Centralised Connectivity-Preserving Transformations by Rotation: 3 Musketeers for all Orthogonal Convex Shapes

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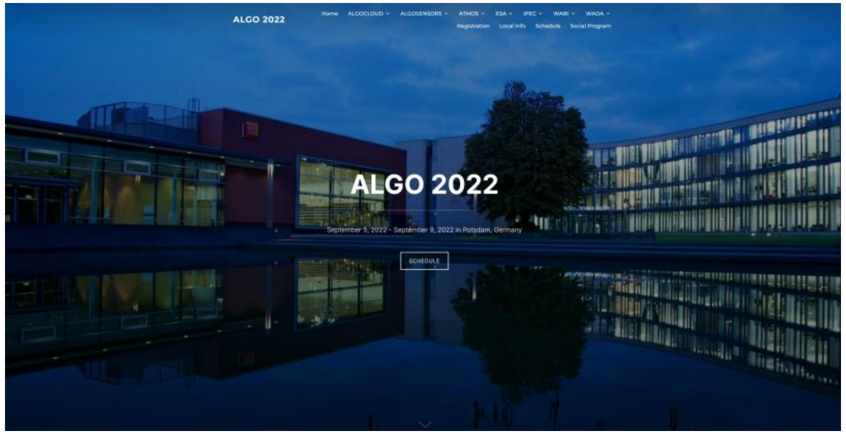
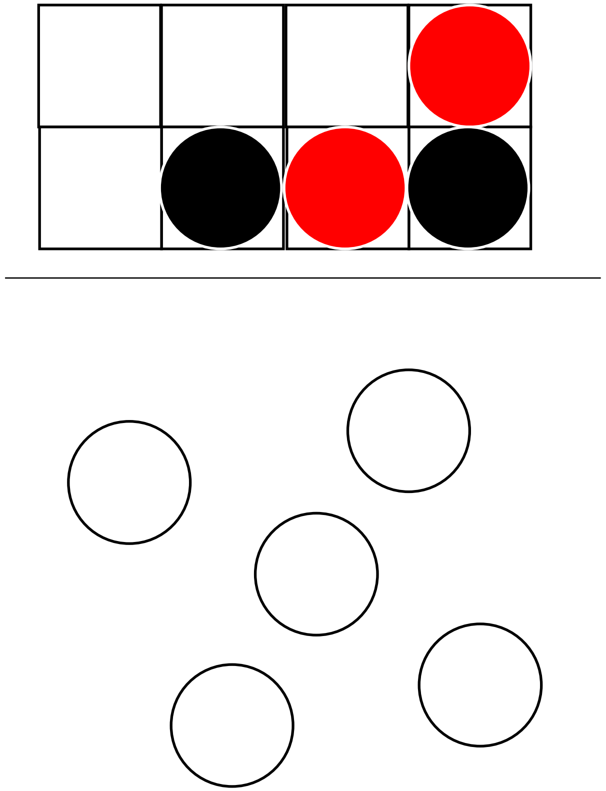
UNIVERSITY OF LIVERPOOL



Programmable matter systems



Multi-Agent systems

Decentralised

Weak

Centralised variant - feasibility

Overview

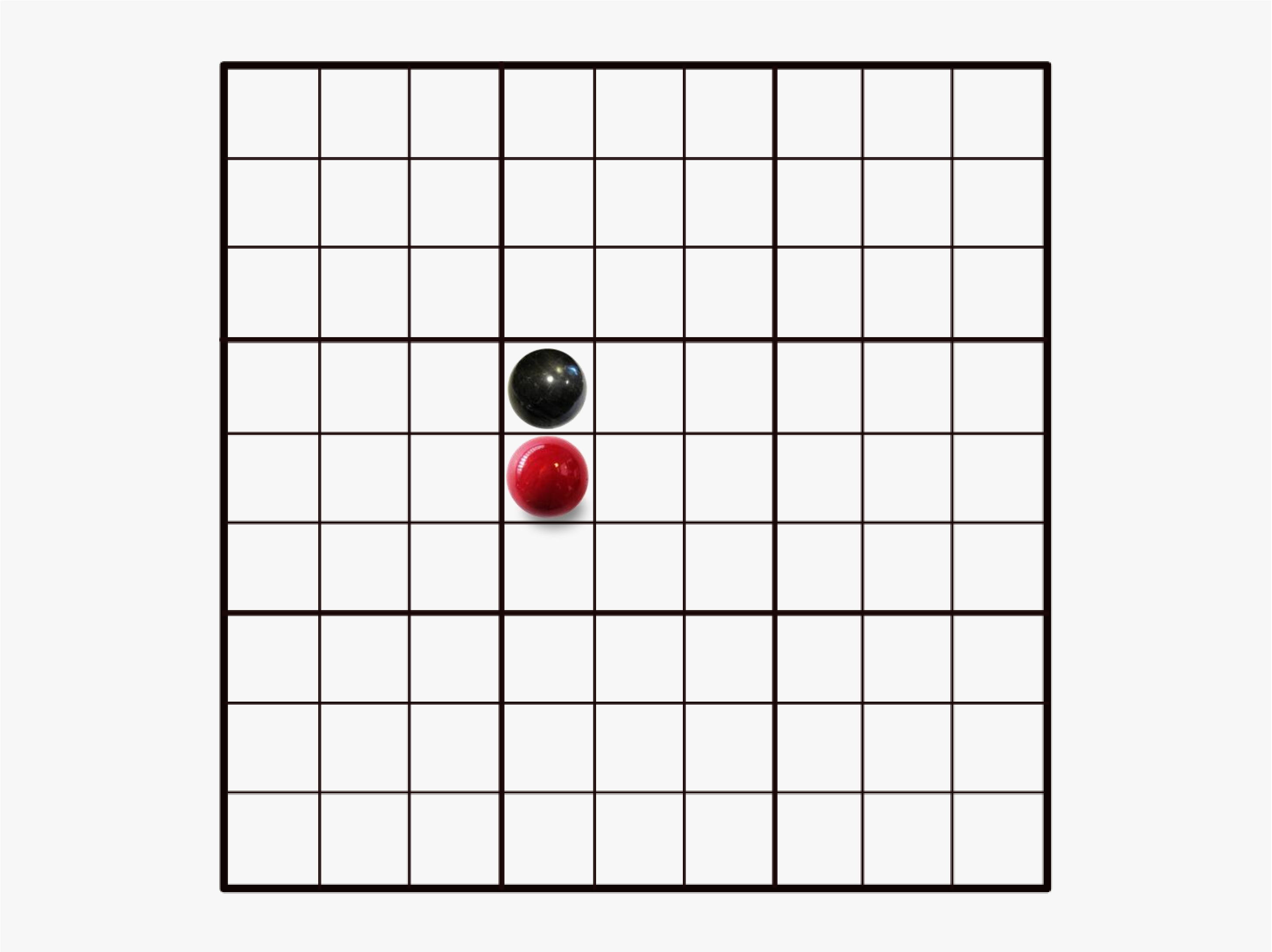
- Model and problem definition
- Orthogonal convex shapes
- 6/7-Robot movement
- Transformation

Model and Problem definitions

- Set S of n agents in the shape A occupying cells in a 2D grid
- Transformation of A into shape B in time t by a series of configurations $C_0 \dots C_t$ each reachable by a single move from a single node
- Centralised model to explore feasibility
- Rotation – movement of one node 90° around another node
- Rot-Transformability – Rotation only
- RotC-Transformability – Rotation only, connectivity must be preserved

Rotation Only

Rotation



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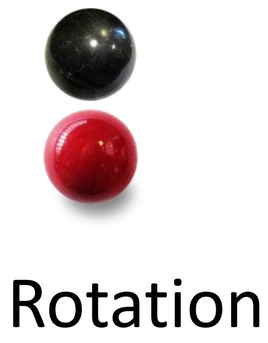

