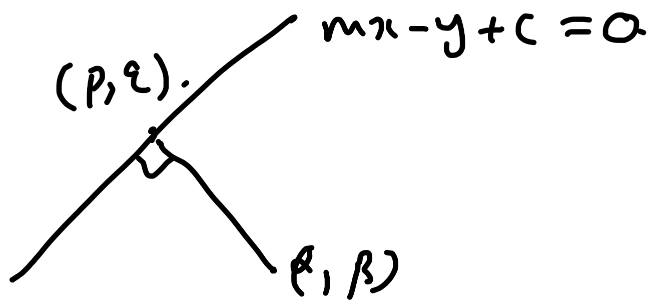
(A).



$$\frac{\beta - q}{\alpha - p} = -\frac{1}{m} \quad \text{— (1)}$$

$$\beta - q = -\frac{1}{m}(\alpha - p)$$

$$q = mp + c$$

$$mp - q + c = 0 \quad \text{— (2)}$$

$$\beta - mp - c = -\frac{1}{m}(\alpha - p)$$

$$\beta - mp - c = -\frac{\alpha}{m} + \frac{p}{m}$$

$$\beta - c + \frac{\alpha}{m} = p\left(m + \frac{1}{m}\right) = \frac{p(m^2+1)}{m}.$$

$$p = \frac{m\beta - mc + \alpha}{1 + m^2} \quad //$$

$$q = \frac{m^2\beta - \cancel{m^2c} + m\alpha + c + \cancel{cm^2}}{1 + m^2}$$

$$= \frac{m\alpha + m^2\beta + c}{1 + m^2} \quad //.$$

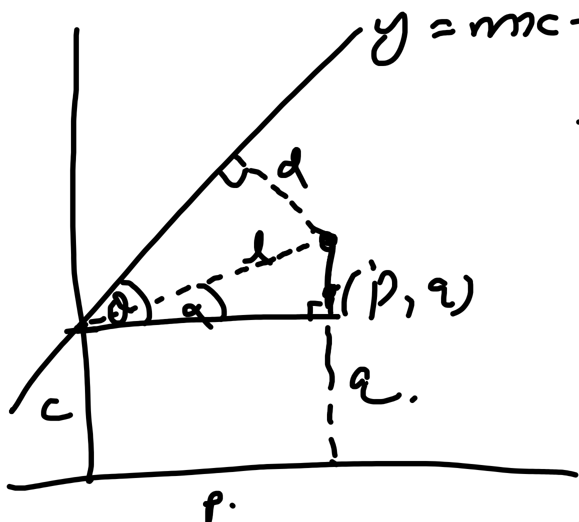
(B) $d^2 = (p-\alpha)^2 + (q-\beta)^2$

$$= \left(\frac{m\beta - mc + \alpha}{1+m^2} - \alpha\right)^2 + \left(\frac{m\alpha + m^2\beta + c}{1+m^2} - \beta\right)^2.$$

$$= \left(\frac{m\beta - mc - \alpha m^2}{1+m^2}\right)^2 + \left(\frac{m\alpha + c - \beta}{1+m^2}\right)^2$$

$$\frac{m^2}{(1+m^2)^2}(\beta - c - \alpha m)^2 + \frac{1}{(1+m^2)^2}(\beta - c - \alpha m)^2$$

$$\frac{(\alpha m - \beta + c)^2}{(1+m^2)^2}(m^2+1) = \frac{(\alpha m - \beta + c)^2}{m^2+1}$$

