Ontology Languages (COMP321)
Solution to Exercise 7

1. **Solution:**

Query	Answer for $\mathcal{I}$	Certain A for $\mathcal{D}_{\mathsf{Nemo}}$	Certain A for $(\mathcal{T}, \mathcal{D}_{\mathsf{Nemo}})$
$\mathsf{Clownfish}(\mathsf{Karl})$	Yes	Yes	Yes
$\mathsf{Clownfish}(\mathsf{Dory})$	No	Don't know	No
$\mathsf{Fish}(\mathsf{Nemo})$	No	Don't know	Yes
$\neg\mathsf{Fish}(\mathsf{Nemo})$	Yes	Don't know	No
$\exists\mathsf{has\_friend}.\top(\mathsf{Nemo})$	Yes	Yes	Yes
$\exists\mathsf{has\_friend}.\mathsf{Fish}(\mathsf{Nemo})$	No	Don't know	Yes
$\mathsf{Clownfish} \sqcap \neg\mathsf{Surgeonfish}(\mathsf{Karl})$	Yes	Don't know	Yes
$\mathsf{Fish}(\mathsf{Dory})$	No	Don't know	Yes
$\mathsf{Surgeonfish} \sqcap \neg\mathsf{Fish}(\mathsf{Dory})$	Yes	Don't know	No
$\exists\mathsf{has\_friend}.\mathsf{Clownfish}(\mathsf{Karl})$	No	Don't know	Don't know

2. Consider the following non-Boolean queries F_i:

   - $F_1(x) = \mathsf{Clownfish}(x)$
   - $F_2(x) = \neg\mathsf{Surgeonfish}(x)$
   - $F_3(x, y) = \mathsf{has\_friend}(x, y)$
   - $F_4(x) = \mathsf{Clownfish}(x) \wedge \neg\mathsf{has\_friend}(x, \mathsf{Dory})$

   For each query F_i, give

   - $\mathsf{answer}(F_i, \mathcal{I})$;
   - $\mathsf{certanswer}(F_i, \mathcal{D}_{\mathsf{Nemo}})$;
   - $\mathsf{certanswer}(F_i, (\mathcal{T}, \mathcal{D}_{\mathsf{Nemo}}))$.

   Solution:

   - $\mathsf{answer}(F_1, \mathcal{I}) = \{\mathsf{Nemo}, \mathsf{Karl}\}$.

- $\mathsf{certanswer}(F_1, \mathcal{D}_{\mathsf{Nemo}}) = \{\mathsf{Nemo}, \mathsf{Karl}\}$.
- $\mathsf{certanswer}(F_1, (\mathcal{T}, \mathcal{D}_{\mathsf{Nemo}})) = \{\mathsf{Nemo}, \mathsf{Karl}\}$.
- $\mathsf{answer}(F_2, \mathcal{I}) = \{\mathsf{Nemo}, \mathsf{Karl}\}$.
- $\mathsf{certanswer}(F_2, \mathcal{D}_{\mathsf{Nemo}}) = \emptyset$.
- $\mathsf{answer}(F_2, (\mathcal{T}, \mathcal{D}_{\mathsf{Nemo}})) = \{\mathsf{Nemo}, \mathsf{Karl}\}$.
- $\mathsf{answer}(F_3, \mathcal{I}) = \{(\mathsf{Nemo}, \mathsf{Dory})\}$.
- $\mathsf{certanswer}(F_3, \mathcal{D}_{\mathsf{Nemo}}) = \{(\mathsf{Nemo}, \mathsf{Dory})\}$.
- $\mathsf{certanswer}(F_3, (\mathcal{T}, \mathcal{D}_{\mathsf{Nemo}})) = \{(\mathsf{Nemo}, \mathsf{Dory})\}$.
- $\mathsf{answer}(F_4, \mathcal{I}) = \{\mathsf{Karl}\}$.
- $\mathsf{certanswer}(F_4, \mathcal{D}_{\mathsf{Nemo}}) = \emptyset$.
- $\mathsf{certanswer}(F_4, (\mathcal{T}, \mathcal{D}_{\mathsf{Nemo}})) = \emptyset$.