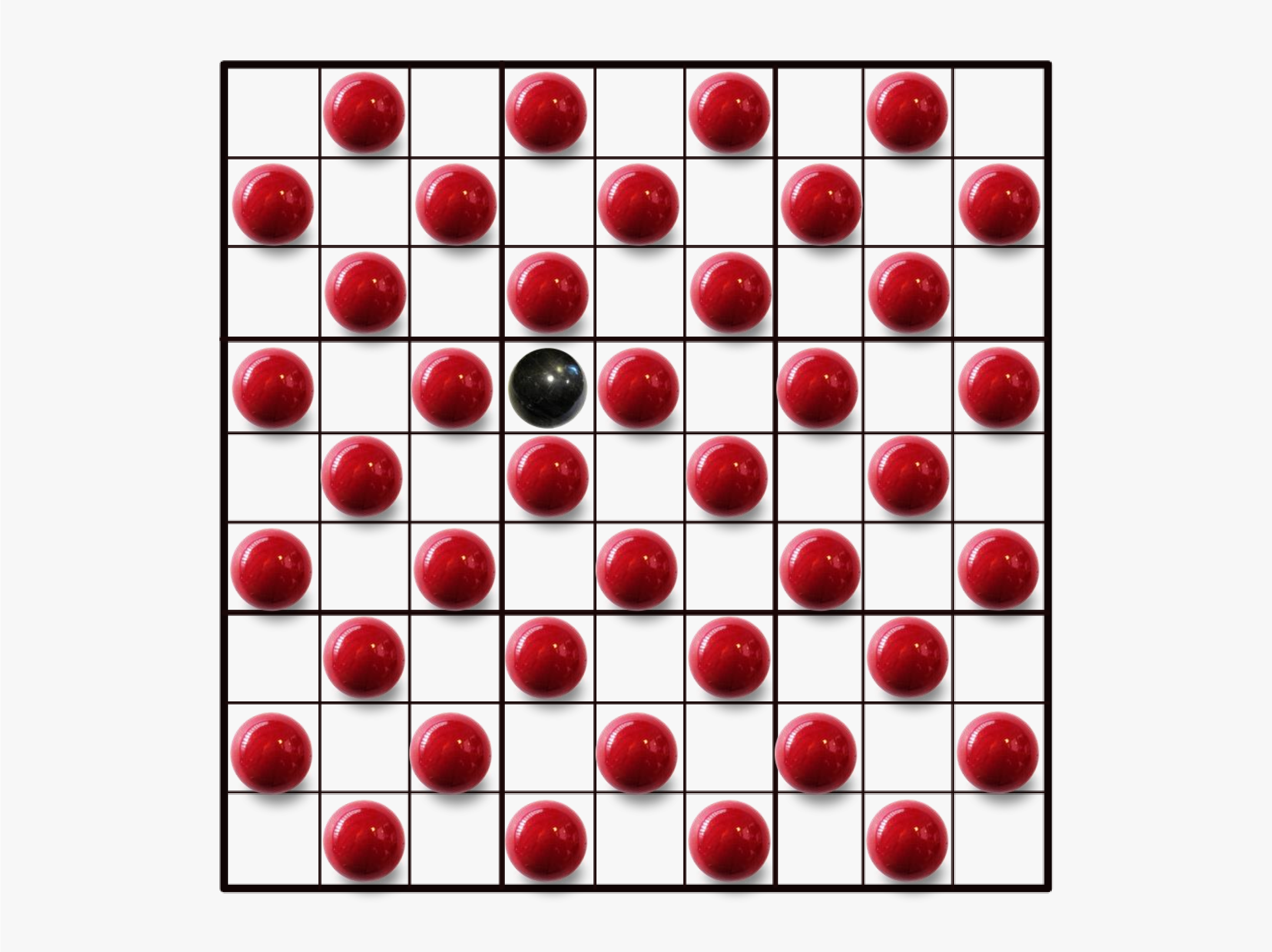

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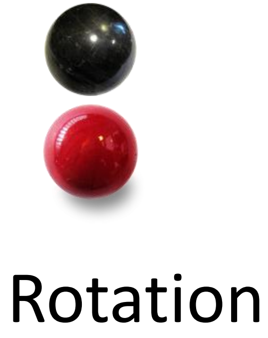
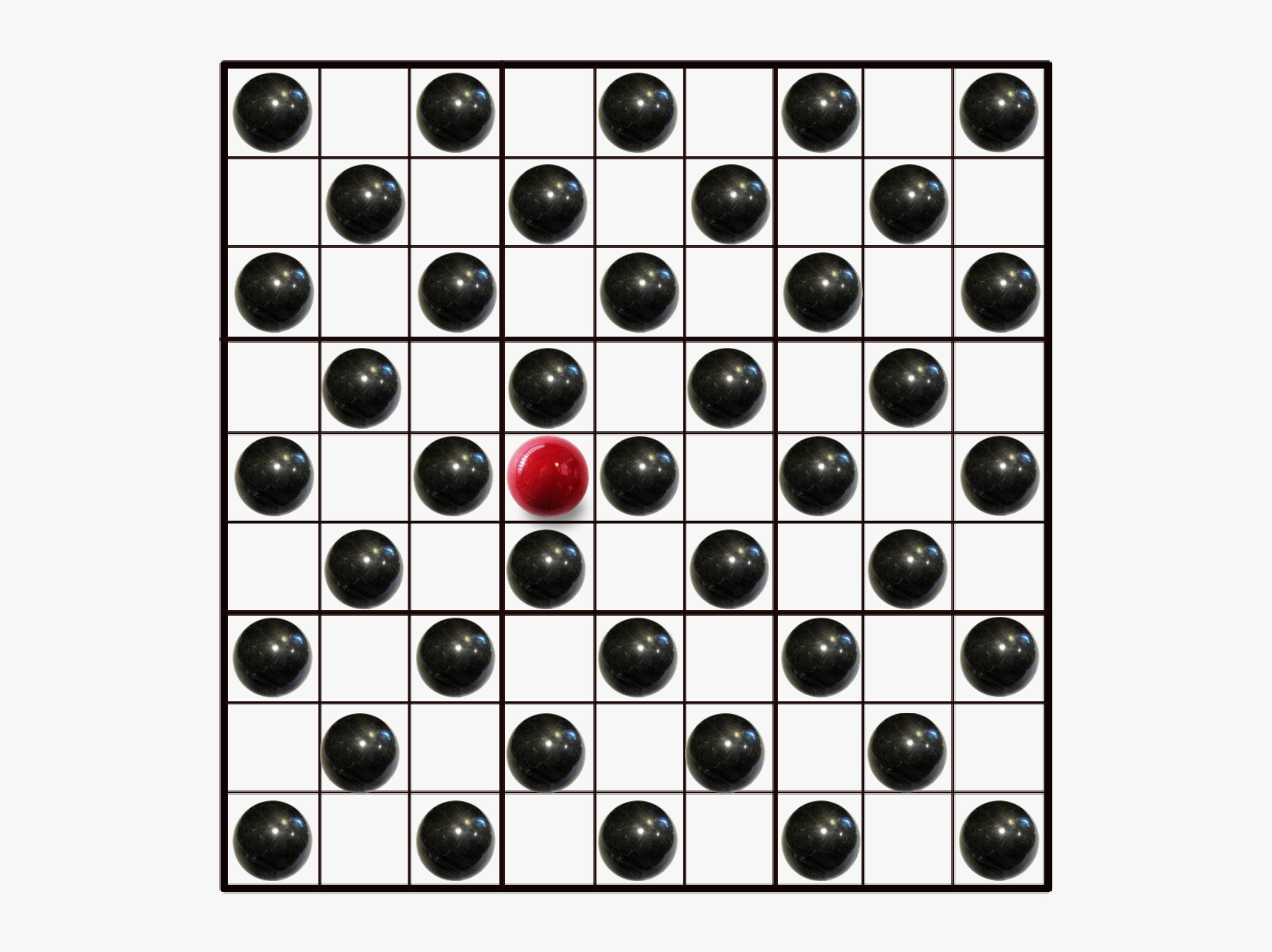
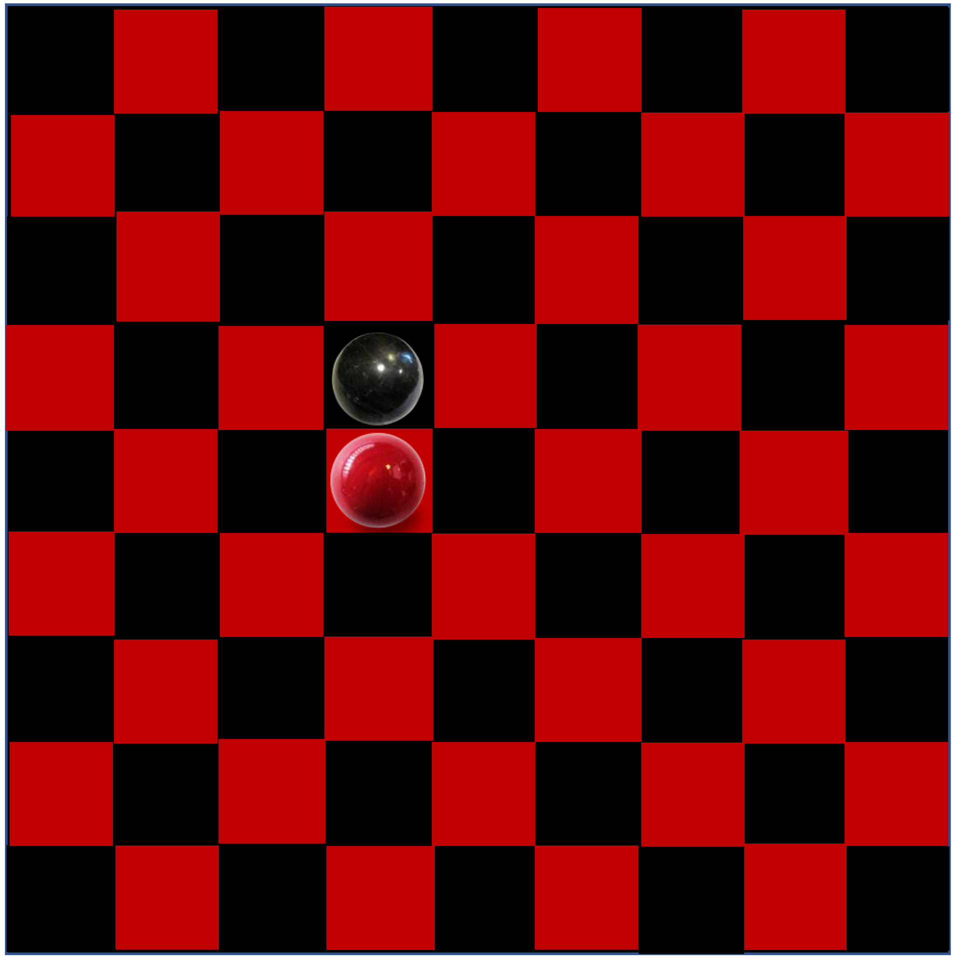




Rotation Only

Rotation



- Our focus – transforming orthogonal convex shapes into each other in the RotC setting
- Rotation only – simple operations are easier to implement in real-world systems, colour restrictions in square/triangular grids technically interesting
- Connectivity – programmable matter, not swarm robotics
- Certain orthogonal convex shapes cannot meaningfully transform in RotC
- Therefore transformations are aided by **seeds** – nodes placed in empty cells neighbouring a shape to create a new shape to aid transformation
- We discard the seed at the end

Related Work

- Various programmable matter models developed e.g. [Dumitrescu, Pach, Symposium on Computational Geometry, 2004]
- Programmable materials developed e.g. [Rothemund, Nature, 2006]

Recent papers on the concept of seed-assisted transformations in programmable matter

- Universal transformation for Rot-Transformability for all unblocked shapes, introduction of RotC-Transformability and seeds, line folding with seeds, impossibility of 5-node line traversal and orthogonal convex idea [Michail *et al.*, JCSS, 2019]
- Any pair of color-consistent *nice shapes* [Almethen, Michail, Potapov, TCS, 2020] *A*, *B* in O(n^2) moves with a 4-seed [C, Michail, Potapov, ALGOSENSORS 2021] – not directly comparable with orthogonal convex
- Universal transformation with connectivity preservation using “leapfrog” and “monkey” movement and a 5-node seed [Akitaya *et al.*, Algorithmica, 2021]

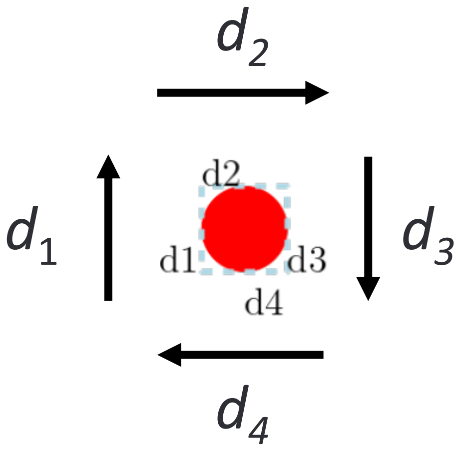
Proposition. A shape S is a connected orthogonal convex shape iff its perimeter satisfies both the following properties:

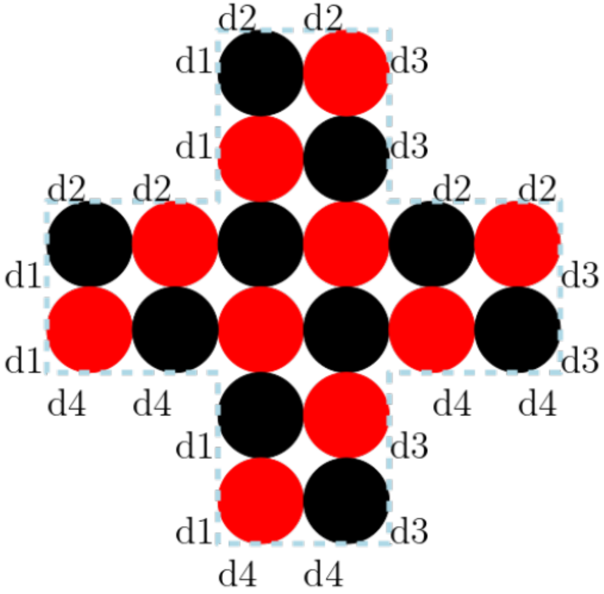
- It is described by the regular expression

$$d_1(d_1 \mid d_2)^* d_2(d_2 \mid d_3)^* d_3(d_3 \mid d_4)^* d_4(d_4 \mid d_1)^*$$

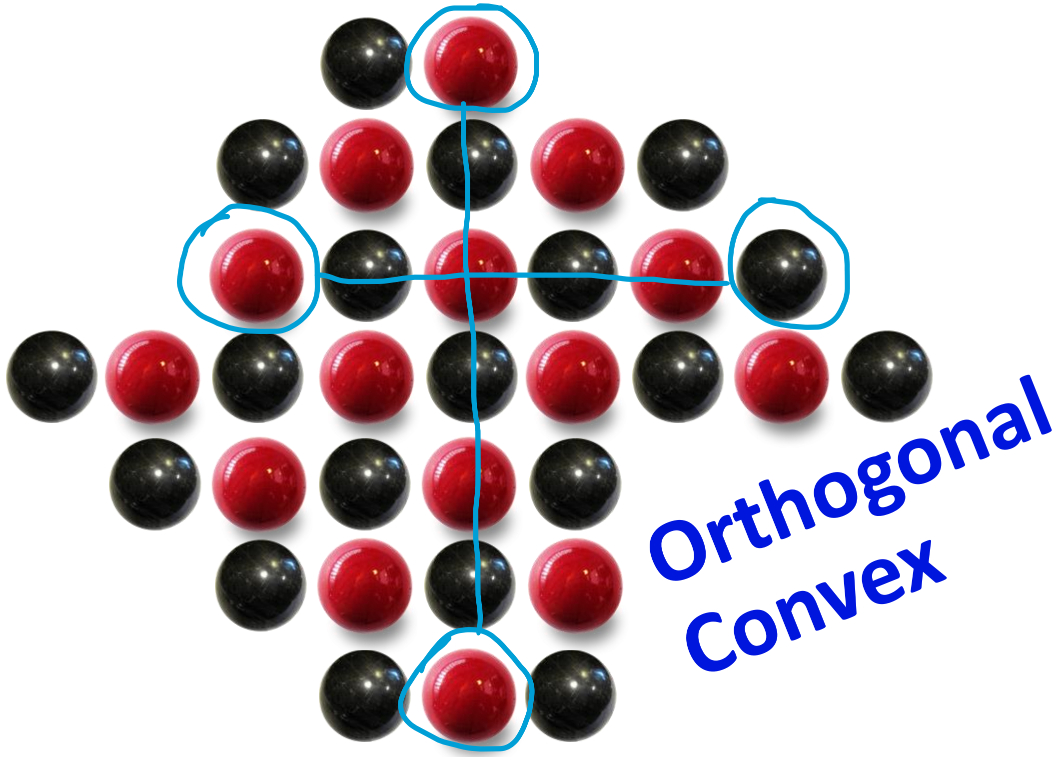
under the additional constraint that $N_1 = N_3$ and $N_2 = N_4$.

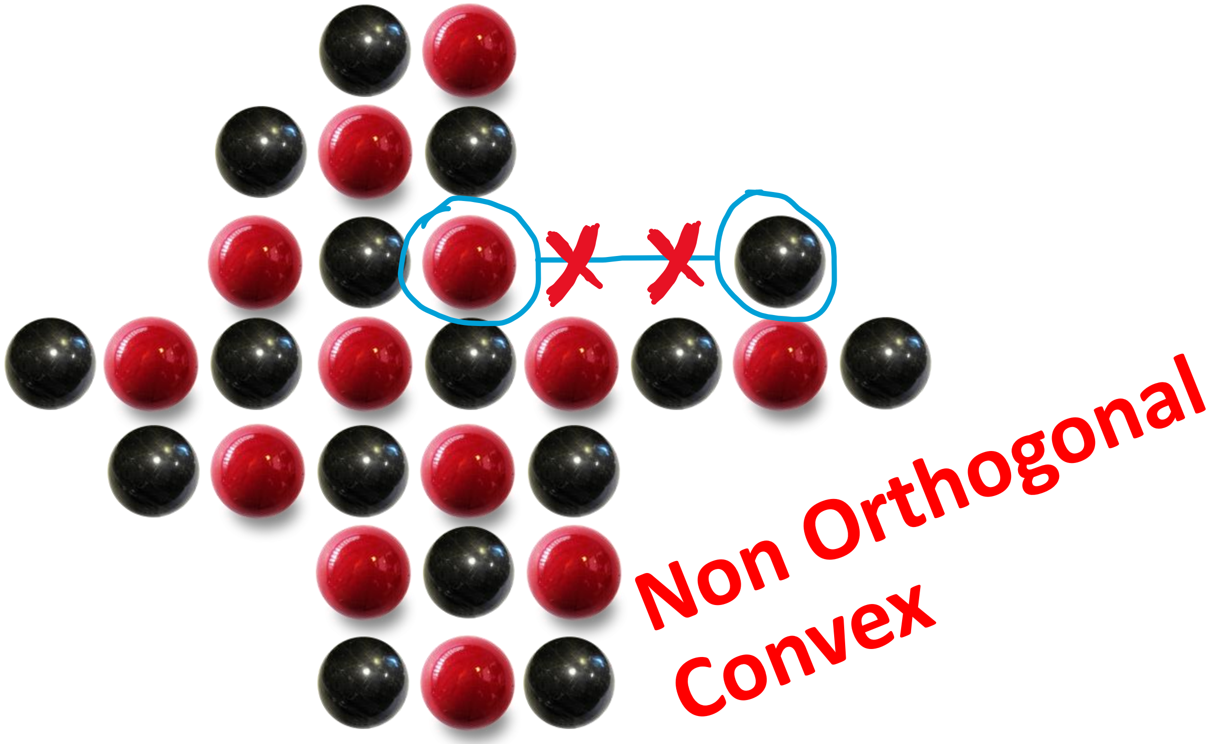
- Its interior has no empty cell.