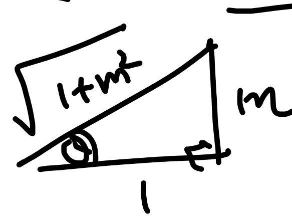


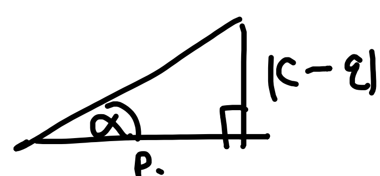
$l\cos\alpha = p.$

$l\sin(\theta - \alpha) = d.$

$$\frac{d}{p} = \frac{\sin(\theta-\alpha)}{\cos\alpha}.$$

$$= \frac{\sin\theta\cos\alpha - \cos\theta\sin\alpha}{\cos\alpha}.$$





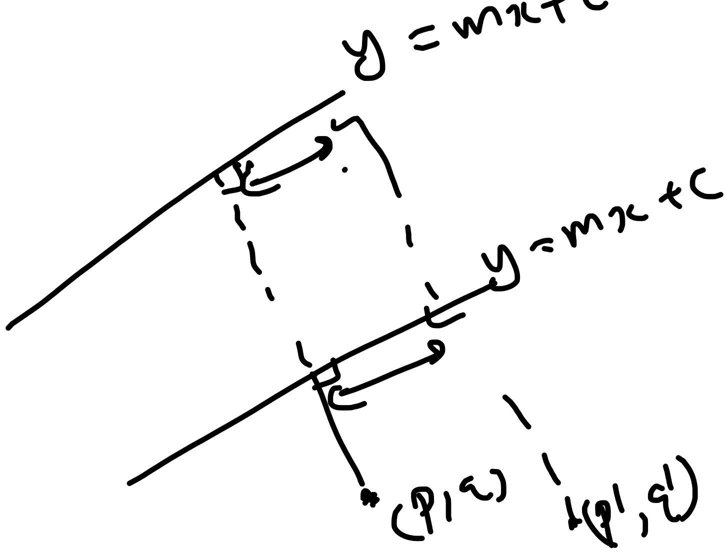
$$\frac{d}{p} = \sin\theta - \cos\theta\tan\alpha.$$

$$\frac{d}{p} = \frac{m}{\sqrt{1+m^2}} - \frac{1}{\sqrt{m^2+1}}\cdot\frac{|c-q|}{p}.$$

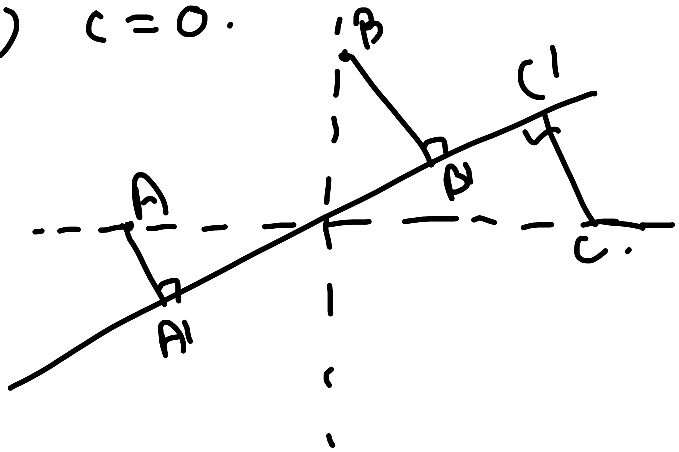
$$\therefore d = \frac{pm - |c-q|}{\sqrt{1+m^2}} = \frac{pm - (q-c)}{\sqrt{1+m^2}}$$

$$d = \frac{pm - q + c}{\sqrt{1+m^2}} \qquad d = \frac{|\alpha m - \beta + c|}{\sqrt{1+m^2}}$$

(C)



(D) $c = 0$.



$$A = \begin{pmatrix} -1 \\ 0 \end{pmatrix} \qquad B = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad C = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\bar{x} = \begin{pmatrix} 0 \\ 1/3 \end{pmatrix}$$

$$S = \frac{1}{N} \sum_{n=1}^{N} (x_n - \bar{x})(x_n - \bar{x})^T$$

$$(A - \bar{x}) = \begin{pmatrix} -1 \\ -1/3 \end{pmatrix} \begin{pmatrix} -1 & -\frac{1}{3} \end{pmatrix} = \begin{pmatrix} 1 & 1/3 \\ \frac{1}{3} & \frac{1}{9} \end{pmatrix}$$

$(A - \bar{x})^T$

$$(B-\bar{x})(B-\bar{x})^T = \begin{pmatrix} 0 \\ 2/3 \end{pmatrix} \begin{pmatrix} 0 & 2/3 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 4/9 \end{pmatrix}$$

$$(C-\bar{x})(C-\bar{x})^T = \begin{pmatrix} +1 \\ -1/3 \end{pmatrix} \begin{pmatrix} 1 & -1/3 \end{pmatrix} = \begin{pmatrix} 1 & -\frac{1}{3} \\ -\frac{1}{3} & \frac{1}{9} \end{pmatrix}$$

$$|S-\lambda I| = 0.$$

$$\begin{vmatrix} 2-\lambda & 0 \\ 0 & \frac{2}{3}-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)\left(\frac{2}{3}-\lambda\right) = 0.$$

$$S = \begin{pmatrix} 2 & 0 \\ 0 & 2/3 \end{pmatrix}$$

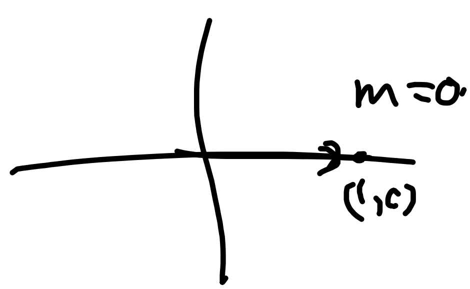
$\therefore$ largest eigenvalue is 2.

