Ontology Languages (COMP321)

Solution to Exercise 6

Consider the database instance $\mathcal{D}_{\mathsf{Nemo}}$ given by

$$\mathsf{Clownfish}(\mathsf{Nemo}), \quad \mathsf{Clownfish}(\mathsf{Karl})$$

$$\mathsf{Surgeonfish}(\mathsf{Dory}), \quad \mathsf{has\_friend}(\mathsf{Nemo}, \mathsf{Dory})$$

We query $\mathcal{D}_{\mathsf{Nemo}}$ under closed world assumption (standard relational database semantics) and under open world assumption. Recall that under the closed world assumption we consider the interpretation $\mathcal{I} := \mathcal{I}_{\mathcal{D}_{\mathsf{Nemo}}}$ defined as follows:

- $\Delta^{\mathcal{I}} = \{\mathsf{Nemo}, \mathsf{Karl}, \mathsf{Dory}\}$;
- $\mathsf{Clownfish}^{\mathcal{I}} = \{\mathsf{Nemo}, \mathsf{Karl}\}$;
- $\mathsf{Surgeonfish}^{\mathcal{I}} = \{\mathsf{Dory}\}$;
- $\mathsf{has\_friend}^{\mathcal{I}} = \{(\mathsf{Nemo}, \mathsf{Dory})\}$.

Consider the following Boolean queries (in description logic notation).

- $\mathsf{Clownfish}(\mathsf{Karl})$
- $\mathsf{Clownfish}(\mathsf{Dory})$
- $\mathsf{Fish}(\mathsf{Nemo})$
- $\neg\mathsf{Fish}(\mathsf{Nemo})$
- $(\exists \mathsf{has\_friend}.\top)(\mathsf{Nemo})$
- $(\exists \mathsf{has\_friend}.\mathsf{Fish})(\mathsf{Nemo})$
- $(\mathsf{Clownfish} \sqcap \neg\mathsf{Surgeonfish})(\mathsf{Karl})$
- $\mathsf{Fish}(\mathsf{Dory})$
- $(\mathsf{Surgeonfish} \sqcap \neg\mathsf{Fish})(\mathsf{Dory})$

- $(\exists \mathsf{has\_friend}.\mathsf{Clownfish})(\mathsf{Karl})$.

(1) Write those Boolean queries in first-order predicate logic (FOPL) notation. (Note that for many queries there is no difference between description logic notation and FOPL notation).

Solution

- $\mathsf{Clownfish}(\mathsf{Karl})$ is already in FOPL
- $\mathsf{Clownfish}(\mathsf{Dory})$ is already in FOPL
- $\mathsf{Fish}(\mathsf{Nemo})$ is already in FOPL
- $\neg\mathsf{Fish}(\mathsf{Nemo})$ is already in FOPL
- $(\exists \mathsf{has\_friend}.\top)(\mathsf{Nemo})$ is in FOPL: $\exists y.\mathsf{has\_friend}(\mathsf{Nemo}, y))$.
- $(\exists \mathsf{has\_friend}.\mathsf{Fish})(\mathsf{Nemo})$ is in FOPL:

$$\exists y.(\mathsf{has\_friend}(\mathsf{Nemo}, y) \wedge \mathsf{Fish}(y))$$

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- $(\mathsf{Clownfish} \sqcap \neg\mathsf{Surgeonfish})(\mathsf{Karl})$ is in FOPL:

$$\mathsf{Clownfish}(\mathsf{Karl}) \wedge \neg\mathsf{Surgeonfish}(\mathsf{Karl})$$

- $\mathsf{Fish}(\mathsf{Dory})$ is already in FOPL.
- $(\mathsf{Surgeonfish} \sqcap \neg\mathsf{Fish})(\mathsf{Dory})$ is in FOPL:

$$\mathsf{Surgeonfish}(\mathsf{Dory}) \wedge \neg\mathsf{Fish}(\mathsf{Dory})$$

- $(\exists \mathsf{has\_friend}.\mathsf{Clownfish})(\mathsf{Karl})$ in in FOPL:

$$\exists y.(\mathsf{has\_friend}(\mathsf{Karl}, y) \wedge \mathsf{Clownfish}(y))$$

(2) Query answering under closed world assumption: check for each Boolean query F whether the answer to the query F given by $\mathcal{D}_{\mathsf{Nemo}}$ is "Yes" or "No". In other words, check whether $\mathcal{I} \models F$ or $\mathcal{I} \models \neg F$.