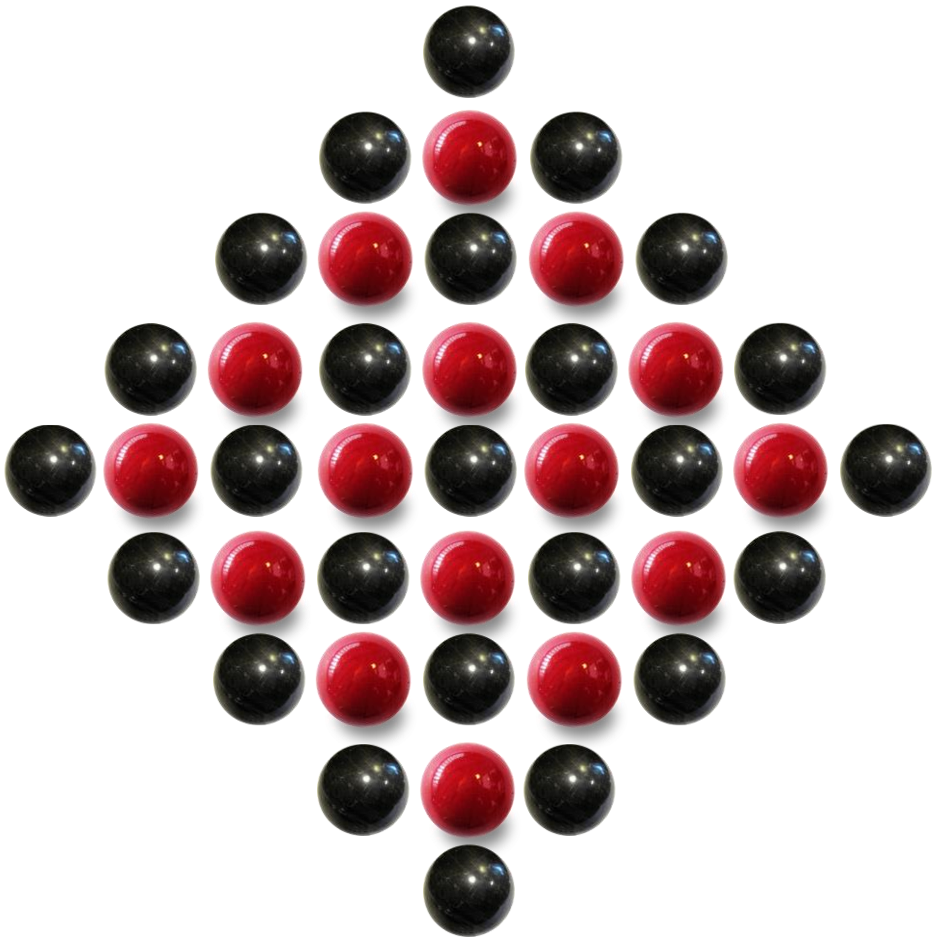


Orthogonal convex shapes





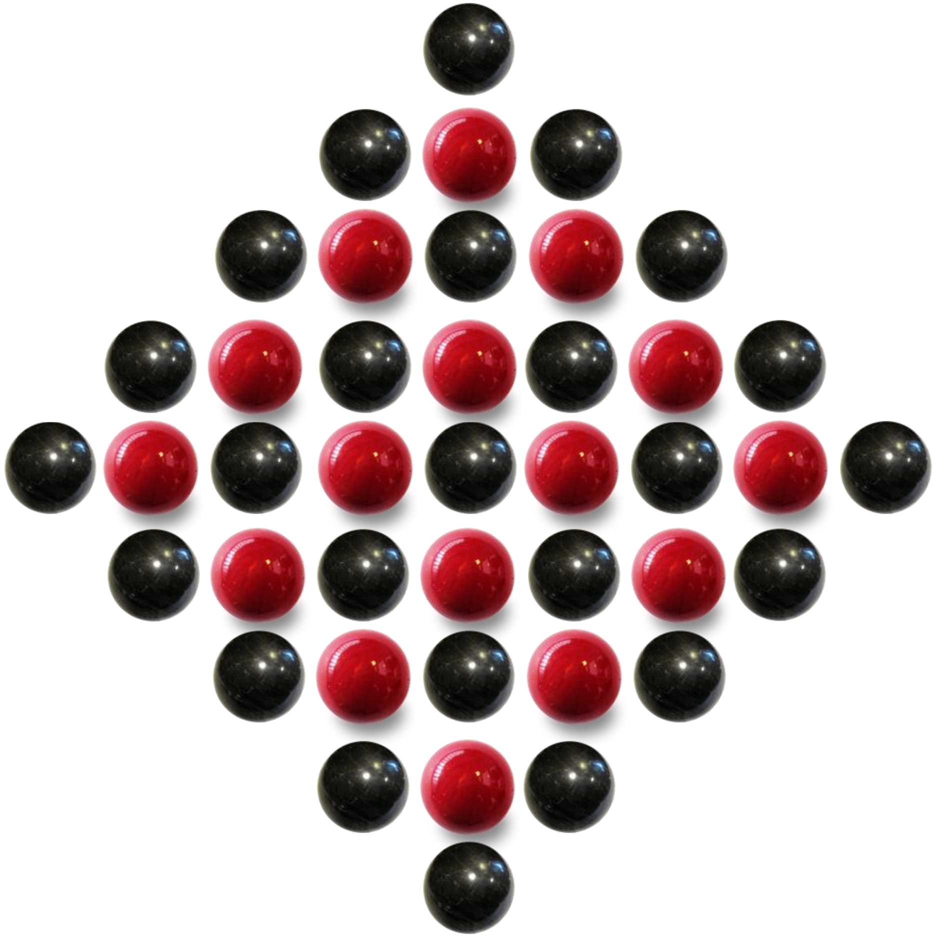
Rotation-only: Blocked Shapes



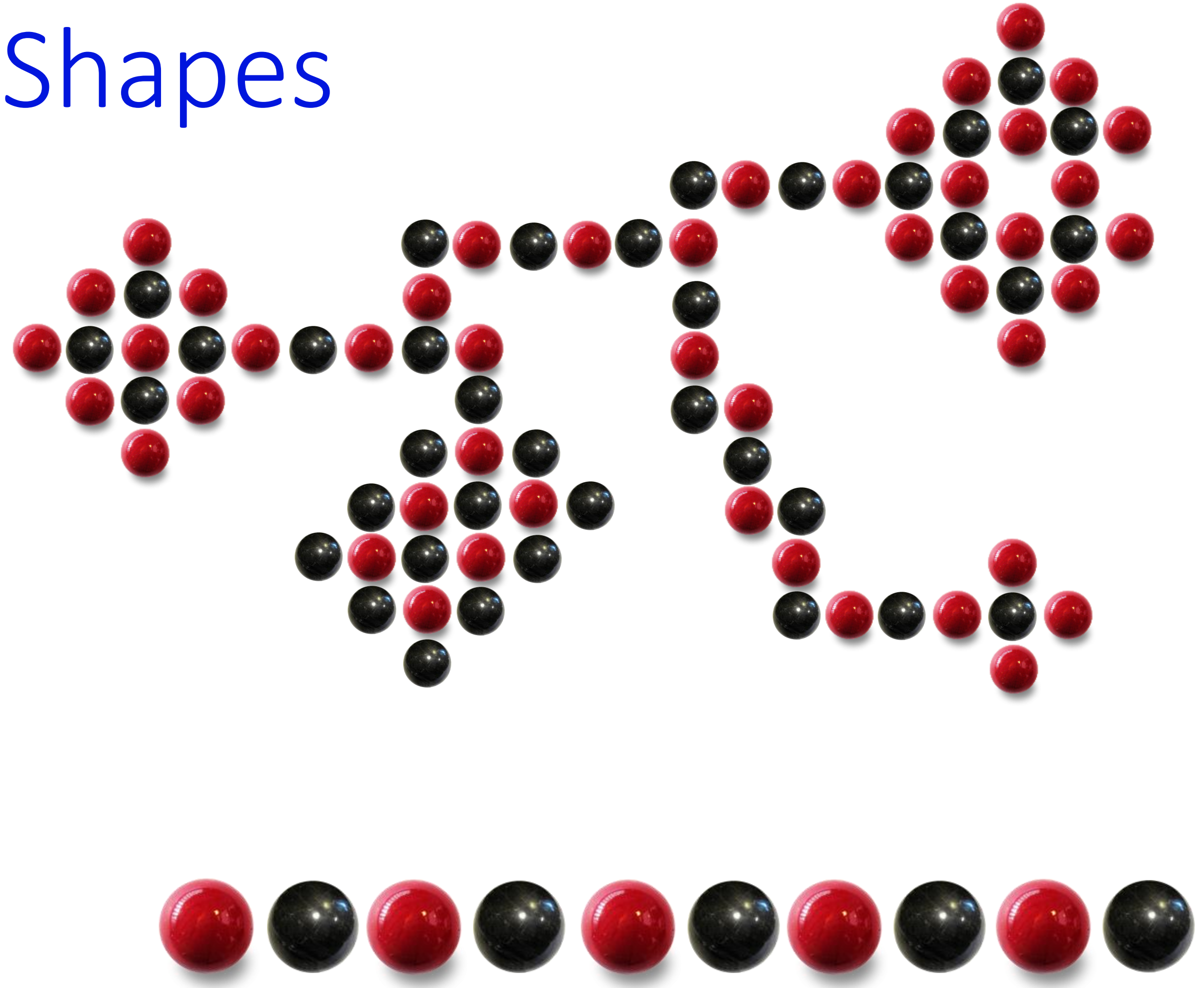
No move: 0-blocked (or blocked)

