# Ontology Languages (COMP321)
## Solution to Exercise 8

1. Query Rewriting. Let

$$\mathcal{T} = \{\mathsf{Player} \equiv \exists \mathsf{plays}.\top, \mathsf{Player} \sqsubseteq \mathsf{Human}, \mathsf{Human} \sqsubseteq \exists \mathsf{has\_father}.\top\}$$

and

$$F(x) = \mathsf{Human}(x), \quad G(x) = \mathsf{Player}(x)$$

Construct queries $F_{\mathcal{T}}(x)$ and $G_{\mathcal{T}}(x)$ such that for all ABoxes $\mathcal{A}$ and the corresponding interpretations $\mathcal{I}_{\mathcal{A}}$ the following holds for all individual names $a$:

$$(\mathcal{T}, \mathcal{A}) \models F(a) \quad \Leftrightarrow \quad \mathcal{I}_{\mathcal{A}} \models F_{\mathcal{T}}(a)$$

$$(\mathcal{T}, \mathcal{A}) \models G(a) \quad \Leftrightarrow \quad \mathcal{I}_{\mathcal{A}} \models G_{\mathcal{T}}(a)$$

**Solution**. We can take

$$F_{\mathcal{T}}(x) = \mathsf{Player}(x) \vee (\exists y.\mathsf{plays}(x, y)) \vee \mathsf{Human}(x)$$

and

$$G_{\mathcal{T}}(x) = \mathsf{Player}(x) \vee (\exists y.\mathsf{plays}(x, y))$$

2. Query Rewriting. Let

$$\mathcal{T} = \{\exists \mathsf{has\_predecessor}.\mathsf{Number} \sqsubseteq \mathsf{Number}\}$$

and let

$$F(x) = \mathsf{Number}(x)$$

Does there exist an FOPL query $F_{\mathcal{T}}(x)$ such that for all ABoxes $\mathcal{A}$ and the corresponding interpretations $\mathcal{I}_{\mathcal{A}}$ the following holds for all individual names $a$:

$$(\mathcal{T}, \mathcal{A}) \models F(a) \quad \Leftrightarrow \quad \mathcal{I}_{\mathcal{A}} \models F_{\mathcal{T}}(a)$$

Give an informal explanation for your answer.

**Solution**. There does not exist such an FOPL query. Note that for the infinite disjunction

$$
\begin{aligned}
F^*_{\mathcal{T}}(x) \;=\; & \mathsf{Number}(x) \vee \\
& (\exists \mathsf{has\_predecessor}.\mathsf{Number})(x) \vee \\
& (\exists \mathsf{has\_predecessor}.\exists \mathsf{has\_predecessor}.\mathsf{Number})(x) \vee \\
& \ldots
\end{aligned}
$$

we have for all ABoxes $\mathcal{A}$, the corresponding interpretations $\mathcal{I}_{\mathcal{A}}$, and all individual names $a$:

$$(\mathcal{T}, \mathcal{A}) \models F(a) \quad \Leftrightarrow \quad \mathcal{I}_{\mathcal{A}} \models F^*_{\mathcal{T}}(a)$$

However, such a disjunction cannot be expressed in FOPL.

3. Axiom Pinpointing. Let

$$\mathcal{T} = \{C \sqsubseteq D, A \sqsubseteq E, E \sqsubseteq \exists r.F, F \sqsubseteq B, H \sqsubseteq B, F \sqsubseteq H\}$$

be a TBox. Then we have that $\mathcal{T} \models A \sqsubseteq \exists r.B$. Determine two sets of axioms that are contained in the pinpointing set $\mathbf{Pin}(\mathcal{T}, A \sqsubseteq \exists r.B)$.

**Solution**. $\mathbf{Pin}(\mathcal{T}, A \sqsubseteq \exists r.B)$ contains

$$\{A \sqsubseteq E, E \sqsubseteq \exists r.F, F \sqsubseteq B\}$$

and

$$\{A \sqsubseteq E, E \sqsubseteq \exists r.F, H \sqsubseteq B, F \sqsubseteq H\}$$