Revenue Maximization via Hiding Item Attributes

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Contents

1. Single item auctions
2. Probabilistic single item auctions
3. Signaling
4. Natural bundles
Auctions

Single item auction

We have an item for sale and there are several buyers
Seller $\rightarrow$ auctioneer
Buyers $\rightarrow$ bidders

1. Maximize the social welfare
2. Maximize auctioneer's revenue
Single item auction

We have an item for sale and there are several buyers
Seller $\rightarrow$ auctioneer
Buyers $\rightarrow$ bidders

1. Maximize the social welfare
2. Maximize auctioneer's revenue
**2^{nd} price auction**

- All bidders submit their bids.
- The bidder with the highest bid takes the item.
- The winner pays the second highest bid to the auctioneer.

**Example**

<table>
<thead>
<tr>
<th>Bidder</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_1$</td>
<td>6</td>
</tr>
<tr>
<td>$b_2$</td>
<td>100</td>
</tr>
<tr>
<td>$b_3$</td>
<td>3</td>
</tr>
</tbody>
</table>
2nd price auction

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<td>$b_3$</td>
<td>3</td>
</tr>
</tbody>
</table>

Bidder $b_2$ takes the item and pays 6 to the auctioneer.
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Revenue Maximization via Hiding Item Attributes
The auctioneer will sell one item
The item can take $m$ different instantiations
$n$ bidders
An instantiation is chosen randomly by nature, according to a common knowledge distribution $p \in \Delta(m)$
Auctioneer sells the item via a $2^{nd}$ price auction

Information asymmetry for the chosen instantiation
Bidders $\rightarrow$ probability distribution over instantiations
Auctioneer $\rightarrow$ actual realization

How the auctioneer can use this extra information in order to extract more revenue?
Single item auctions
Probabilistic single item auctions
Signaling
Natural bundles

Probabilistic single item auctions

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- The item can take \( m \) different instantiations
- \( n \) bidders
- An instantiation is chosen randomly by nature, according to a common knowledge distribution \( p \in \Delta(m) \)
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Information asymmetry for the chosen instantiation

Bidders → probability distribution over instantiations
Auctioneer → actual realization

How the auctioneer can use this extra information in order to extract more revenue?

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Probabilistic single item auctions

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- $n$ bidders
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Information assymetry for the chosen instantiation

Bidders $\rightarrow$ probability distribution over instantiations
Auctioneer $\rightarrow$ actual realization

How the auctioneer can use this extra information in order to extract more revenue?
An example

Suppose that the item has 4 possible instantiations, \(i_1, i_2, i_3, i_4\) and each one is chosen with probability 1/4.

<table>
<thead>
<tr>
<th>Bidders</th>
<th>(i_1)</th>
<th>(i_2)</th>
<th>(i_3)</th>
<th>(i_4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b_1)</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(b_2)</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(b_3)</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>(b_4)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

For \(i_1\) bidder \(b_1\) bids 1 and all other bidders bid 0, so \(r(i_1) = 0\).

For \(i_2\) bidder \(b_2\) bids 1 and all other bidders bid 0, so \(r(i_2) = 0\).

For \(i_3\) bidder \(b_3\) bids 1 and all other bidders bid 0, so \(r(i_3) = 0\).

For \(i_4\) bidder \(b_4\) bids 1 and all other bidders bid 0, so \(r(i_4) = 0\).

Expected revenue

\[
r = \frac{1}{4} \times r(i_1) + \frac{1}{4} \times r(i_2) + \frac{1}{4} \times r(i_3) + \frac{1}{4} \times r(i_4) = 0
\]
An example

Suppose that the item has 4 possible instantiations, $i_1, i_2, i_3, i_4$ and each one is chosen with probability 1/4.

<table>
<thead>
<tr>
<th>Bidders</th>
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<th>$i_2$</th>
<th>$i_3$</th>
<th>$i_4$</th>
</tr>
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<tbody>
<tr>
<td>$b_1$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$b_2$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$b_3$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$b_4$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

The auctioneer always reveals the chosen instantiation.

For $i_1$ bidder $b_1$ bids 1 and all other bidders bid 0, so $r(i_1) = 0$.

For $i_2$ bidder $b_2$ bids 1 and all other bidders bid 0, so $r(i_2) = 0$.

For $i_3$ bidder $b_3$ bids 1 and all other bidders bid 0, so $r(i_3) = 0$.

For $i_4$ bidder $b_4$ bids 1 and all other bidders bid 0, so $r(i_4) = 0$.

Expected revenue

$$r = \frac{1}{4} \cdot r(i_1) + \frac{1}{4} \cdot r(i_2) + \frac{1}{4} \cdot r(i_3) + \frac{1}{4} \cdot r(i_4) = 0$$
An example

No matter which item is chosen, the auctioneer does not reveal any information.