If connectivity must be preserved: *k*-blocked

Rotation-only: Blocked Shapes

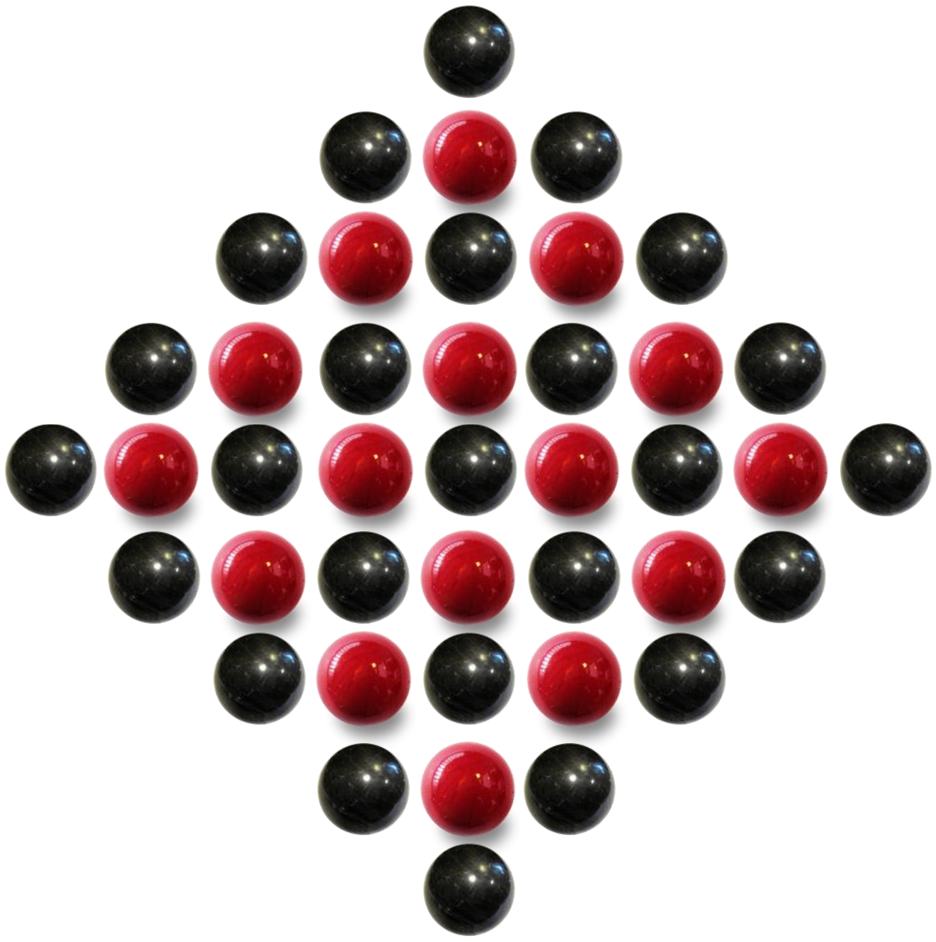


No move: 0-blocked

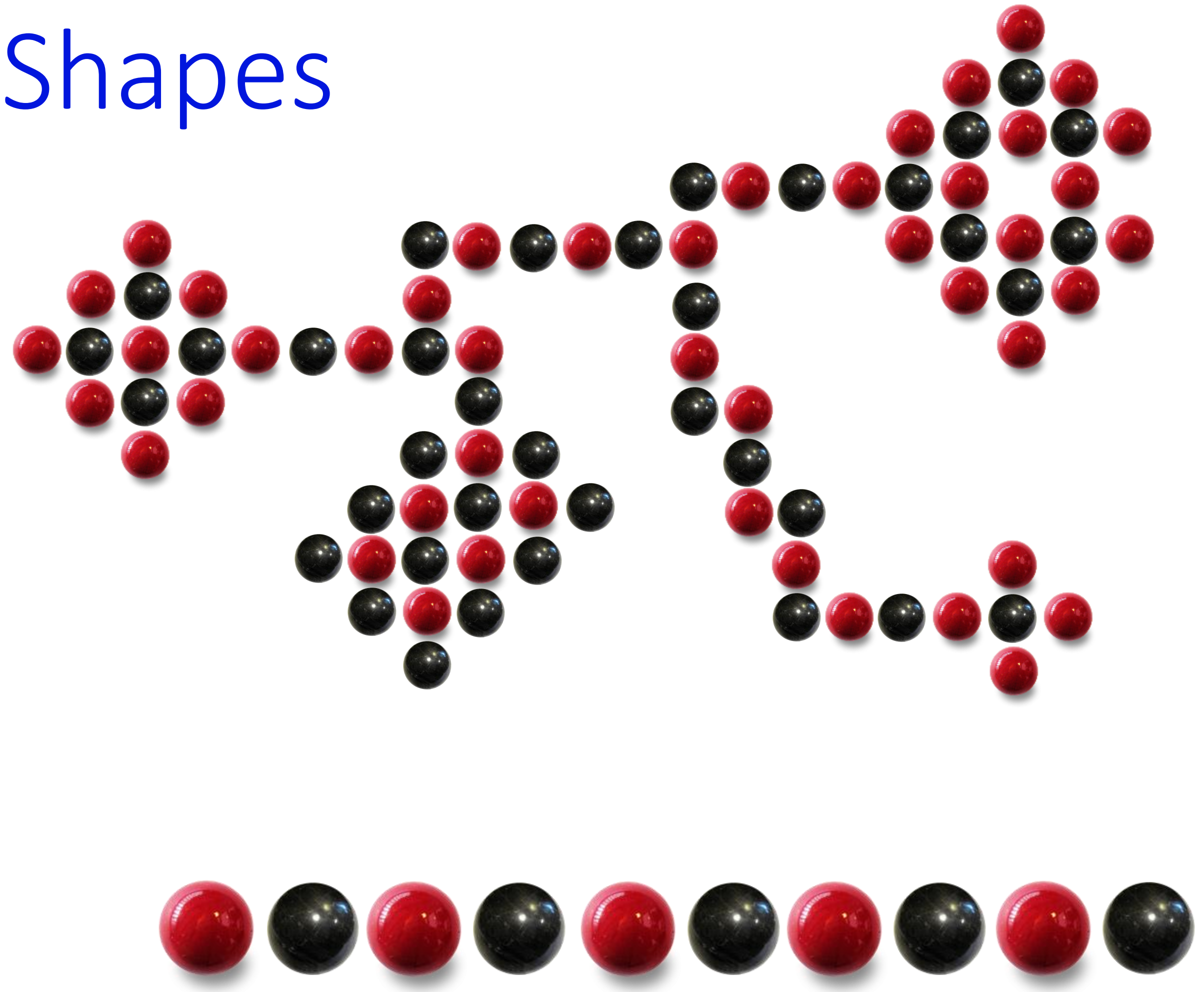


If connectivity must be preserved: *k*-blocked

Rotation-only: Blocked Shapes

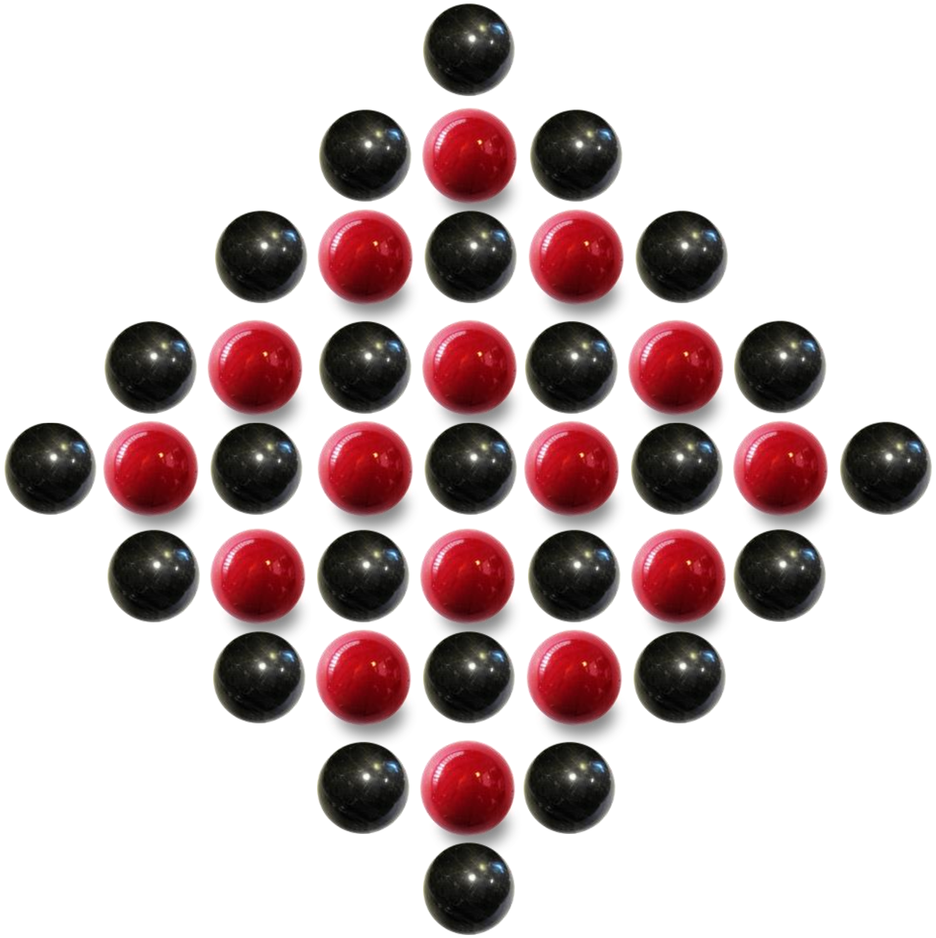


No move: 0-blocked

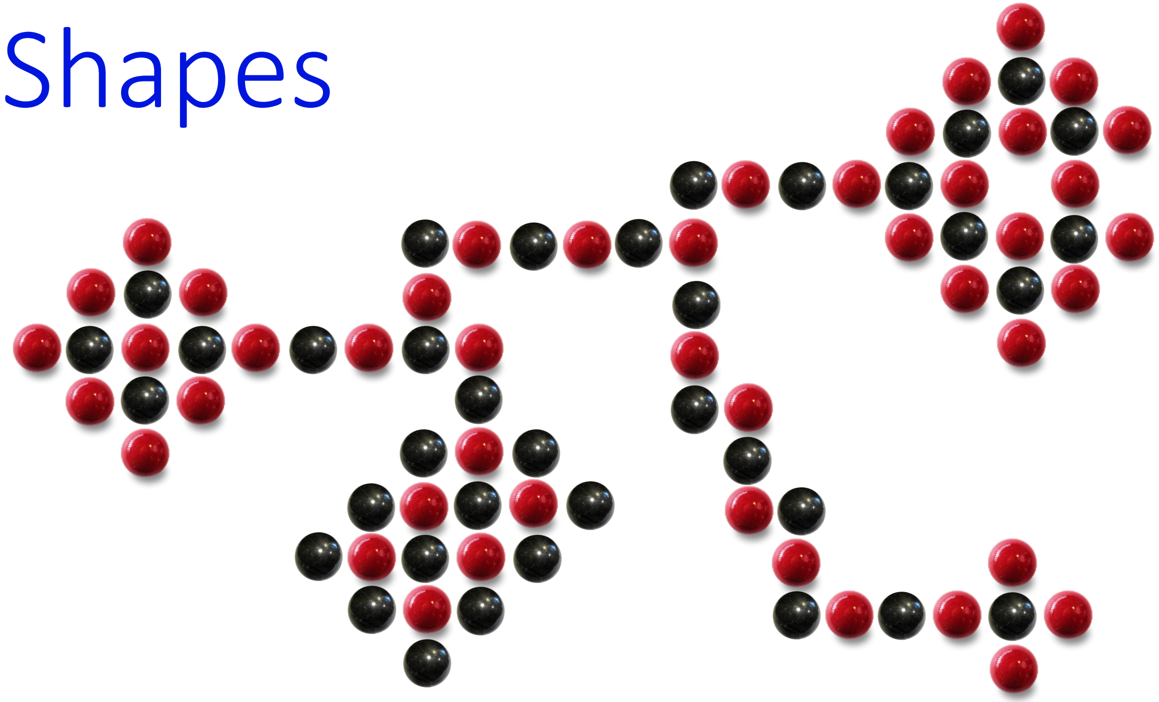


If connectivity must be preserved: *k*-blocked

Rotation-only: Blocked Shapes



No move: 0-blocked



If connectivity must be preserved: *k*-blocked

Main Results

- **Theorem** 1. For any orthogonal convex shape S, a 6-robot is capable of traversing the perimeter of S.
- **Theorem** 2. For any orthogonal convex shape S, a 7-robot is capable of traversing the perimeter of S.
- **Theorem** 3. Let S and S' be connected colour-consistent orthogonal convex shapes. Then there is a connected shape M of 3 nodes (the 3 musketeers) and an attachment of M to the bottom-most row of S, such that $S \cup M$ can reach the configuration S' in $O(n^2)$ time steps.

6-robot movement

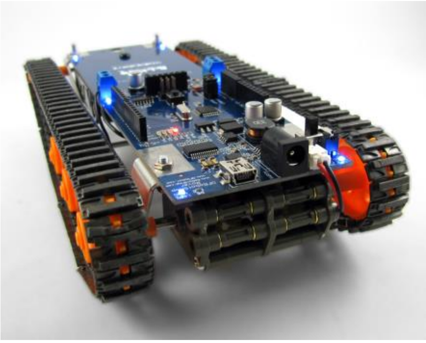
6 node block

Set C of corner cases depending on height and width

Invariant – robot in new location with same structure

Solve for one quadrant, the rest follow by rotation

6-robot



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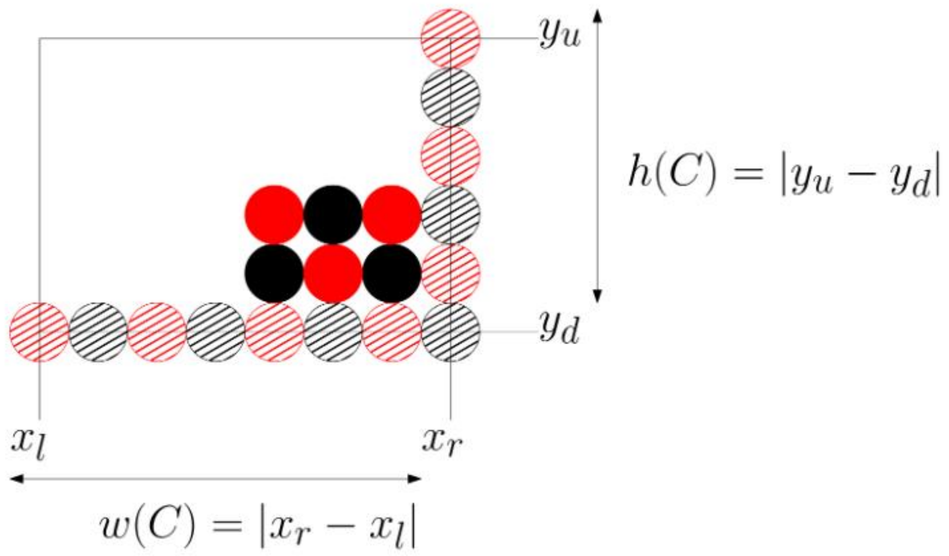
Cases



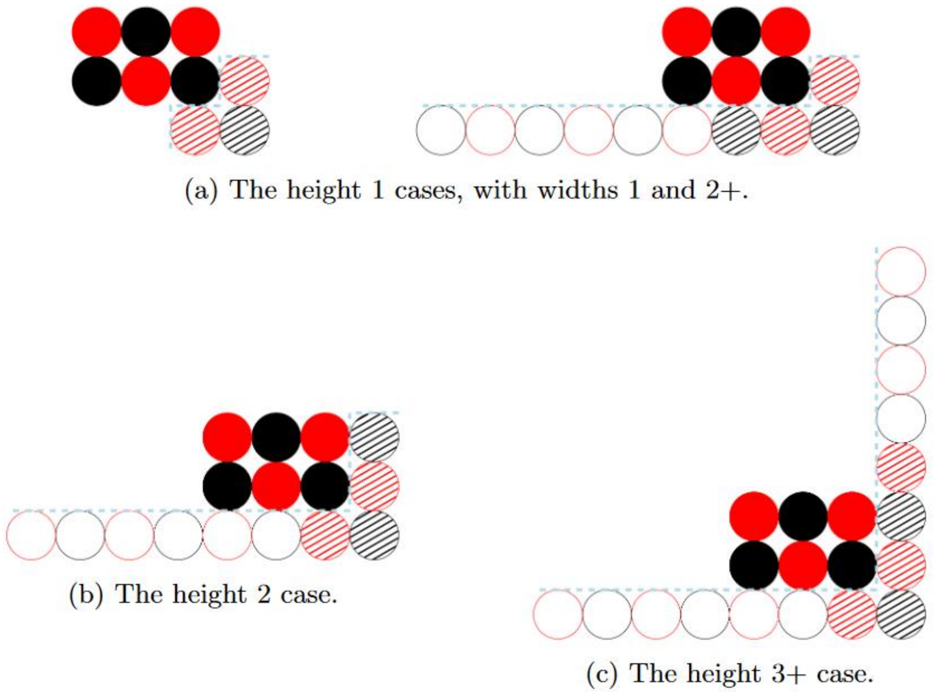
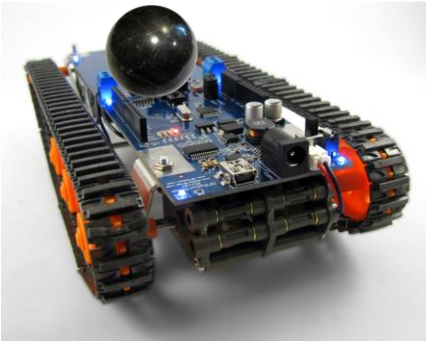


Fig. 8: The four basic corner scenarios of $\mathcal{C}$.

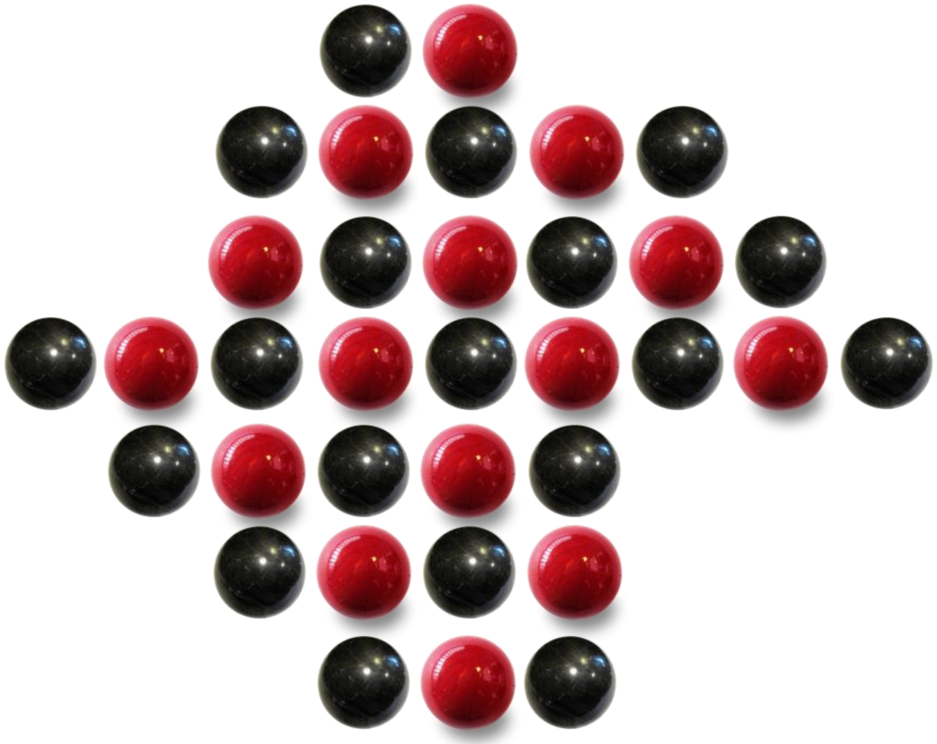
7-robot movement

- Represents a 6-robot carrying an extra node (the load)
- Mostly the same as 6-robot movement
- Key difference – two positions for the extra node
- Double the cases!