3. Consider the $\mathcal{EL}$ TBox $\mathcal{T}_0$:

$$\begin{aligned}
\mathsf{Footballplayer} &\sqsubseteq \exists \mathsf{plays_for}.\mathsf{Team} \\
\mathsf{Basketballplayer} &\sqsubseteq \exists \mathsf{plays_for}.\mathsf{Team} \\
\mathsf{Handballplayer} &\sqsubseteq \exists \mathsf{plays_for}.\mathsf{Team} \\
\mathsf{Team} &\sqsubseteq \exists \mathsf{managed_by}.\mathsf{Manager} \\
\mathsf{Manager} &\sqsubseteq \mathsf{Employee} \\
\mathsf{Manager} &\sqsubseteq \exists \mathsf{managed_by}.\mathsf{Manager}
\end{aligned}$$

and the ABox $\mathcal{A}_0$:

$$\mathsf{Footballplayer}(\mathsf{bob}), \quad \mathsf{Basketballplayer}(\mathsf{john}),$$

$$\mathsf{Handballplayer}(\mathsf{peter}), \quad \mathsf{Team}(\mathsf{redsocks})$$

$$\mathsf{managed\_by}(\mathsf{redsocks}, \mathsf{sue})$$

Compute the interpretation $\mathcal{I}_{\mathcal{T}_0,\mathcal{A}_0}$ as described in the Comp321 Lecture Notes.

Solution:

The initial assignment is:

$$
\begin{aligned}
S(d_{\mathsf{Footballplayer}}) &= \{\mathsf{Footballplayer}\} \\
S(d_{\mathsf{Basketballplayer}}) &= \{\mathsf{Basketballplayer}\} \\
S(d_{\mathsf{Handballplayer}}) &= \{\mathsf{Handballplayer}\} \\
S(d_{\mathsf{Team}}) &= \{\mathsf{Team}\} \\
S(d_{\mathsf{Manager}}) &= \{\mathsf{Manager}\} \\
S(d_{\mathsf{Employee}}) &= \{\mathsf{Employee}\} \\
S(\mathsf{bob}) &= \{\mathsf{Footballplayer}\} \\
S(\mathsf{john}) &= \{\mathsf{Basketballplayer}\} \\
S(\mathsf{peter}) &= \{\mathsf{Handballplayer}\} \\
S(\mathsf{redsocks}) &= \{\mathsf{Team}\} \\
S(\mathsf{sue}) &= \emptyset \\
R(\mathsf{plays\_for}) &= \emptyset \\
R(\mathsf{managed\_by}) &= \{(\mathsf{redsocks}, \mathsf{sue})\}
\end{aligned}
$$

- Update R using (rightR) six times:

$$
\begin{aligned}
R(\mathsf{plays\_for}) = \{ & (d_{\mathsf{Footballplayer}}, d_{\mathsf{team}}), (d_{\mathsf{Basketballplayer}}, d_{\mathsf{Team}}), \\
& (d_{\mathsf{Handballplayer}}), d_{\mathsf{Team}}), (\mathsf{bob}, d_{\mathsf{Team}}), \\
& (\mathsf{john}, d_{\mathsf{Team}}), (\mathsf{peter}, d_{\mathsf{Team}})\}
\end{aligned}
$$

- Update S using (simpleR):

$$S(d_{\mathsf{Manager}}) = \{\mathsf{Manager}, \mathsf{Employee}\}$$

- Update R using (rightR) three times:

$$
\begin{aligned}
R(\mathsf{managed\_by}) = \{ & (\mathsf{redsocks}, \mathsf{sue}), (d_{\mathsf{Team}}, d_{\mathsf{Manager}}), \\
& (d_{\mathsf{Manager}}, d_{\mathsf{Manager}}), (\mathsf{redsocks}, d_{\mathsf{Manager}})\}
\end{aligned}
$$

Thus, the interpretation $\mathcal{I}_{\mathcal{T}_0,\mathcal{A}_0}$ is given by:

- The domain $\Delta^{\mathcal{I}_{\mathcal{T}_0,\mathcal{A}_0}}$ consists of $d_{\mathsf{Footballplayer}}$, $d_{\mathsf{Basketballplayer}}$, $d_{\mathsf{Handballplayer}}$, d_{Team}, d_{Manager}, d_{Employee}, bob, john, peter, redsocks, sue.

