A Refined Resolution Calculus for CTL, $R_{\text{CTL}}$

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Computational Tree Logic (CTL)

- CTL extends propositional logic by modal operators $E\bigcirc$ and $EU$
- Other operators can be defined in terms of these, e.g.

$$A\bigcirc\varphi \equiv \neg E\bigcirc\neg\varphi$$
$$A\Box\varphi \equiv \neg E(true U \neg\varphi)$$
$$A(\varphi W \psi) \equiv \neg E(\neg\psi U (\neg\varphi \land \neg\psi))$$

- Informally, the semantics is given by tree structure on which formulae are interpreted as follows

- $\neg\varphi$ \hspace{1cm} $\varphi$ is false in a node
- $\varphi \land \psi$ \hspace{1cm} $\varphi$ and $\psi$ are true in a node
- $E\bigcirc\varphi$ \hspace{1cm} there exits a path such that $\varphi$ is true in the next node
- $E(\varphi U \psi)$ \hspace{1cm} there exists a path such that $\varphi$ is true until $\psi$ is true
Resolution Calculus

- Normal form transformation:

\[ A □ (∧_{i=1}^{n} l_i \Rightarrow E ○ (∨_{j=1}^{k} m_j)_{\langle ind \rangle} ) \]

E-step clause

- Resolution rules: \( R_{CTL} \) consists of step resolution rules (SRES1 to SRES8) and eventuality resolution rules (ERES1 and ERES2).

**SRES2**

\[
\begin{align*}
P &\Rightarrow E ○ (C ∨ l)_{\langle ind \rangle} \\
Q &\Rightarrow A ○ (D ∨ \neg l)
\end{align*}
\]

\[
\frac{P \land Q}{P \land Q \Rightarrow E ○ (C ∨ D)_{\langle ind \rangle}}
\]

**ERES1**

\[
\begin{align*}
\tilde{P} &\Rightarrow E ○ E □ l \\
Q &\Rightarrow A □ \neg l \\
Q &\Rightarrow A (\neg \tilde{P} \vee \neg l)
\end{align*}
\]

where \( ind \) is an index and \( l \) is a literal, \( P \) and \( Q \) are conjunctions of literals, \( C \) and \( D \) are disjunctions of literals, and \( \tilde{P} \) is a disjunction of conjunctions of literals.
While there are many calculi/methods for CTL theorem proving, there are almost no implementation systems

- **Reduce implementation effort by re-using an FOL resolution theorem prover**
  - Requires representation of CTL normal form in FOL
  - Step resolution inferences can be emulated by ordered first-order resolution
  - Eventuality resolution inferences require a so-called loop search algorithm to be added to the FOL resolution theorem prover
\( R_{\text{CTL}} \) is based on a previous calculus by Bolotov (2000).

- **New normal form for CTL,** \( \text{SNF}_{\text{CTL}}^g \)
  - Formal semantics of \( \text{SNF}_{\text{CTL}}^g \)
    - Nicer proofs
  - Fewer clauses generated by normal form transformation and resolution
  - More amenable to implementation
  - The only side effect is worked around by our first-order implementation

- **Complexity of** \( R_{\text{CTL}} \) **is** EXPTIME

- **New completeness proof**
  - Fewer eventuality resolution rules required

- **Implementation via** FOL
Thank You

Questions?