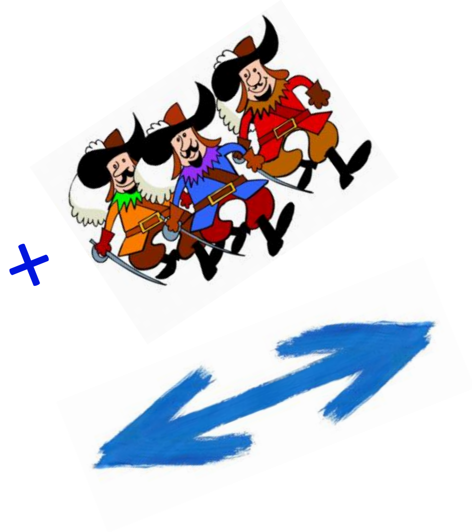
7-robot

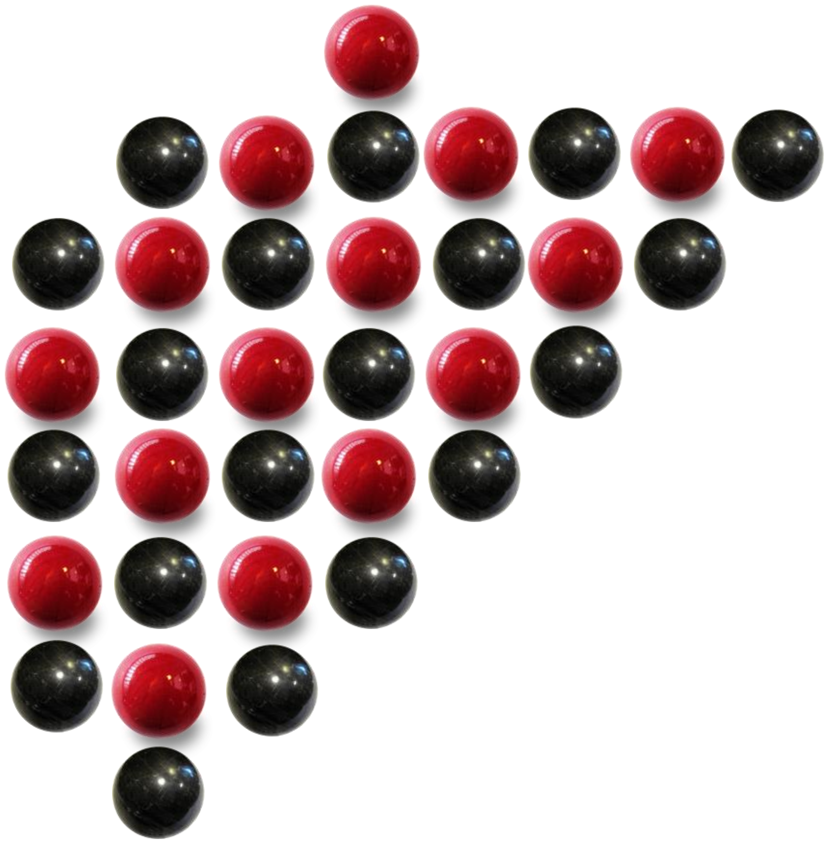


Any pair of color-consistent *orthogonal convex shapes A*, *B* in $O(n^2)$ moves with a 3-seed



Shape *A*

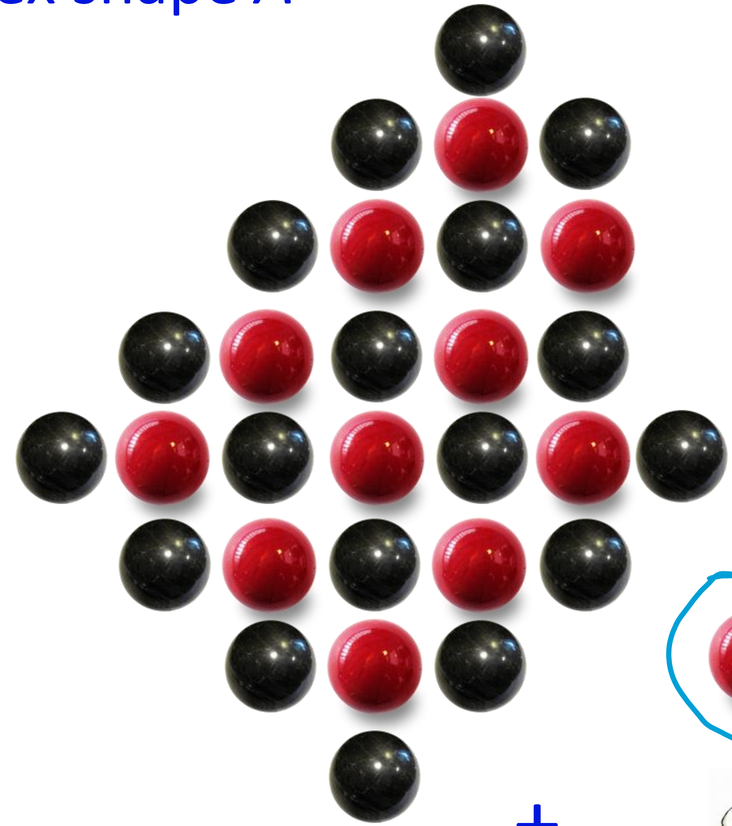




Shape *B*

3 musketeers to 6-robot

Orthogonal-convex shape *A*



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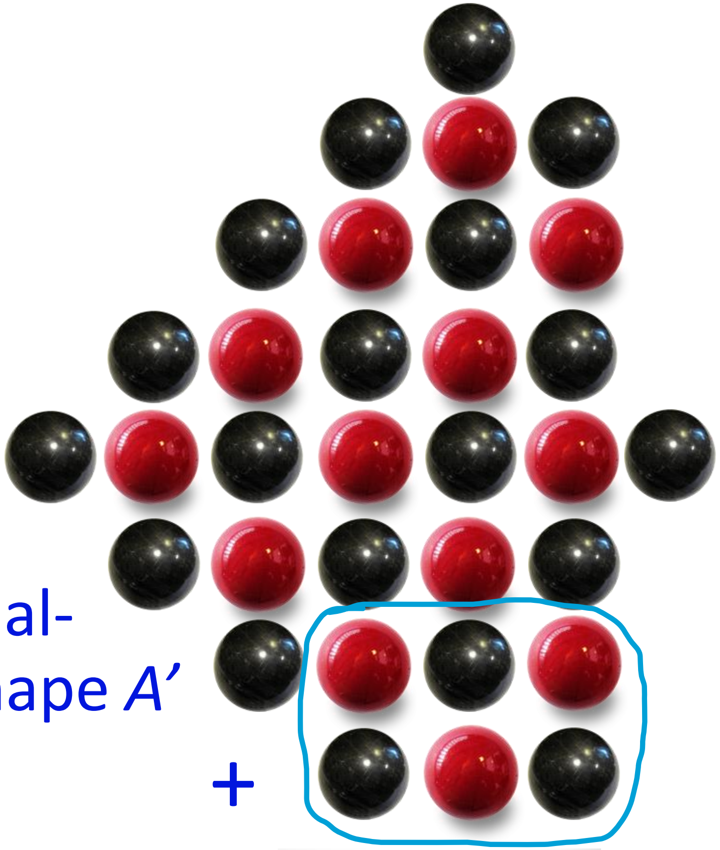

Orthogonal-convex shape *A'*

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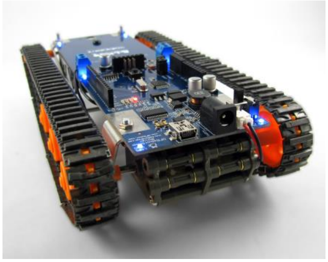
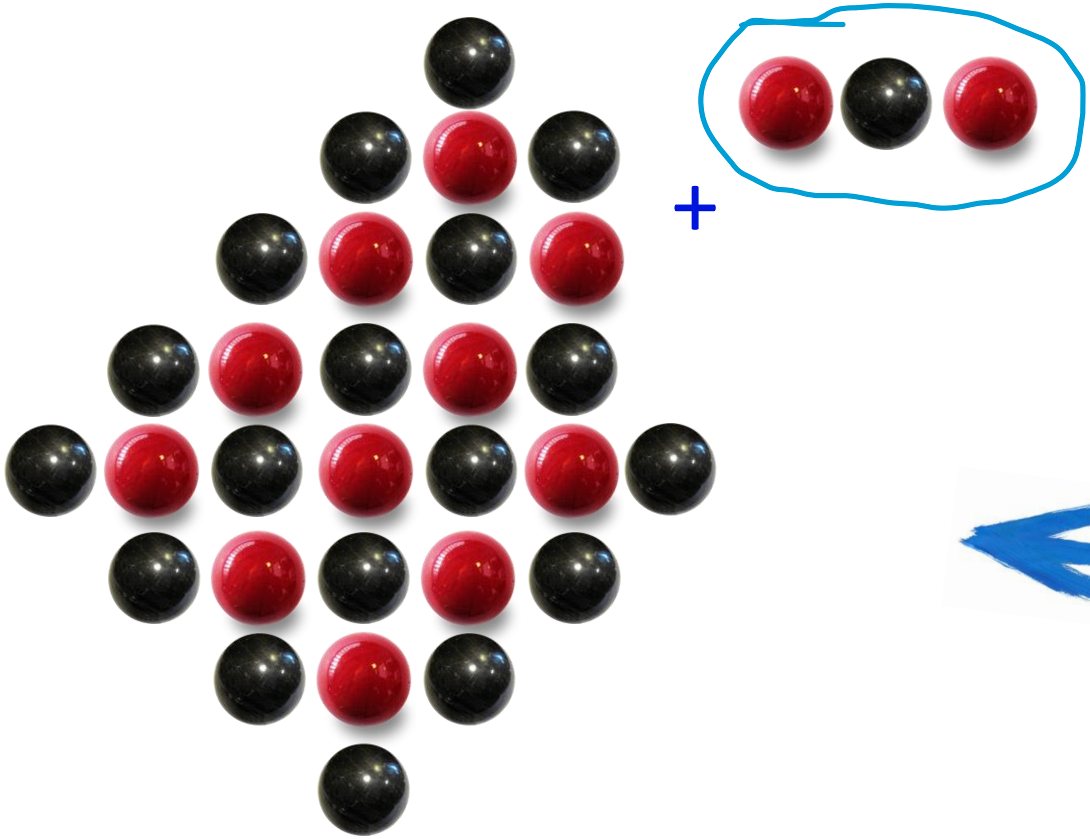


The transformation process

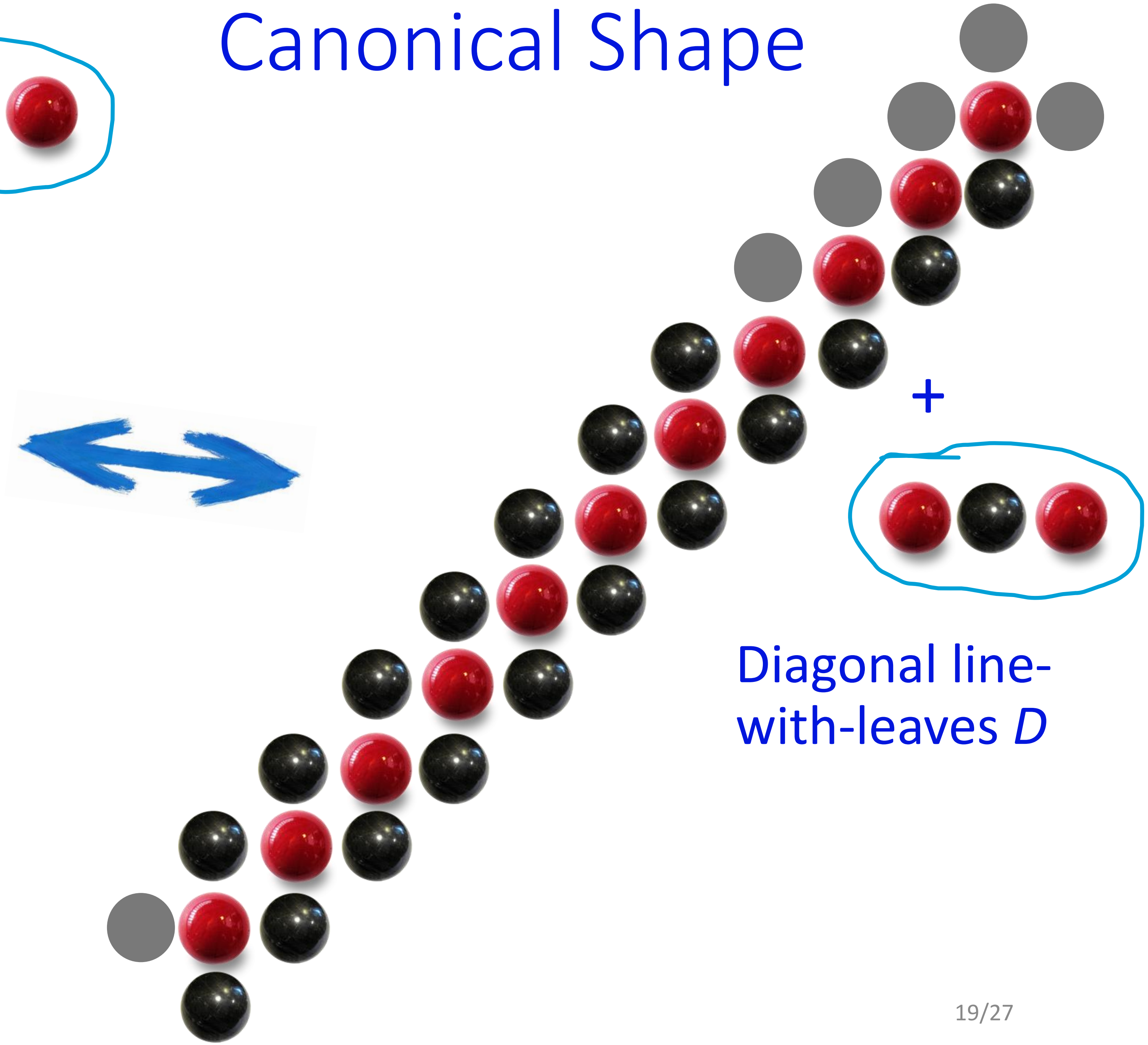
We now have the main structure of our transformation process:

- Add a 3-node seed to create a 6-robot
- Move the 6-robot around the shape
- Remove a node according to a shape elimination sequence
- Move the resulting 7-robot
- Place the node according to a shape generation sequence