$$(S-2I) = \begin{pmatrix} 0 & 0 \\ 0 & -\frac{4}{3} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$\therefore$ corresponding eigen vector u_1 is given by

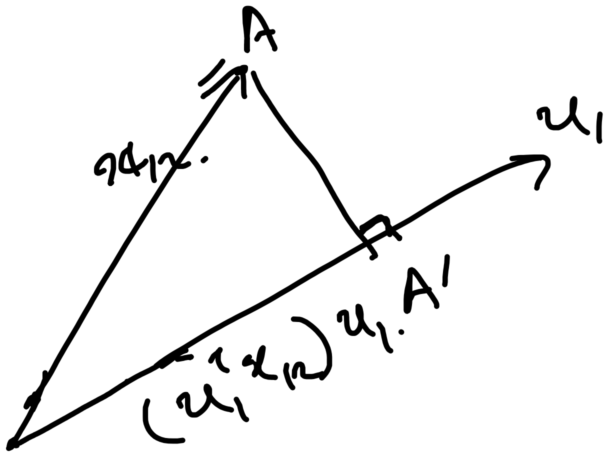
$$-\frac{4}{3}x_2 = 0 \qquad x_2 = 0$$

$$u_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$



Alternative approach

Minimum error formulation



$$AA' = x_n - (u_1^T x_n) u_1$$

$$|AA'|^2 = \| x_n - (u_1^T x_n) u_1 \|_2^2$$

$$= (x_n - (u_1^T x_n) u_1)^T (x_n - (u_1^T x_n) u_1)$$

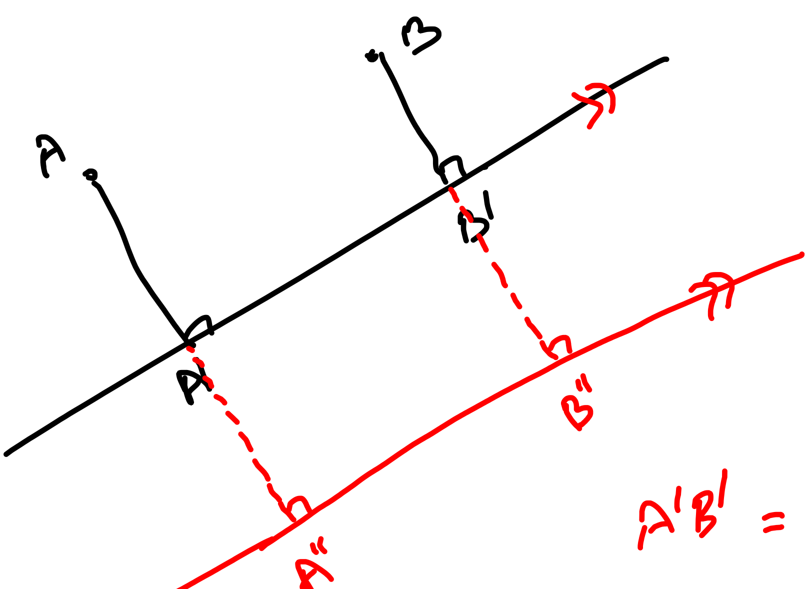
$$\bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_n$$

$$= \| x_n \|^2 - x_n^T u_1 (u_1^T x_n) - (u_1^T x_n)(u_1^T x_n) + (u_1^T x_n) \| u_1 \|_2^2$$

$$= \frac{1}{N} \sum_{n=1}^{N} \| x_n \|^2 - 2 (x_n^T u_1)^2 + (x_n^T u_1)$$

$$= \quad \| \bar{x} \|_2^2 = \| x_1 \|_2^2 + \| x_2 \|_2^2 + \cdots + \| x_N \|_2^2$$

