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APPLICATIONS OF MODAL LOGIC IN LINGUISTICS

Lawrence S. Moss and Hans-Jörg Tiede

1 INTRODUCTION

That logic and language are closely related is almost true by definition. Logic is concerned with the study of valid inferences in arguments, and these are most commonly defined in terms of truth in models. Symbolic logic studies formal languages (logics) as models of certain aspects of natural languages, such as quantification, while abstracting away from certain other aspects of natural languages, such as ambiguity, as models typically do. Linguistics studies the structure of natural languages as well as the relation of language to other areas of cognitive science. The roles that logic in general, and modal logic in particular, play in linguistics are quite varied, as we shall see.

In linguistic semantics, logic is used to formalize, or interpret, an *object language*. We take as given that we want to study the semantics of some natural language, and in this chapter the language that we shall deal with is English. Above all else, we would like to directly interpret English sentences in some formally specified model. So even at this point we can see some connection to the Kripke semantics of modal operators: just as all of the other phenomena in this "applied" section of the handbook have been modeled with mathematical structures involving possible worlds, so too have these been used in semantic applications. For example, all manner of linguistic phenomena involving time have led to proposals for using the models from temporal logic. More generally, the main models of all types of *intensional* phenomena are closely related to the models in modal logic.

But so far we have only considered the matter of interpreting natural language directly. Usually, this is difficult or even impossible. (For example, consider the famous quantifier scope ambiguities in sentences like *every handbook has a famous editor*. The ambiguity is neatly expressed in logical notation as $\exists\forall$ vs $\forall\exists$: is there one person, let's call him Dov, who edits all the handbooks, or is it merely that every handbook has some editor or other? One lesson to take from such ambiguities is that it is impossible to associate a function from (English $\times$ models) to truth values in a way that respects our intuitions.) So one way or another, we translate natural language to some artificial language and then interpret that other language, in such a way that ambiguous sentences will be translated into multiple logical formulas. And here is a second place modal logic comes in: the language of higher-order modal logic has been used extensively to drive this translation process, as we shall see when we discuss Montague semantics.

We next turn to syntax, a field in which one finds several different uses of logic. There are syntactic frameworks which are heavily proof-theoretic, so the question of whether a given string is a sentence or not boils down to whether a related (formal) sentence is a theorem in some logical system. This proof-theoretic move is especially prominent in categorial grammar. Another quite different use of logic is as a *meta language* in which one formalizes a linguistic formalism declaratively. This is the move of *model theoretic syntax*, a research program we consider in depth in the second half our chapter. This application relates logic to linguistics in the same way that logic can be applied to formalize theories of other sciences, like set theory. However, the aims of this formalization are somewhat different from those of other areas, since model theoretic syntax is particularly interested in using *decidable* logics for this formalization so that matters can be implemented. This is of course one of the reasons why modal logics are attractive in this context, although much of the focus has been on monadic second-order logic of trees, which is decidable as well.

Applications of logic in linguistics have traditionally not been too concerned with meta-results. The main uses of modal logic in semantics are independent from the main concerns of modal logicians: completeness and correspondence. We are not aware of any serious application of the basic theory of modal logic in semantics, let alone the advanced theory that is showcased in various chapters of this handbook. The only exception is definability theory, interest in which is motivated by trying to find a logic for linguistic applications that has the right kind of "expressiveness." For example, the fact that *most A are B* is not first-order definable is of some importance for semantics. On the other side, the application of logic in syntax has led to more applications of sophisticated meta-results, for example proof theoretical results like cut-elimination or normalization in categorial grammar. It is interesting to note that definability is also of importance in model theoretic syntax, due to its relation to descriptive complexity theory. A related point: because so many current syntactic frameworks are designed with a hope of implementation, sharper theoretical results about them are called for.

In this chapter, we only survey applications of modal logic to the syntax and semantics of natural languages. We concentrate on these two applications because of the historical importance of modal logic in the development of natural language semantics and because of the significance of model theoretic syntax in current research in mathematical linguistics. There are many areas of applications of logic in linguistics that we do not mention, some of which are surveyed in the *Handbook of Logic & Language* [4].

2 SEMANTICS

Linguistic semantics studies meaning in natural languages. The central assumption of current semantic theory is that meaning should be studied model theoretically, in the same way that semantics of logics are studied. Thus, the study of *meaning* is tied to the concept of *truth*. Of course, there are other ways to pursue the project of understanding meaning, most notably to tie it to *action* in some way. As it happens, for some purposes possible worlds semantics is even better for this second purpose than for the first; see, for example, [69].

The interpretation of logical formulas usually involves the interpretation of subformulas in some systematic fashion. For instance, in propositional logic we have interpretational clauses like

$$[\![\varphi \wedge \psi]\!] = [\![\wedge]\!]([\![\varphi]\!], [\![\psi]\!])$$

where $[\![\wedge]\!]$ is the boolean *and* function. The methodological principle that stipulates that all interpretations of complex expressions should involve the interpretations of its parts is called the "principle of compositionality," and it plays a central role in linguistic semantics. Whether that principle is in fact a meaningful restriction on semantic theory or whether it is vacuous is a point of ongoing debate. For one source that discusses the matter at length, see Janssen [41].

Since natural language semantics applies model theoretic methods, the role of modal logic in this context involves the application of possible worlds semantics to natural languages, mainly to model *intensional* phenomena. However, in order to follow the principle of compositionality uniformly, the meanings of some expressions are modeled using higher-order logic. Thus, the most influential, systematic application of modal logic to linguistic semantics, usually referred to as *Montague semantics* after its founder, involved *higher-order intensional logic*. Although Montague's application of higher-order intensional logic to natural language semantics yielded many important results, almost all of the contemporary research is concerned with finding suitable alternatives to this framework. Many of these are surveyed in the *Handbook of Logic and Language* [4]. Another handbook in this area, with a more empirical and linguistic, as opposed to theoretical and logical, slant is the *Handbook of Contemporary Semantic Theory* [57]. There are many introductory textbooks in linguistic semantics, including [15, 22, 36].

2.1 Possible worlds in semantics

The major use of modal logic in semantics stems from possible worlds semantics. Indeed, this is the only kind of application we are considering in this chapter.