Canonical Shape

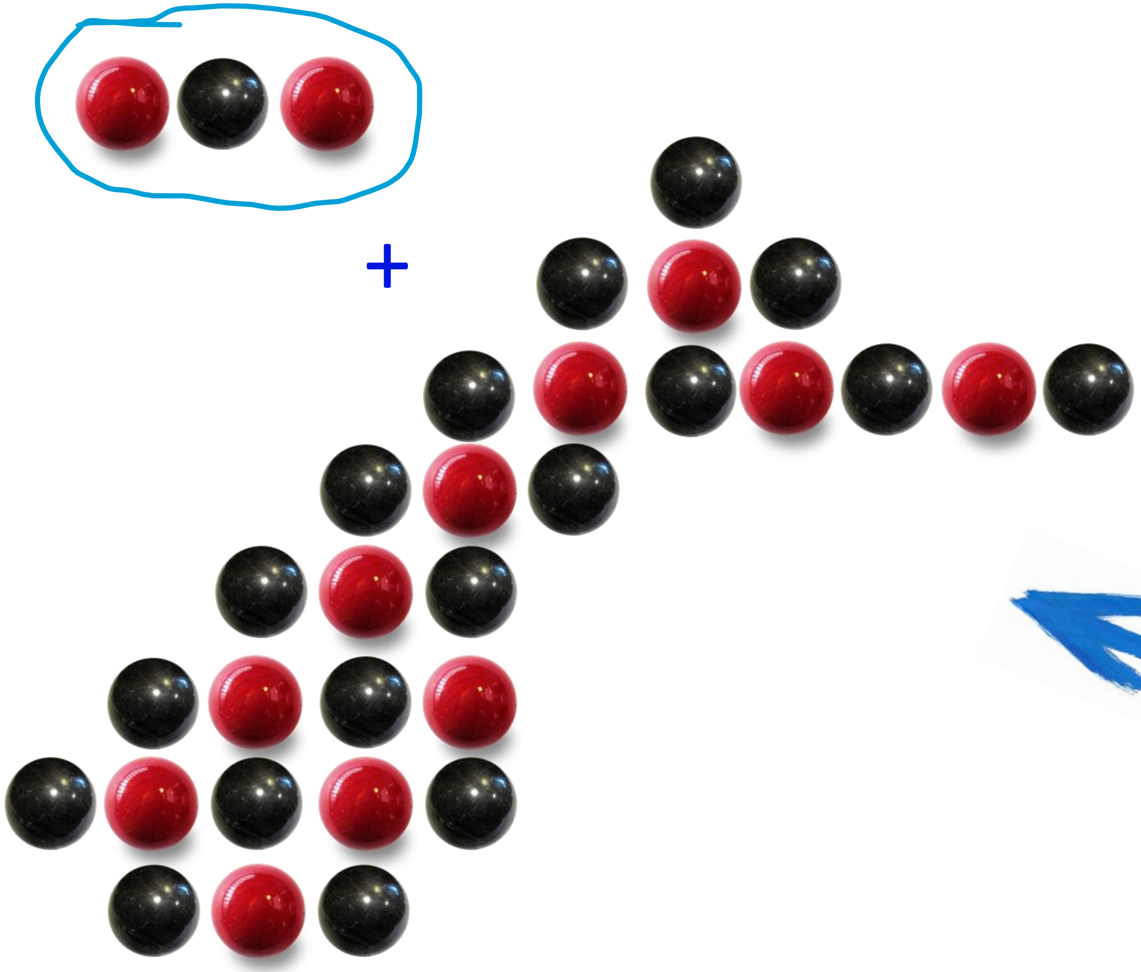


Shape A

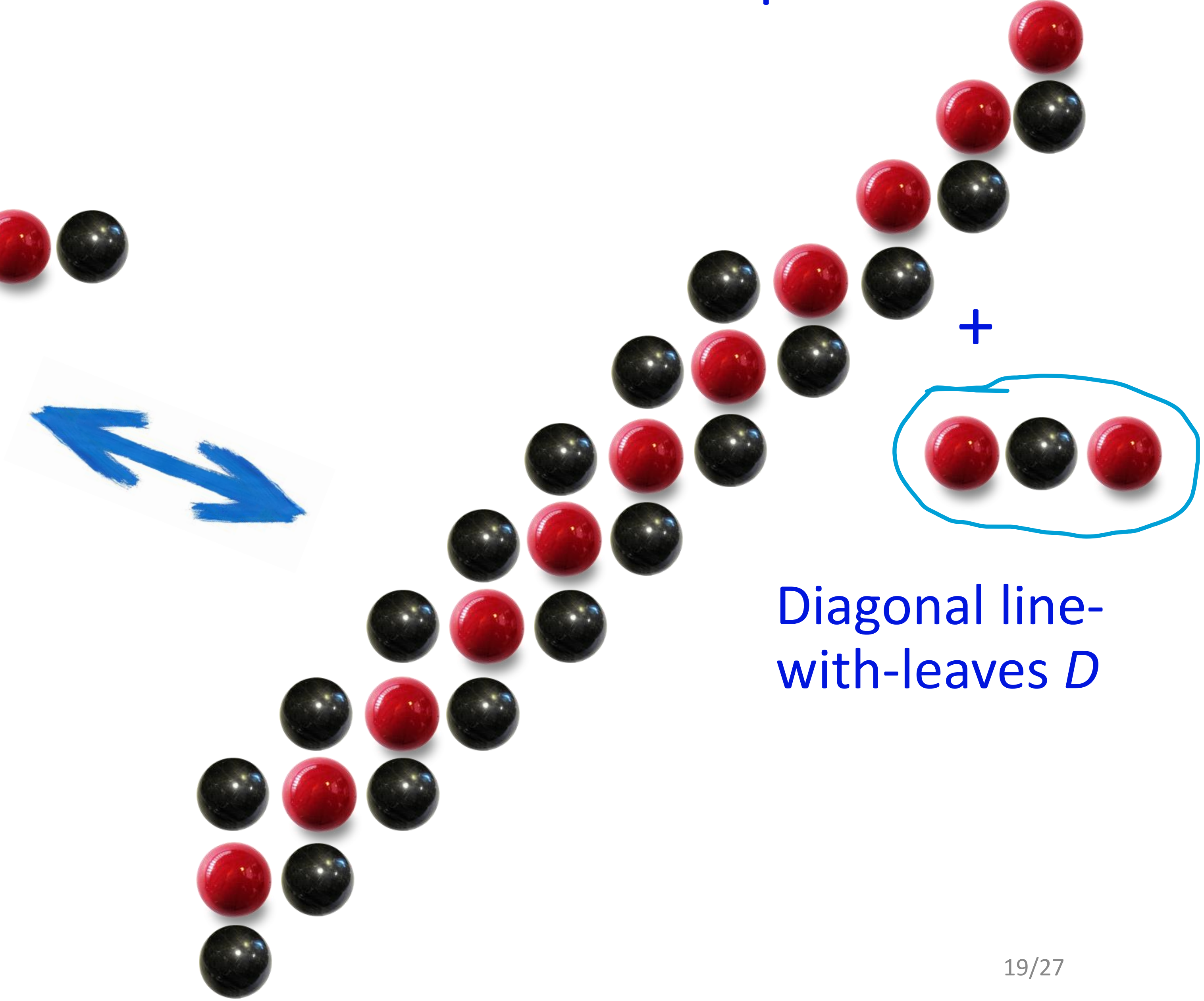


Diagonal line-with-leaves D

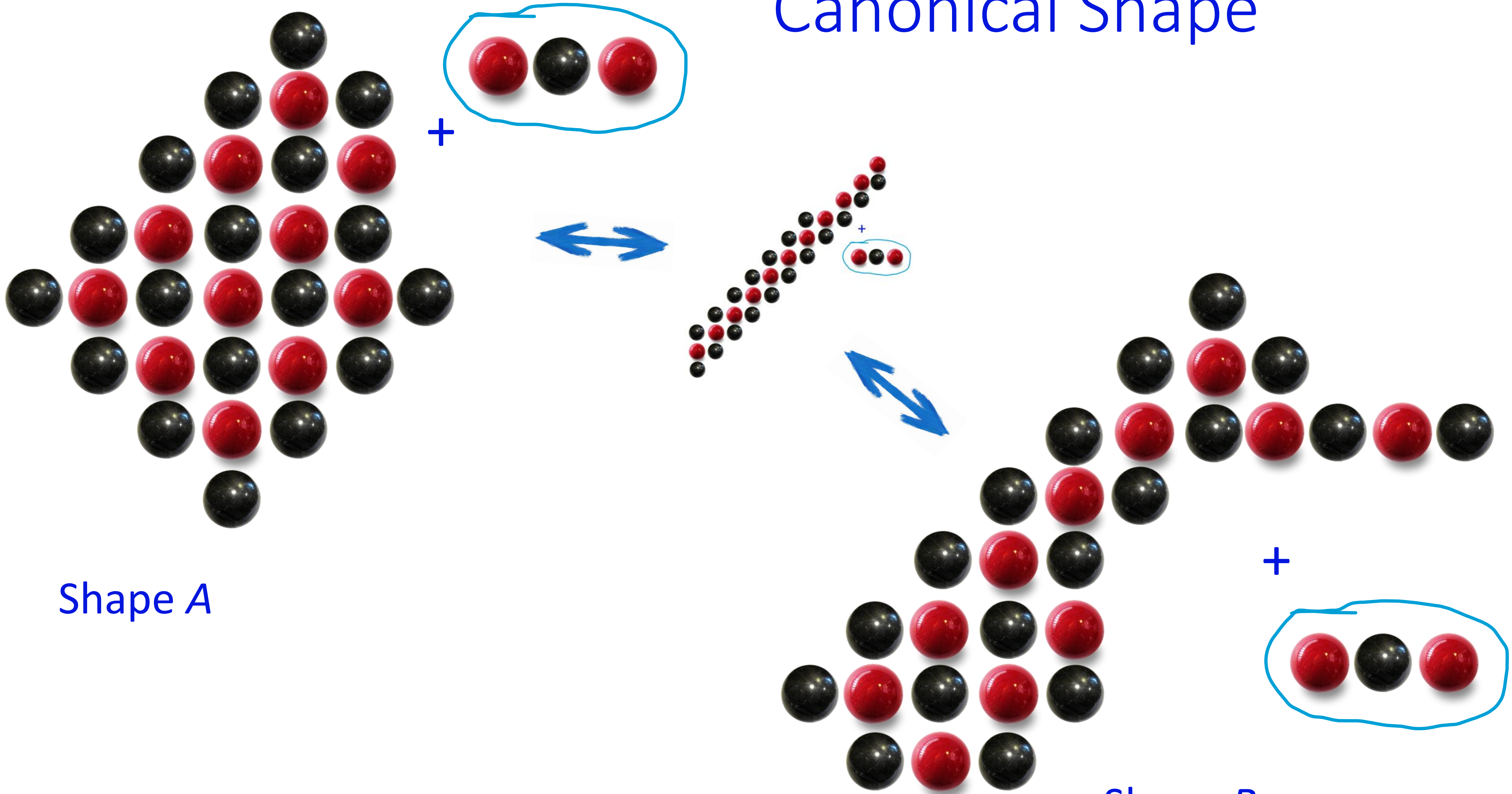
Canonical Shape



Shape *B*



Canonical Shape



Shape *A*

Shape *B*

Algorithm 2 OConvexToDLL(S, M)

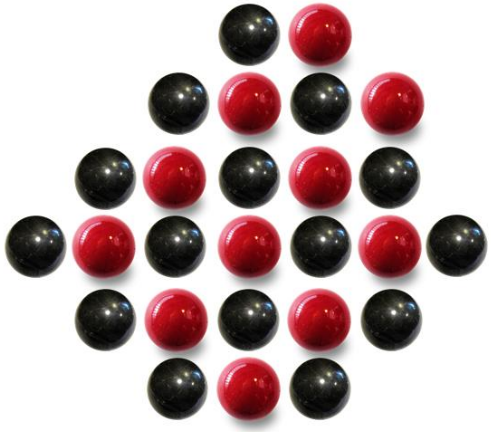
Input: shape $S \cup M$, where S is a connected orthogonal convex shape of n nodes and M is a 3-node seed on the cell perimeter of S, row elimination sequence $\sigma = (u_1, u_2, \ldots, u_n)$ of S, extended staircase generation sequence of $W \cup T = \sigma' = (u'_1, u'_2, \ldots, u'_n)$ which is colour-order preserving w.r.t. σ, shape elimination sequence $\sigma = (u_1, u_2, \ldots, u_{|T|})$ of T, shape generation sequence of $X = \sigma' = (u'_1, u'_2, \ldots, u'_{|T|})$ which is colour-order preserving w.r.t. σ

Output: shape $G = W \cup X \cup M$, where G is a diagonal line-with-leaves and M is a connected 3-node shape on the cell perimeter of S.

$R \leftarrow$ GenerateRobot(S, M)
$\sigma \leftarrow$ rowEliminationSequence(S)
$\sigma' \leftarrow$ ExtendedStaircase(σ)
$W \cup T \leftarrow$ OConvexToExtStaircase(S, R, σ, σ')
$\sigma \leftarrow$ repsEliminationSequence($W \cup T$)
$\sigma' \leftarrow$ stairExtensionSequence($W \cup T$)
$G \leftarrow$ ExtStaircaseToDLL($W \cup T, R, \sigma, \sigma'$)
TerminateRobot(G, R)

Proof Overview

Any orthogonal-convex shape



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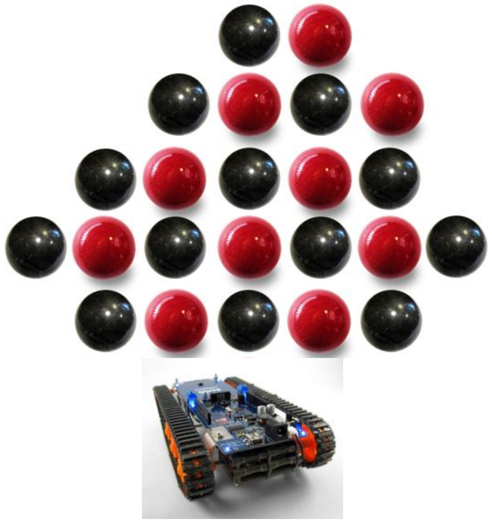
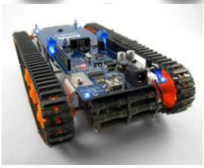


Lemma 15. *Let S be a connected orthogonal convex shape. Then there is a connected shape M of 3 nodes (the 3 musketeers) and an attachment of M to the bottom-most row of S, such that $S \cup M$ can reach a configuration $S' \cup M'$ satisfying the following properties. $S' = S \setminus \{u_1, u_2, u_3\}$, where $\{u_1, u_2, u_3\}$ is the 3-prefix of a row elimination sequence σ of S starting from the bottom-most row of S. M' is a 6-robot on the perimeter of S'.*

Orthogonal-convex shape + 6-robot

Proof Overview



Lemma 2. *Every connected orthogonal convex shape S has a row (and column) elimination sequence σ.*

Lemma 3. *Let σ be a bicoloured sequence of nodes that fulfills all the following conditions:*

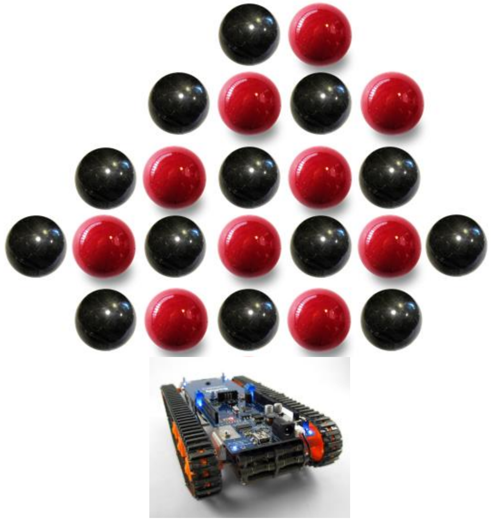
- *The set of the first two nodes in σ is not single-coloured.*
- *The third node of σ is black.*
- *σ does not contain a single-coloured 3-sub-sequence.*

Then there is an extended staircase generation sequence $\sigma' = (u'_1, u'_2, \ldots, u'_n)$ which is colour-order preserving with respect to σ.