$$+ 2 x_1^T x_2 + \cdots$$



$A'B' = A''B''$

$AA' + BB'$

$AA'' + BB''$

$(AA' + A'A'') + (BB' + B'B'')$

$$d = \frac{|\alpha m - \beta|}{\sqrt{1+m^2}}.$$

$A = (-1, 0)$

$B = (0, 1)$

$C = (1, 0)$

$$d_A = \frac{|-m|}{\sqrt{1+m^2}} = \frac{m}{\sqrt{1+m^2}}.$$

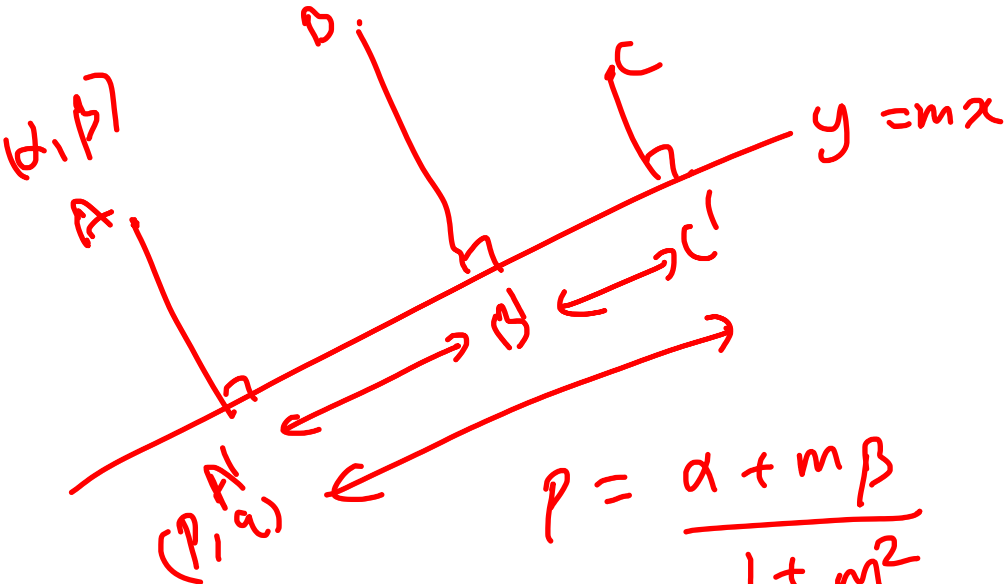
$$d_B = \frac{1}{\sqrt{1+m^2}}.$$

$$d_C = \frac{m}{\sqrt{1+m^2}}.$$

$$d^2 = d_A^2 + d_B^2 + d_C^2 = \frac{m^2 + 1 + m^2}{1+m^2}$$

$$d^2 = \frac{2m^2+1}{m^2+1} = 2 - \frac{1}{m^2+1}$$

d_{min} by $m = 0$.



$$p = \frac{\alpha + m\beta}{1+m^2} \qquad q = \frac{m\alpha + m^2\beta}{1+m^2}$$

$$A' = \left(\frac{-1}{1+m^2}, \frac{-m}{1+m^2}\right)$$

$$B' = \left(\frac{m}{1+m^2}, \frac{m^2}{1+m^2}\right)$$

$$C' = \left(\frac{1}{1+m^2}, \frac{m}{1+m^2}\right)$$

$$d^2 = (A'B')^2 + (B'C')^2 + (A'C')^2$$

$$d^2 = \frac{(1+m)^2 + (m+m^2)^2 + (m-1)^2 + (m^2-m)^2 + \cancel{4} + 4m^2}{(1+m^2)^2}$$

$$d^2 = (1+m)^2 + m^2(1+m)^2 + (m-1)^2 + m^2(m-1)^2 + 4(1+m^2)$$

$$\overline{\qquad\qquad}$$
$$(1+m^2)^2.$$

$$= \frac{(1+m)^2 + m^2(1+m)^2 + (m-1)^2(1+m^2) + 4(1+m^2)}{(1+m^2)^2.}$$

$$= \frac{\cancel{(1+m^2)}(1+m)^2 + \cancel{(1+m^2)}(m-1)^2 + 4\cancel{(1+m^2)}}{(1+m^2)^{\cancel{2}}}$$

$$= \frac{2m^2+2+4}{1+m^2} = \frac{2m^2+6}{1+m^2} = \frac{2(m^2+1)+4}{m^2+1}$$

$$= 2 + \frac{4}{m^2+1}$$