A 40 min Intro to Quantum Mechanics

There is no quantum world.
There is only an abstract physical description.

Niels Bohr.
What is Physics?

Give me matter and motion, and I will construct the Universe.
René Descartes

By 1900, quite a lot was known about two kinds of matter and its motion:

Particles: Mechanics

Waves: Electrodynamics
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Mechanics: Particles

- Mass \( m \)
- Coordinate \( x(t) \in \mathbb{R}^3 \)
- Momentum \( p(t) = mx'(t) \)
- Energy \( E(x, p) = \frac{p^2}{2m} + V(x) \)

Law of motion: \( p'(t) = F(t) \)
Electromagnetic Fields

Electric field: $F = qE$

Sources of field: charges or ...

Magnetic Field: $F = qv \times B$

Sources of field: currents or ...
Waves

... or fields can produce fields !!!

\[
\begin{align*}
\partial_t E & \sim \nabla \times B \quad \text{(Ampere’s Law)} \\
\partial_t B & \sim \nabla \times E \quad \text{(Faraday’s Law)}
\end{align*}
\]

Wave equation

\[
\partial_t E = c^2 \nabla^2 E
\]

\[
\nabla = (\partial_{x_1}, \partial_{x_2}, \partial_{x_3})^+ \quad \nabla^2 = \partial_{x_1}^2 + \partial_{x_2}^2 + \partial_{x_3}^2
\]
Planar waves

\[ \partial_{tt} E = c^2 \partial_{xx} E \]

\[ E \sim A \sin(\omega t - kx) \]

Wavenumber \( k = \frac{2\pi}{\lambda} \)

Frequency \( \omega = \frac{2\pi}{T} \)

Period \( T \)  Wavelength \( \lambda \)

Phase velocity \( c = \frac{\omega}{k} \)
## Two worlds

<table>
<thead>
<tr>
<th>Particles</th>
<th>Waves</th>
</tr>
</thead>
<tbody>
<tr>
<td>Momentum</td>
<td>Frequency</td>
</tr>
<tr>
<td>Energy</td>
<td>Wavelength</td>
</tr>
<tr>
<td>Interaction: collisions (elastic, non-elastic, etc)</td>
<td>Interaction: Interference</td>
</tr>
</tbody>
</table>
First failure of classics: photoelectric effect
First failure of classics: photoelectric effect
First failure of classics: photoelectric effect

Current depends on the frequency of light...
Another failure: Electron diffraction

Particle diffraction
Another failure: Electron diffraction

Particle diffraction

Wave diffraction
Another failure: Electron diffraction

Particle diffraction

Wave diffraction

Interference
Another failure: Electron diffraction

Particle diffraction

Wave diffraction

Electron diffraction

“Wavelength” of electrons ????
Wave-particle duality

EM-wave: prescribe energy being a multiple of \( \hbar \omega \)

Particles: prescribe wavelength \( \lambda = \frac{h}{p} \)

Planck constant \( h \approx 6.6 \times 10^{-34} \) \( \quad \hbar = \frac{h}{2\pi} \)

Energy \quad \longleftrightarrow \quad Frequency
Momentum \quad \longleftrightarrow \quad Wavelength

Bullet 0.01 kg, 600 m/s: wavelength = \( 10^{-34} \text{ m} \) \( \), X-rays \( 10^{-10} \text{ m} \)
Wave function

In QM, every state of particle is *completely* described by its wave function

$$\Psi(x, t) : \mathbb{R}^4 \rightarrow \mathbb{C}$$

$$|\Psi(x, t)|^2$$ is the probability to find the particle at $$(x, t)$$

Normalization: $$\int_{\mathbb{R}^3} |\Psi(x, t)|^2 \, dx = 1$$

This *naturally* defines a Hilbert space of square integrable functions
Measurements in QM

To every measurable quantity ("observable"), there is a corresponding operator, which acts on the wave function:

- Coordinate operator: $x$
- Momentum operator: $p = -i\hbar\nabla$
- Energy operator: $H = \frac{p^2}{2m} + V = -\frac{\hbar^2}{2m}\nabla^2 + V$

Measurements are essentially invasive!