- $\mathsf{Footballplayer}^{\mathcal{I}_{\mathcal{T}_0,\mathcal{A}_0}} = \{d_{\mathsf{Footballplayer}}, \mathsf{bob}\}$;
- $\mathsf{Basketballplayer}^{\mathcal{I}_{\mathcal{T}_0,\mathcal{A}_0}} = \{d_{\mathsf{Basketballplayer}}, \mathsf{john}\}$;
- $\mathsf{Handballplayer}^{\mathcal{I}_{\mathcal{T}_0,\mathcal{A}_0}} = \{d_{\mathsf{Handballplayer}}, \mathsf{peter}\}$;
- $\mathsf{Team}^{\mathcal{I}_{\mathcal{T}_0,\mathcal{A}_0}} = \{d_{\mathsf{Team}}, \mathsf{redsocks}\}$;
- $\mathsf{Manager}^{\mathcal{I}_{\mathcal{T}_0,\mathcal{A}_0}} = \{d_{\mathsf{Manager}}\}$;
- $\mathsf{Employee}^{\mathcal{I}_{\mathcal{T}_0,\mathcal{A}_0}} = \{d_{\mathsf{Manager}}, d_{\mathsf{Employee}}\}$;
- $\mathsf{plays\_for}^{\mathcal{I}_{\mathcal{T}_0,\mathcal{A}_0}}$ consists of $(d_{\mathsf{Footballplayer}}, d_{\mathsf{team}})$, $(d_{\mathsf{Basketballplayer}}, d_{\mathsf{Team}})$, $(d_{\mathsf{Handballplayer}}), d_{\mathsf{Team}})$ $(\mathsf{bob}, d_{\mathsf{Team}})$, $(\mathsf{john}, d_{\mathsf{Team}})$, $(\mathsf{peter}, d_{\mathsf{Team}})\}$.
- $\mathsf{managed\_by}^{\mathcal{I}_{\mathcal{T}_0,\mathcal{A}_0}}$ consists of $(\mathsf{redsocks}, \mathsf{sue})$, $(d_{\mathsf{Team}}, d_{\mathsf{Manager}})$, $(d_{\mathsf{Manager}}, d_{\mathsf{Manager}})$, $(\mathsf{redsocks}, d_{\mathsf{Manager}})$.

For $\mathcal{EL}$ concept queries, we know that $\mathcal{I}_{\mathcal{T}_0,\mathcal{A}_0}$ gives the answer "Yes" if, and only if, $(\mathcal{T}_0, \mathcal{A}_0)$ gives the answer "Yes". Check this for the queries:

- $\exists \mathsf{plays\_for}.\mathsf{Team}(\mathsf{peter})$;
- $\exists \mathsf{managed\_by}.\mathsf{Manager}(\mathsf{peter})$;
- $\exists \mathsf{plays\_for}.\exists \mathsf{managed\_by}.\mathsf{Manager}(\mathsf{peter})$.

Solution.

- $\exists \mathsf{plays\_for}.\mathsf{Team}(\mathsf{peter})$: in both cases the answer is "Yes".
- $\exists \mathsf{managed\_by}.\mathsf{Manager}(\mathsf{peter})$: in both cases the answer is not "Yes". For $\mathcal{I}_{\mathcal{T}_0,\mathcal{A}_0}$ it is "No"; for $(\mathcal{T}_0, \mathcal{A}_0)$ it is "Don't know".
- $\exists \mathsf{plays\_for}.\exists \mathsf{managed\_by}.\mathsf{Manager}(\mathsf{peter})$: in both cases the answer is "Yes".

For more complex queries, $\mathcal{I}_{\mathcal{T}_0,\mathcal{A}_0}$ can give the answer "Yes" even if $(\mathcal{T}_0, \mathcal{A}_0)$ does not give the answer "Yes". Check this for

- $F(x, y) = \exists z.(\mathsf{plays\_for}(x, z) \wedge \mathsf{plays\_for}(y, z))$.
- $F = \exists x.\mathsf{managed\_by}(x, x)$.

Solution.

- john and peter play for d_{Team} in $\mathcal{I}_{\mathcal{T}_0,\mathcal{A}_0}$. Thus $\mathcal{I}_{\mathcal{T}_0,\mathcal{A}_0} \models F(\mathsf{john}, \mathsf{peter})$. But $(\mathcal{T}_0, \mathcal{A}_0) \not\models F(\mathsf{john}, \mathsf{peter})$.
- $(d_{\mathsf{Manager}}, d_{\mathsf{manager}}) \in \mathsf{managed\_by}^{\mathcal{I}_{\mathcal{T}_0,\mathcal{A}_0}}$. Thus $\mathcal{I}_{\mathcal{T}_0,\mathcal{A}_0} \models F$. However, $(\mathcal{T}_0, \mathcal{A}_0) \not\models F$.