Reverse-Fit a 2-approximation for strip-packing

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Introduction
The problem 2D strip-packing
Next-Fit
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Discussion
2D strip-packing

**Problem:** Given a list $L$ of rectangles and a bin with width 1.

**Goal:** A non overlapping packing with minimal height.

**Restriction:** No rotation.
Next-Fit

NextFit(L)

- Sort the rectangles by height.
Next-Fit

NextFit(L)

- Sort the rectangles by height.
- Pack a level from left to right.
Reverse-Fit a 2-approximation for strip-packing

Introduction

Next-Fit

NextFit(L)

- Sort the rectangles by height.
- Pack a level from left to right.
- Construct a new groundline.
Next-Fit

NextFit(L)

- Sort the rectangles by height.
- Pack a level from left to right.
- Construct a new groundline.
- Repeat 2nd and 3rd step.
Example
Analysis of Next-Fit

**Theorem**

*Let* $L$ *be an instance for 2D strip-packing.*

$$NF(L) \leq 2 \cdot OPT(L) + h_{\text{max}}$$
Introduction

Analysis of Next-Fit

Theorem

Let $L$ be an instance for 2D strip-packing. 

$$NF(L) \leq 2 \cdot OPT(L) + h_{\text{max}}$$

Proof.

Define for all levels $i > 1$ an area $A_i$ with $A_i \geq \frac{1}{2} h_i$. 

Then $OPT(L) \geq \sum_{i=2}^{N} A_i \geq \sum_{i=2}^{N} \frac{1}{2} h_i \geq \frac{1}{2} NF(L) - \frac{1}{2} h_{\text{max}}$. 

\[ \square \]
Definition of the areas
The algorithm Reverse-Fit
Reverse-Fit a 2-approximation for strip-packing

Reverse-Fit

The algorithm

Stack the width rectangles.
Reverse-Fit

- Stack the width rectangles.
- Sort the remaining rectangles by height.
Reverse-Fit

- Stack the width rectangles.
- Sort the remaining rectangles by height.
- Pack 1st level from left to right.
Reverse-Fit

- Stack the width rectangles.
- Sort the remaining rectangles by height.
- Pack 1st level from **left** to **right**.
- Pack 2nd level from **right** to **left**.
Reverse-Fit

- Stack the width rectangles.
- Sort the remaining rectangles by height.
- Pack 1st level from **left** to **right**.
- Pack 2nd level from **right** to **left**.
- Lower the 2nd level.
Reverse-Fit

- Stack the width rectangles.
- Sort the remaining rectangles by height.
- Pack 1st level from left to right.
- Pack 2nd level from right to left.
- Lower the 2nd level.
- If the endpoints of the touching line is less than $\frac{1}{2}$, then lower again.
Reverse-Fit

- Stack the width rectangles.
- Sort the remaining rectangles by height.
- Pack 1st level from \textbf{left} to \textbf{right}.
- Pack 2nd level from \textbf{right} to \textbf{left}.
- Lower the 2nd level.
- If the endpoints of the touching line is less than $\frac{1}{2}$, then lower again.
- If the distance of this lowering is deeper than the last rectangle; move it in the 3rd level.
Reverse-Fit

- Stack the width rectangles.
- Sort the remaining rectangles by height.
- Pack 1st level from left to right.
- Pack 2nd level from right to left.
- Lower the 2nd level.
- If the endpoints of the touching line is less than $\frac{1}{2}$, then lower again.
- If the distance of this lowering is deeper than the last rectangle; move it in the 3rd level.
- Pack the following level with a modifyNext-Fit.
Lowering
1st case: $m_2 \geq \frac{1}{2}$
2nd case: $m_2 < \frac{1}{2}$
2nd lowering
3rd case: 2nd lowering is deeper than last rectangle
2nd lowering

\[ y \]

\[ H_0 \]

\[ H_1 \]

\[ x = \frac{1}{2} \]
Move last rectangle in the 3rd level
Following levels with Next-Fit
Analysis of Reverse-Fit

**Theorem**

$$RF(L) \leq 2 \cdot OPT$$

**Proof.**

For all levels there is an area $A_i$, filled with rectangles, and with

$$A_i \geq \frac{1}{2} h_i.$$  

Then $OPT \geq \sum_{i=0}^{N} A_i \geq \frac{1}{2} \sum_{i=0}^{N} h_i = \frac{1}{2} RF(L)$. \qed
Area $A_0$
Area $A_1$
Area $A_2$
Area $A_3$
Area $A_3$

The diagram illustrates the area $A_3$ with $H_0$, $H_1$, $H_3$, and $r_k = r_q$. The diagram shows a reverse-fit for strip-packing analysis.
All areas
For all levels there is an area $A_i$ with $A_i \geq \frac{1}{2} h_i$.
So the bin is half filled and it follows:
$RF(L) \leq 2 \cdot OPT$
Discussion

Open questions:

- Could the ratio be improved with the use of First-Fit?
Discussion

Open questions:

- Could the ratio be improved with the use of First-Fit?
- Could the ratio be improved with more reverse levels?
Thank you for your attention!
Any questions?