After a measurement, the wave function of the particle is the eigenfunction of the measured operator. Measured values are the eigenvalues of the operator.
Measurements are uncertain

Observable \( \Phi \) two possible outcomes of measurement \( a_{12} \)

Eigenvectors \( \varphi_{12} \)

Initial states \( \Psi_{12} \)

in (a), outcome \( a_1 \) is more likely

in (b), both outcomes are equally likely
Trying to measure two quantities ..?

Two observables $\Phi_1$ $\Phi_2$
with different sets of eigenvectors

Measurement of blue observable makes the following red measurement *uncertain*, and vice versa
Coordinate-momentum uncertainty

\[ \Delta p \Delta x \geq \frac{\hbar}{2} \]

Also plausible in classical mechanics:

\[ c = \frac{\Delta x}{\Delta t} \]
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Schrödinger Equation

\[ i\hbar \frac{\partial}{\partial t} \Psi = H \Psi \]

\[ H = \frac{p^2}{2m} + V = -\frac{\hbar^2}{2m} \nabla^2 + V \]

Time-independent states

\[ \partial_t |\Psi(x, t)|^2 = 0 \]

obtain eigenvalue problem:

\[ H \Psi = E \Psi \]

Energy values \( E \) are the eigenvalues of \( H \)

Time-independent = with constant energy
Particle in a 1D box

\[
\begin{bmatrix}
-\frac{\hbar^2}{2m} \nabla^2 + V
\end{bmatrix} \Psi = E \Psi
\]
Particle in a 1D box

\[ \left( -\frac{\hbar^2}{2m} \nabla^2 + V \right) \Psi = E \Psi \]

\[ -\frac{\hbar^2}{2m} \partial_x^2 \Psi = E \Psi \]

\[ E = E_0, E_1, E_2, \ldots \]

\[ E_n \propto n^2 \]

\[ \Psi_n \propto \sin\left( \frac{n\pi x}{L} \right) \]
Square 2D box

Two quantum numbers

\[ E = E_{n_x, n_y} \]

Degeneracy:

\[ E_{n_x, n_y} = E_{n_y, n_x} \]
Hydrogen atom

\[
\begin{bmatrix}
-\frac{\hbar^2}{2m} \nabla^2_{3D} - \frac{\gamma}{r}
\end{bmatrix} \Psi = E \Psi
\]

\[
\nabla^2_{3D} \Psi = \left( \frac{1}{r^2} \partial_r (r^2 \partial_r \Psi) + \frac{1}{r^2 \sin \theta} \partial_\theta (\sin \theta \partial_\theta \Psi) + \frac{1}{r^2 \sin^2 \theta} \partial^2_\theta \Psi \right)
\]

Surprisingly, it can be solved analytically (typically the last 3 lectures in a QM course)

\[
E = E_{n,l,m}
\]

Three quantum numbers, energy depends only on \( n \)
Spin

Stern-Gerlach experiment

2x-degeneracy of the outer electron

Silver: one electron in the outer shell, it has no orbital angular momentum
Spin uncertainty
Spin uncertainty

\[ z \text{ and } x \text{ cannot be measured simultaneously!!!} \]
Postulates of QM

1. The state of a QM system is completely described by its wave function

2. To every observable corresponds a linear self-adjoint operator

3. In any measurement, the observed values are the eigenvalues of that operator

4. Probability to measure a value is given by the projection of the state on the corresponding eigenfunction of the operator

5. Wave function evolves in time according to the Schrödinger equation.
Nobel prize history of QM

- 1906, Thomson, *discovery of electron*
- 1918, Planck, *discovery of energy quanta, black body radiation*
- 1921, Einstein, *photoelectric Effect*
- 1927 Compton, *Compton effect*
- 1929 de Broglie, *wave nature of electrons*
- 1932 Heisenberg, *creation of Quantum Mechanics*
- 1933 Schrödinger & Dirac, *discovery of new forms of atomic theory*
- 1937 Davisson & Thomson, *diffraction of electrons*
- 1943 Stern, *magnetic moment of proton*
- 1945 Pauli, *Pauli’s principle*
- 1954 Born, *statistical interpretation of the wavefunction*
Tunnel effect

States with energy $E < V_0$

Classically, the wall is impenetrable and particle is bounced back

In QM, there is a non-zero probability to get through the potential wall