Solution

- $\mathcal{I} \models \mathsf{Clownfish}(\mathsf{Karl})$? Yes
- $\mathcal{I} \models \mathsf{Clownfish}(\mathsf{Dory})$? No
- $\mathcal{I} \models \mathsf{Fish}(\mathsf{Nemo})$? No
- $\mathcal{I} \models \neg\mathsf{Fish}(\mathsf{Nemo})$? Yes
- $\mathcal{I} \models (\exists \mathsf{has\_friend}.\top)(\mathsf{Nemo})$? Yes
- $\mathcal{I} \models (\exists \mathsf{has\_friend}.\mathsf{Fish})(\mathsf{Nemo})$? No
- $\mathcal{I} \models (\mathsf{Clownfish} \sqcap \neg\mathsf{Surgeonfish})(\mathsf{Karl})$? Yes
- $\mathcal{I} \models \mathsf{Fish}(\mathsf{Dory})$? No
- $\mathcal{I} \models (\mathsf{Surgeonfish} \sqcap \neg\mathsf{Fish})(\mathsf{Dory})$? Yes
- $\mathcal{I} \models (\exists \mathsf{has\_friend}.\mathsf{Clownfish})(\mathsf{Karl})$? No

(3) Query answering under open world assumption: check for each Boolean query F whether the certain answer to F given by $\mathcal{D}_{\mathsf{Nemo}}$ is "Yes", "No", or "Don't know". In other words, check whether $\mathcal{D} \models F$ or $\mathcal{D} \models \neg F$ or neither of these two hold.

Solution

- $\mathsf{Clownfish}(\mathsf{Karl})$? Yes
- $\mathsf{Clownfish}(\mathsf{Dory})$? Don't know
- $\mathsf{Fish}(\mathsf{Nemo})$? Don't know
- $\neg\mathsf{Fish}(\mathsf{Nemo})$? Don't know
- $(\exists \mathsf{has\_friend}.\top)(\mathsf{Nemo})$? Yes
- $(\exists \mathsf{has\_friend}.\mathsf{Fish})(\mathsf{Nemo})$? Don't know
- $(\mathsf{Clownfish} \sqcap \neg\mathsf{Surgeonfish})(\mathsf{Karl})$? Don't know
- $\mathsf{Fish}(\mathsf{Dory})$? Don't know
- $(\mathsf{Surgeonfish} \sqcap \neg\mathsf{Fish})(\mathsf{Dory})$? Don't know
- $(\exists \mathsf{has\_friend}.\mathsf{Clownfish})(\mathsf{Karl})$? Don't know

Consider the following non-Boolean queries F_i:

- $F_1(x) = \mathsf{Clownfish}(x)$
- $F_2(x) = \neg\mathsf{Surgeonfish}(x)$
- $F_3(x, y) = \mathsf{has\_friend}(x, y)$
- $F_4(x) = \mathsf{Clownfish}(x) \wedge \neg\mathsf{has\_friend}(x, \mathsf{Dory})$

(4) For each query F_i, give $\mathsf{answer}(F_i, \mathcal{I}_N)$.
Solution

- $\mathsf{answer}(F_1, \mathcal{D}_{\mathsf{Nemo}}) = \{\mathsf{Nemo}, \mathsf{Karl}\}$.
- $\mathsf{answer}(F_2, \mathcal{D}_{\mathsf{Nemo}}) = \{\mathsf{Nemo}, \mathsf{Karl}\}$.
- $\mathsf{answer}(F_3, \mathcal{D}_{\mathsf{Nemo}}) = \{(\mathsf{Nemo}, \mathsf{Dory})\}$.
- $\mathsf{answer}(F_4, \mathcal{D}_{\mathsf{Nemo}}) = \{\mathsf{Karl}\}$.

(5) For each query F_i, give $\mathsf{certanswer}(F_i, \mathcal{D}_{\mathsf{Nemo}})$.
Solution

- $\mathsf{certanswer}(F_1, \mathcal{D}_{\mathsf{Nemo}}) = \{\mathsf{Nemo}, \mathsf{Karl}\}$.
- $\mathsf{certanswer}(F_2, \mathcal{D}_{\mathsf{Nemo}}) = \emptyset$.
- $\mathsf{certanswer}(F_3, \mathcal{D}_{\mathsf{Nemo}}) = \{(\mathsf{Nemo}, \mathsf{Dory})\}$.
- $\mathsf{certanswer}(F_4, \mathcal{D}_{\mathsf{Nemo}}) = \emptyset$.