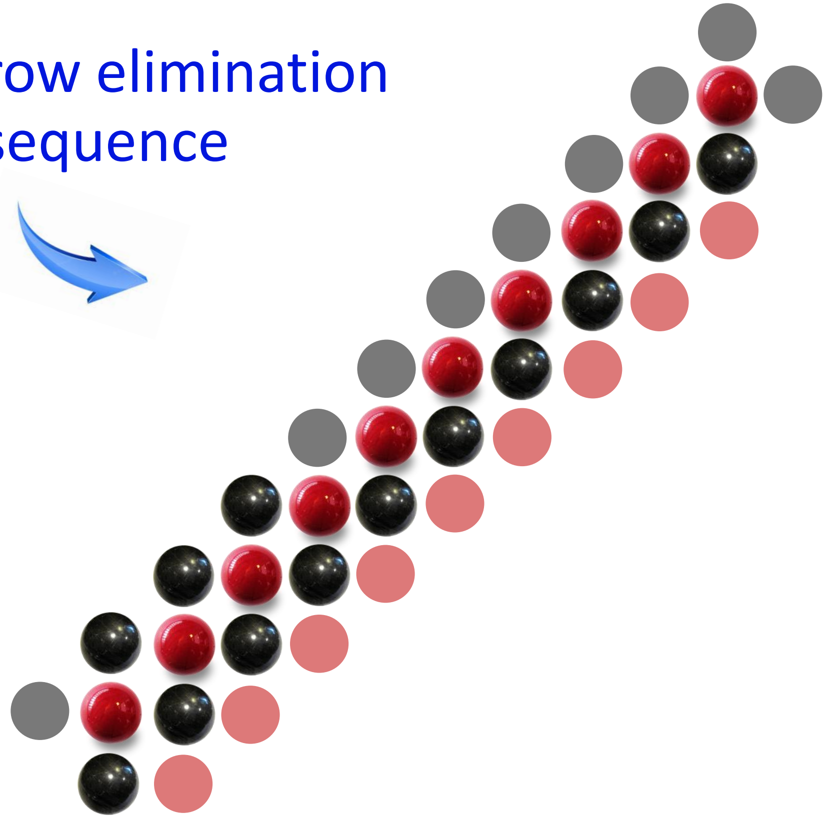
Lemma 4. *For any connected orthogonal convex shape S of n nodes, given a row elimination sequence $\sigma = (u_1, u_2, \ldots, u_n)$ of S where the set of the first two nodes in σ is not single-coloured and u_3 is black, there is an extended staircase generation sequence $\sigma' = (u'_1, u'_2, \ldots, u'_n)$ which is colour-order preserving w.r.t σ and such that, for all $1 \leq i \leq |\sigma|$, $D_i = \{u'_1, u'_2, \ldots, u'_i\}$ is a connected orthogonal convex shape.*

