Combinatorial Properties of the Stable Marriage Problem

The Stable Marriage Problem is a classical combinatorial and algorithmic problem. The problem was first formally stated in 1962 by David Gale and Lloyd Shapley[1], although hospitals had been using similar procedures to determine intern assignments before this time.

Simply stated, consider a set of n women and n men. Each woman has her own strict preference rating for the men, and similarly each man has one for the women. A *marriage* is a pairing of the men and women into n pairs (where each man and women are used once, of course in the pairings). The marriage is called *stable* if there does not exist a woman W and a man M such that W prefers M over her current partner and M prefers W over his current partner (else W and M would drop their partners and pair up themselves).

It is well-known that every set of preference lists admits at least one stable marriage, and there are efficient algorithms to find one. There are many variants of this problem such as to the case of roommates [2] (i.e. pairings need not be male/female, in which case it is not necessarily the case that a stable assignment exists), matching three sets (such as men, women, and dogs), allowing preference lists to have ties, etc. In these matchings, issues such as "fairness" also arise, such as not favoring one sex over another in determining who is "happiest" in the pairings. (The classical marriage algorithm described by Gale and Shapley is inherently biased in that one of the two sexes, namely the ones doing the proposing, ends up happier than the other, i.e. the "proposers" have their best possible partners over all possible stable marriages.)

The focus of this project is to first implement some related algorithms for these matching/marriage problems, and use them to explore the combinatorial properties of the structures that arise from them.

Questions we may explore include: What is the average rank of partners in the male-optimal or female-optimal marriages (over a large set of marriages with random preference lists)? What is the average rank or partners in the *egalitarian matching* (which satisfies some well-defined property)?

We are also interested in the set of "rotations" (which described how stable marriages are related to each other, i.e. swap a set of partners in a well-defined manner to move from one marriage to another); how many rotations are there in an "average" marriage instance? In general, finding the set of rotations can be performed in polynomial time. While we may use the collection of rotations to generate all stable marriages, it is interesting to note that counting the *number* of stable marriages is a #P-complete problem. [3]

Viable candidates for this project will have a firm understanding of the some high-level programming language (such as C++, Java, or PHP) and data structures. Appropriate reference material will be provided.

References

[1] D. Gale and L.S. Shapley. College admissions and the stability of marriage. *The American Mathematical Monthly* **69** (1962), pp. 9-15.

[2] R.W. Irving. An efficient algorithm for the "Stable Roommates" problem. *J. Algorithms* **6** (1985), pp. 577–595.

[3] R. Irving and P. Leather. The compelxity of counting stable marriages. *SIAM J. Computing* **15** (1986), pp. 655–667.

[4] D.E. Knuth. Stable Marriage and its Relation to Other Combinatorial Problems (English version). American Mathematical Society, Providence, RI, 1997.