- **Item $j$ is chosen.**
  - With probability $\frac{1}{4}$ is $i_1$, with $\frac{1}{4}$ is $i_2$, $\frac{1}{4}$ is $i_3$, with $\frac{1}{4}$ is $i_4$.
  - Bidder $b_1$ has expected value for the item
    \[ \frac{1}{4} v(i_1) + \frac{1}{4} v(i_2) + \frac{1}{4} v(i_3) + \frac{1}{4} v(i_4) = \frac{1}{4} (1 + 0 + 0 + 0) = 1/4, \]
    so he bids $\frac{1}{4}$.
  - Bidder $b_2$ has expected value for the item
    \[ \frac{1}{4} v(i_1) + \frac{1}{4} v(i_2) + \frac{1}{4} v(i_3) + \frac{1}{4} v(i_4) = \frac{1}{4} (0 + 1 + 0 + 0) = 1/4, \]
    so he bids $\frac{1}{4}$.
  - ...

### Expected revenue

\[ r = r(i_1) \times \frac{1}{4} + r(i_2) \times \frac{1}{4} + r(i_3) \times \frac{1}{4} + r(i_4) \times \frac{1}{4} = \frac{1}{4} \left( \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} \right) = 1/4 \]
An example

No matter which item is chosen, the auctioneer does not reveal any information.

**Item \( j \) is chosen.**

- With probability \( \frac{1}{4} \) is \( i_1 \), with \( \frac{1}{4} \) is \( i_2 \), \( \frac{1}{4} \) is \( i_3 \), with \( \frac{1}{4} \) is \( i_4 \).
- Bidder \( b_1 \) has expected value for the item
  \[
  \frac{1}{4} v(i_1) + \frac{1}{4} v(i_2) + \frac{1}{4} v(i_3) + \frac{1}{4} v(i_4) = \frac{1}{4} (1 + 0 + 0 + 0) = 1/4,
  \]
  so he bids \( \frac{1}{4} \).
- Bidder \( b_2 \) has expected value for the item
  \[
  \frac{1}{4} v(i_1) + \frac{1}{4} v(i_2) + \frac{1}{4} v(i_3) + \frac{1}{4} v(i_4) = \frac{1}{4} (0 + 1 + 0 + 0) = 1/4,
  \]
  so he bids \( \frac{1}{4} \).

\[
\ldots
\]

**Expected revenue**

\[
r = r(i_1) \times \frac{1}{4} + r(i_2) \times \frac{1}{4} + r(i_3) \times \frac{1}{4} + r(i_4) \times \frac{1}{4} = \frac{1}{4} \left( \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} \right) = 1/4
\]
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Bundling

Bundling scheme

- The auctioneer a-priori declares a partition for the set of instantiations into bundles, and once an instantiation is randomly chosen, she reveals the cluster that contains it to the bidders.
- Then the bidders submit their bids.
- The winner and the payment for each cluster is determined according to the $2^{nd}$ price auction.

Optimal signaling problem

Find the partition that maximizes auctioneer's revenue.
An example - Optimal Bundling

With probability $\frac{1}{2}$ the signal is $s_1$
- $b_1$ bids $1/2$, $b_2$ bids $1/2$, $b_3$ bids $0$ and $b_4$ bids $0$.
- $r(s_1) = 1/2$

With probability $\frac{1}{2}$ the signal is $s_2$
- $b_1$ bids $0$, $b_2$ bids $0$, $b_3$ bids $1/2$ and $b_4$ bids $1/2$.
- $r(s_2) = 1/2$

Expected revenue

$$r = r(s_1) \times \frac{1}{2} + r(s_2) \times \frac{1}{2} = \frac{1}{2}$$
An example - Optimal Bundling

<table>
<thead>
<tr>
<th>Bidder</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bidder 1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Bidder 2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Bidder 3</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Bidder 4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

- With probability $\frac{1}{2}$ the signal is $s_1$
  - $b_1$ bids 1/2, $b_2$ bids 1/2, $b_3$ bids 0 and $b_4$ bids 0.
  - $r(s_1) = \frac{1}{2}$
- With probability $\frac{1}{2}$ the signal is $s_2$
  - $b_1$ bids 0, $b_2$ bids 0, $b_3$ bids 1/2 and $b_4$ bids 1/2.
  - $r(s_2) = \frac{1}{2}$

**Expected revenue**

$$r = r(s_1) \times \frac{1}{2} + r(s_2) \times \frac{1}{2} = \frac{1}{2}$$
Results so far

- [Ghosh'07] It is NP-hard to determine how to optimally partition the instantiations into arbitrary bundles, even if we know every bidder’s valuation for every instantiation.
- [Emek et al.'12, Miltersen, Sheffet '12] Randomized bundling can be solved by linear programming
- [Emek et al.'12] 0-1 valuation $\rightarrow$ solved using matching
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Natural bundles for instantiations with attributes

**Setting**
- Item is identified by $k$ attributes.
- Each attribute $a_i, i \in [k]$ can take $C_i$ different values.
- Total number of possible instantiations: $\prod_i C_i$.

