) * \frac{1}{2} + r(s_2) * \frac{1}{2} = 1/2$

An example - Optimal Bundling

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Bidder 1	1	0	0	0
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- With probability $\frac{1}{2}$ the signal is s_2
  - b_1 bids 0, b_2 bids 0, b_3 bids 1/2 and b_4 bids 1/2.
  - $r(s_2) = 1/2$

Expected revenue

$$r = r(s_1) * \frac{1}{2} + r(s_2) * \frac{1}{2} = 1/2$$

Results so far

- [Ghosh'07] It is NP - hard to determine how to optimally partition the instantiations into arbitrary bundles, even if we know every bidder's valuation for every instantiation.
- [Emek et al.'12, Miltersen, Sheffet '12] Randomized bundling can be solved by linear programming
- [Emek et al.'12] 0-1 valuation $\rightarrow$ solved using matching

Contents

Natural bundles for instantiations with attributes

Setting

- Item is identified by k attributes.
- Each attribute $a_i, i \in [k]$ can take C_i different values.
- Total number of possible instantiations: $\prod_i C_i$.

Natural bundles

A signal is called natural if it is created only by hiding some attributes (or none).

- Hidden attribute $\rightarrow ?$

Bundling scheme

A partition for the instantiations

- Using only natural bundles.
- Every instantiations belongs to exactly one bundle.
- It is possible to sell an instantiation as a singleton.

Natural bundles for instantiations with attributes 2

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- Hidden attribute $\rightarrow ?$

Example: two binary attributes

- Items: 00, 01, 10, 11

Bundle	Items
00	00
01	01
10	10
11	11
0?	00, 01
1?	10, 11
?0	00, 10
?1	01, 11
??	00, 01, 10, 11

Natural bundles for instantiations with attributes

bundling scheme

A partition for the instantiations

- Using only natural bundles.
- Every instantiation belongs to exactly one bundle.
- It is possible to sell an instantiation as a singleton.

Example: two binary attributes

- Items: 00, 01, 10, 11

Signal	Items
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10	10
11	11
0?	00, 01
1?	10, 11
?0	00, 10
?1	01, 11
??	00, 01, 10, 11

Number of Bundles

- Let m be the number of possible instantiations.
- The number of arbitrary bundles is 2^m.
- The number of natural bundles is less than m^2
  - k attributes, attribute i takes C_i possible values
  - $m = C_1 C_2 \dots C_k$
  - Number of natural bundles $(C_1 + 1)(C_2 + 1) \dots (C_k + 1)$

Our results

- It is NP-hard to compute the optimal bundling scheme, even if we allow only natural bundles.[Guo, D.]
- It is NP-hard even when bidders are single minded and each attribute can take 8 different values.
- We proposed two heuristic algorithms
  - Matching based
  - Tree structured

Matching algorithm

Construct the graph $G = (V, E)$

- For every item construct a vertex
- There is the edge $v_i v_j$ iff items v_i, v_j differ in only one attribute and moreover the auctioneer gains more revenue if she hides that attribute.
- $Rev(\{v_i v_j\}) > Rev(v_i) + Rev(v_j)$

Find the maximum weighted matching for graph G

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Find the maximum weighted matching for graph G

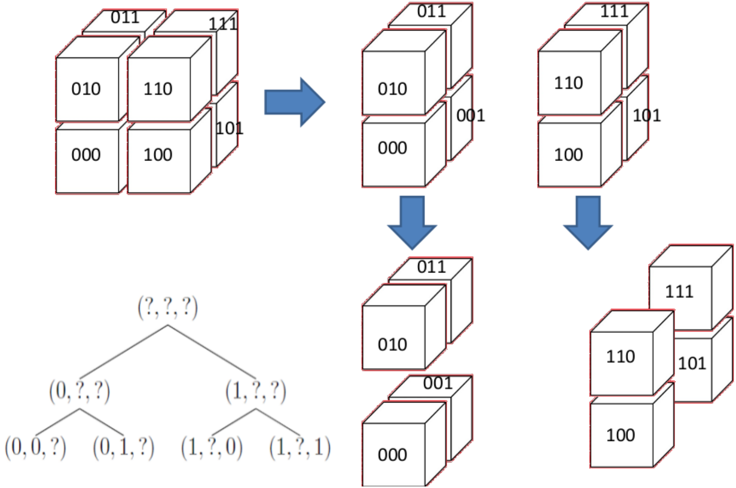
Facts

- Apply only for binary attributes.
- Competitive ratio equals to infinity
- Finds the best signaling scheme, if only one attribute is allowed to be hidden.

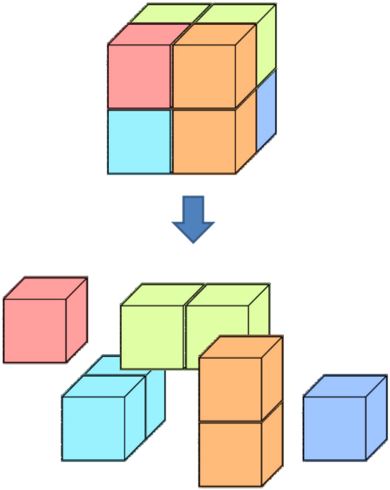
Tree-Structured Hiding Schemes

- Start from signal $(?, \ldots, ?)$
- Do recursive splitting by revealing the attribute that gives the highest revenue
- Stop when you cannot gain more revenue with splitting

Tree-Structured Hiding Schemes - example



Tree-Structured Hiding Schemes - bad case



Future plans - open questions

- What is the complexity for computing the optimal signaling scheme when each bidder is interested in exactly one natural bundle?
- Bayesian case?
- What is the competitive ratio for the Tree-structured algorithm?
- Search for heuristic algorithms for signaling when the attributes' values have some strucure (tree structure, intervals).
- Instead of hiding attributes, can we find a pricing scheme for each attribute?