Mesh connected networks.
Sorting algorithms

Efficient Parallel Algorithms
COMP308

Any nontrivial problem has no NC-algorithm on the mesh, since the diameter is too large. The problem of size $n = k^2$ can be solved in time at least $2k-1$, since this is the number of steps for communication between opposite corners.

For example we cannot compute on a mesh minimum of $n$ numbers in polylogarithmic time since the communication between opposite corners of the mesh takes more time than that.

Sorting on the 1-dimensional mesh

Algorithms on meshes can be considered "systolic" algorithms, systolic means very regular and well synchronized.

Sorting on the 1-dimensional mesh $k$ phases,
- in the first phase compare-exchange $(1,2), (3,4), (5,6), \ldots$
- in the second phase compare-exchange $(2,3), (4,5), (6,7), \ldots$ etc.
Sorting on 2-dim mesh

- The first attempt to sort 2-dim mesh is to sort all rows simultaneously then sort all columns simultaneously.
- However, the whole table is not sorted in the sense that elements of lower rows are all larger than elements of upper rows.

Shearsort Algorithm

**Shearsort of a k x k mesh**

- 2 \( \log_2 k \)+1 phases
- In odd-numbered phases, sort independently all rows.
- In even-numbered phases, sort independently the columns.

Correctness of Shearsort

- Applying 0-1 principle, consider only 0-1 sequences.
- Define the row to be dirty iff it contains at least one 0 and at least one 1, other rows are called clean.

Main property

- The number of dirty rows shrinks at least by half after applying two phases (row sort & column sort).
Correctness of Shearsort

Consider two adjacent dirty rows, assume w.l.o.g that the first is an odd row the second an even row after sorting these 2 rows according to their order we have the following 3 possibilities:

- Start sorting columns by first comparing-exchanging between of dirty rows decreases by half (one row per each pair) these 2 rows, then one of them becomes clean, so the number. Then these two rows will become (3 cases)

independently how we sort columns later the number of dirty rows does not increase, the final result of sorting columns does not depend of the way of sorting them, the result always the same so we could started with making exchanges between these 2 rows.