**Natural bundles**
A signal is called natural if it is created only by hiding some attributes (or none).
- Hidden attribute $\rightarrow$?

**Bundling scheme**
A partition for the instantiations
- Using only natural bundles.
- Every instantiations belongs to exactly one bundle.
- It is possible to sell an instantiation as a singleton.
Natural bundles for instantiations with attributes 2

Setting

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- Total number of possible instantiations: \( \prod_i C_i \).

Natural signals

A bundle is called natural if it is created only by hiding some attributes.

- Hidden attribute \( \rightarrow \)?

Example: two binary attributes

- Items: 00, 01, 10, 11

<table>
<thead>
<tr>
<th>Bundle</th>
<th>Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>00</td>
<td>00</td>
</tr>
<tr>
<td>01</td>
<td>01</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>0?</td>
<td>00, 01</td>
</tr>
<tr>
<td>1?</td>
<td>10, 11</td>
</tr>
<tr>
<td>?0</td>
<td>00, 10</td>
</tr>
<tr>
<td>?1</td>
<td>01, 11</td>
</tr>
<tr>
<td>??</td>
<td>00, 01, 10, 11</td>
</tr>
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Natural bundles for instantiations with attributes

**bundling scheme**

A partition for the instantiations
- Using only natural bundles.
- Every instantiation belongs to exactly one bundle.
- It is possible to sell an instantiation as a singleton.

**Example: two binary attributes**

- **Items**: 00, 01, 10, 11

<table>
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<tr>
<td>00</td>
<td>00</td>
</tr>
<tr>
<td>01</td>
<td>01</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>0?</td>
<td>00, 01</td>
</tr>
<tr>
<td>1?</td>
<td>10, 11</td>
</tr>
<tr>
<td>?0</td>
<td>00, 10</td>
</tr>
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</tr>
<tr>
<td>??</td>
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Number of Bundles

- Let $m$ be the number of possible instantiations.
- The number of arbitrary bundles is $2^m$.
- The number of natural bundles is less than $m^2$
  - $k$ attributes, attribute $i$ takes $C_i$ possible values
  - $m = C_1C_2 \ldots C_k$
  - Number of natural bundles $(C_1 + 1)(C_2 + 1) \ldots (C_k + 1)$
Our results

- It is NP-hard to compute the optimal bundling scheme, even if we allow only natural bundles. [Guo, D.]
- It is NP-hard even when bidders are single minded and each attribute can take 8 different values.
- We proposed two heuristic algorithms
  - Matching based
  - Tree structured
Matching algorithm

Construct the graph $G = (V, E)$

- For every item construct a vertex
- There is the edge $v_i v_j$ iff items $v_i, v_j$ differ in only one attribute and moreover the auctioneer gains more revenue if she hides that attribute.
- $\text{Rev}(\{v_i, v_j\}) > \text{Rev}(v_i) + \text{Rev}(v_j)$

Find the maximum weighted matching for graph $G$
Matching algorithm

Construct the graph $G = (V, E)$
- For every item construct a vertex
- There is the edge $v_iv_j$ iff items $v_i, v_j$ differ in only one attribute and moreover the auctioneer gains more revenue if she hides that attribute.

$\text{Rev(}\{v_iv_j\} > \text{Rev}(v_i) + \text{Rev}(v_j)$

Find the maximum weighted matching for graph $G$

Facts
- Apply only for binary attributes.
- Competitive ratio equals to infinity
- Finds the best signaling scheme, if only one attribute is allowed to be hidden.
Tree-Structured Hiding Schemes

- Start from signal (?,?,...,?)
- Do recursive splitting by revealing the attribute that gives the highest revenue
- Stop when you cannot gain more revenue with splitting
Tree-Structured Hiding Schemes - example
Tree-Structured Hiding Schemes - bad case

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Future plans - open questions

- What is the complexity for computing the optimal signaling scheme when each bidder is interested in exactly one natural bundle?
- Bayesian case?
- What is the competitive ratio for the Tree-structured algorithm?
- Search for heuristic algorithms for signaling when the attributes' values have some structure (tree structure, intervals).
- Instead of hiding attributes, can we find a pricing scheme for each attribute?
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