Proof Overview

Orthogonal-convex shape + 6-robot



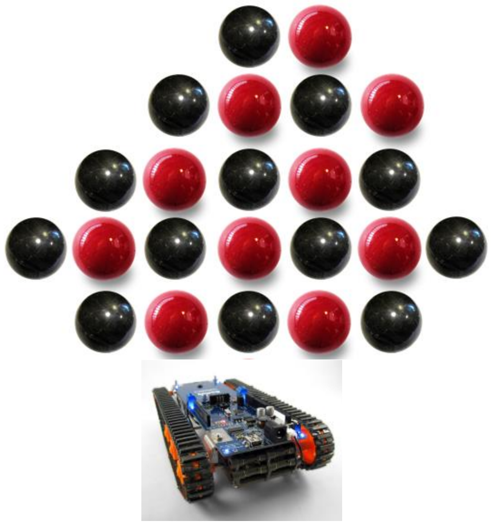
row elimination sequence



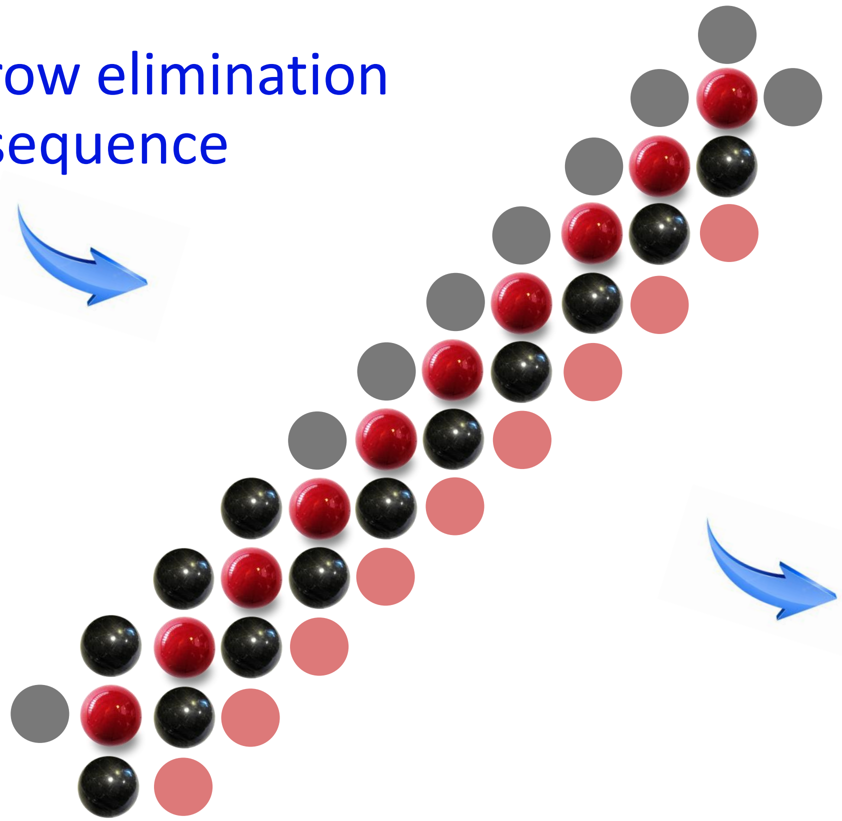
Extended staircase

Proof Overview

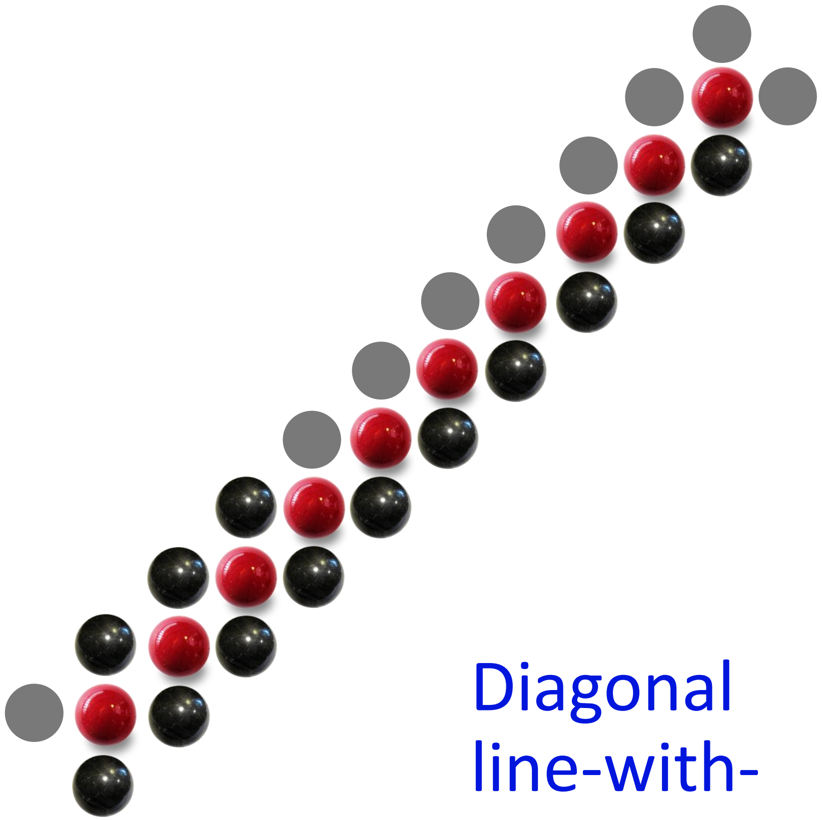
Orthogonal-convex shape + 6-robot



row elimination sequence



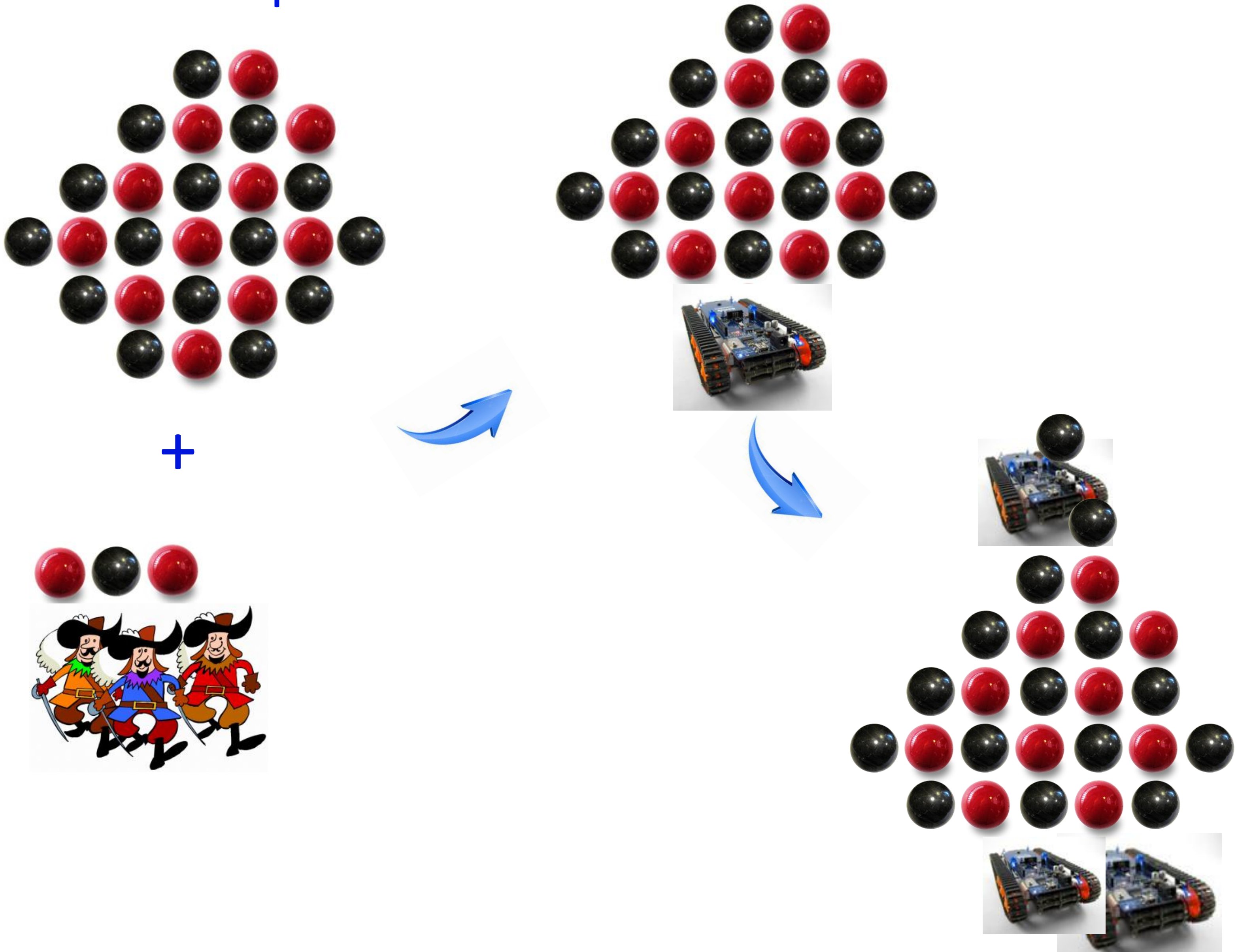
Extended staircase



Diagonal line-with-leaves

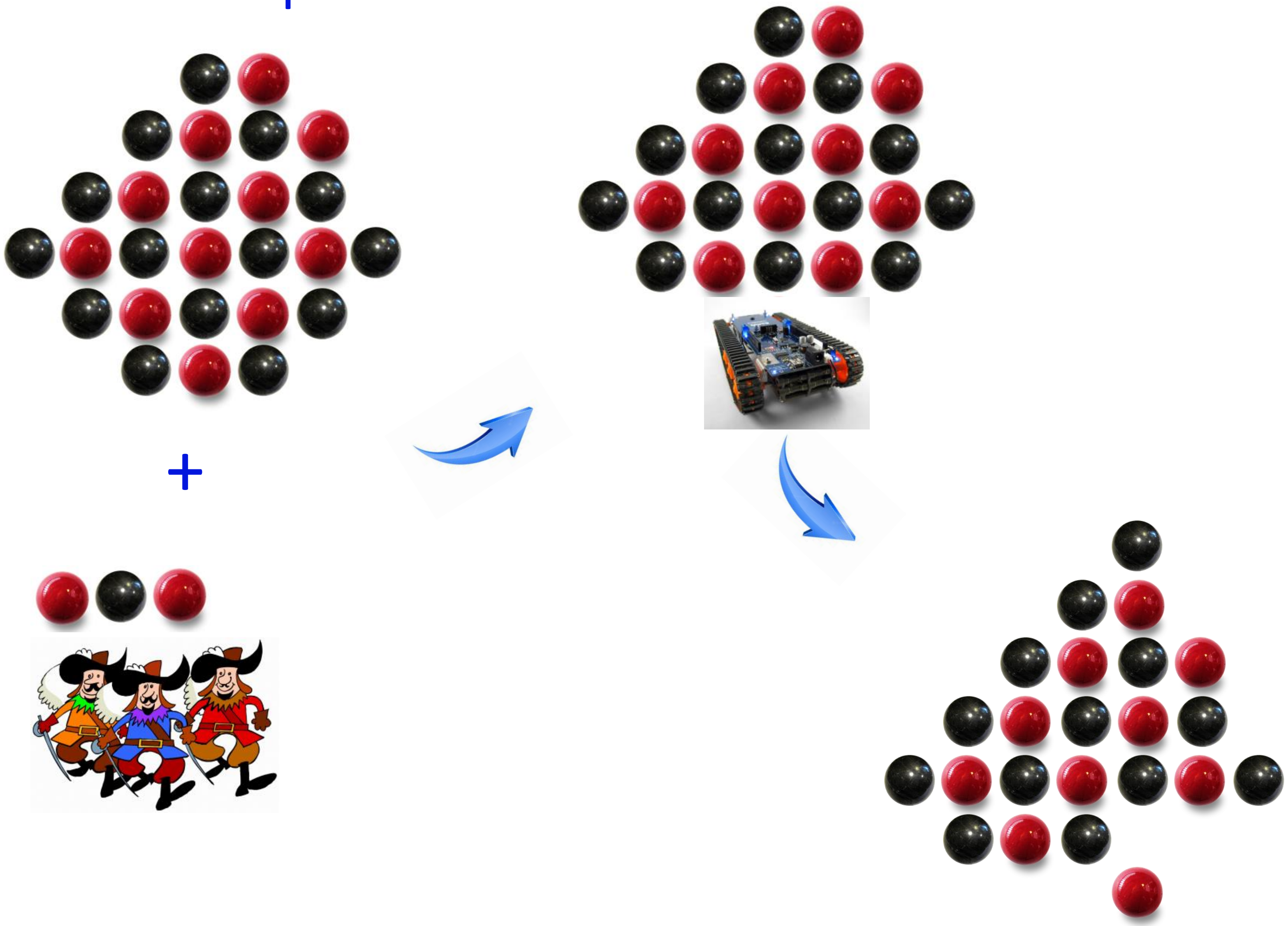
Proof Overview

Any orthogonal-convex shape



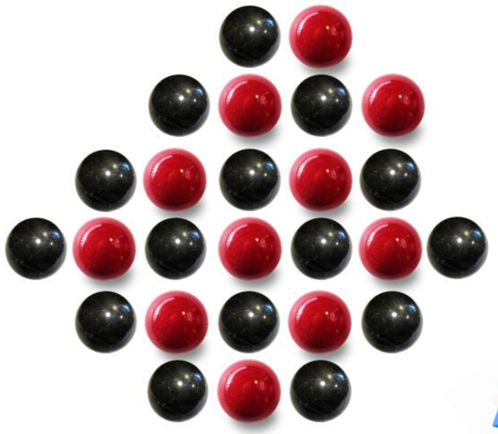
Proof Overview

Any orthogonal-convex shape



Proof Overview

Any orthogonal-convex shape



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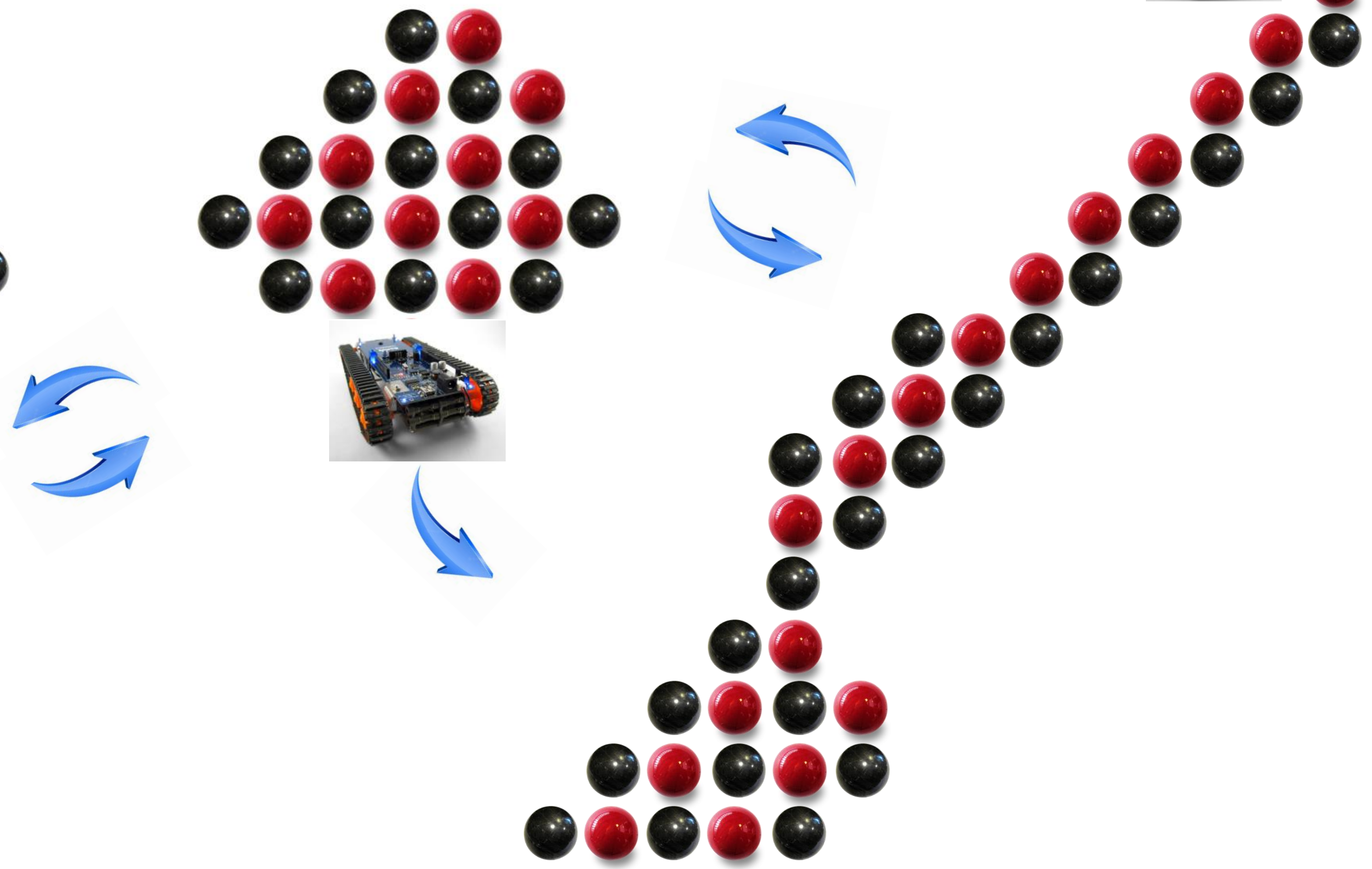
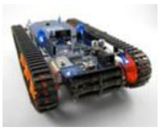
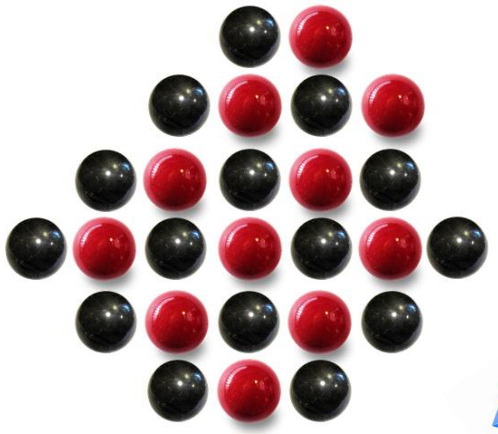
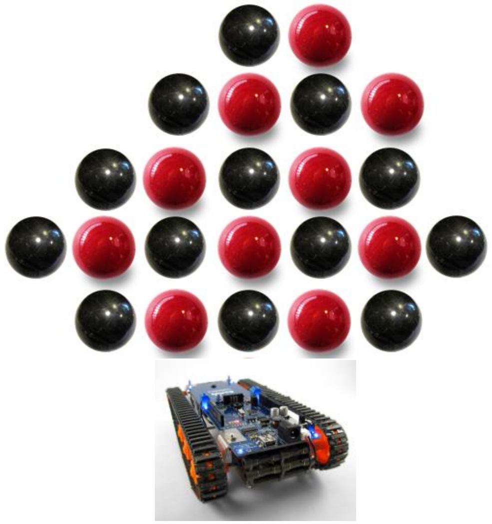


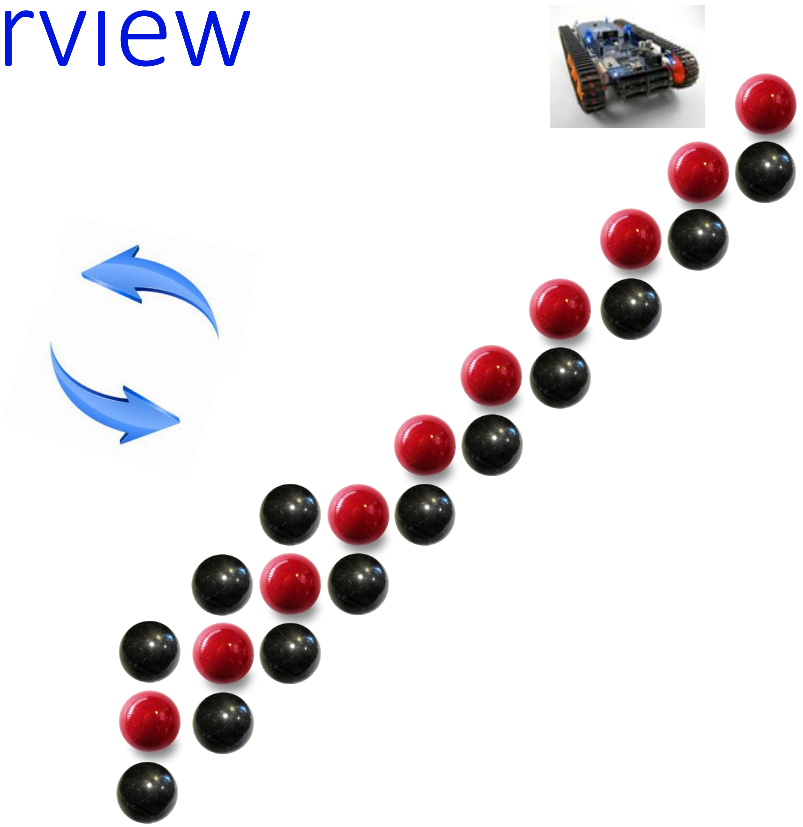


Any orthogonal-convex shape

Proof Overview







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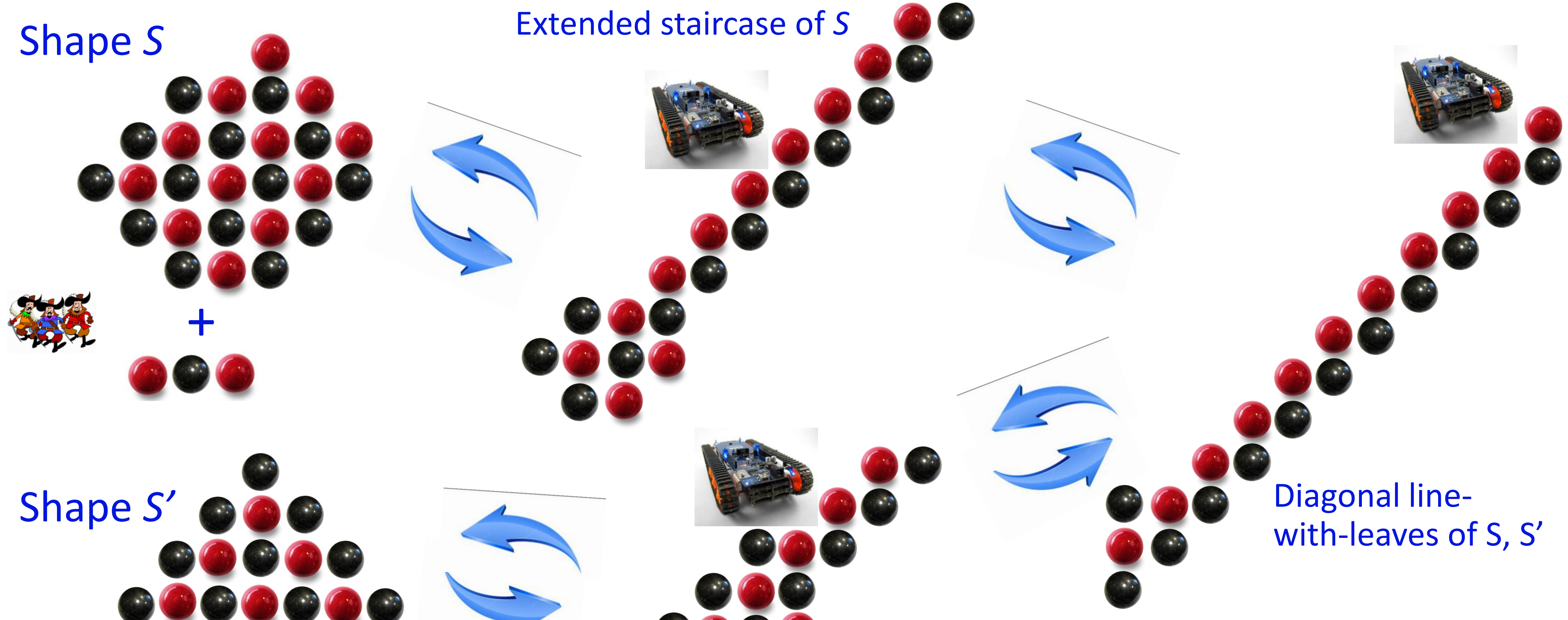
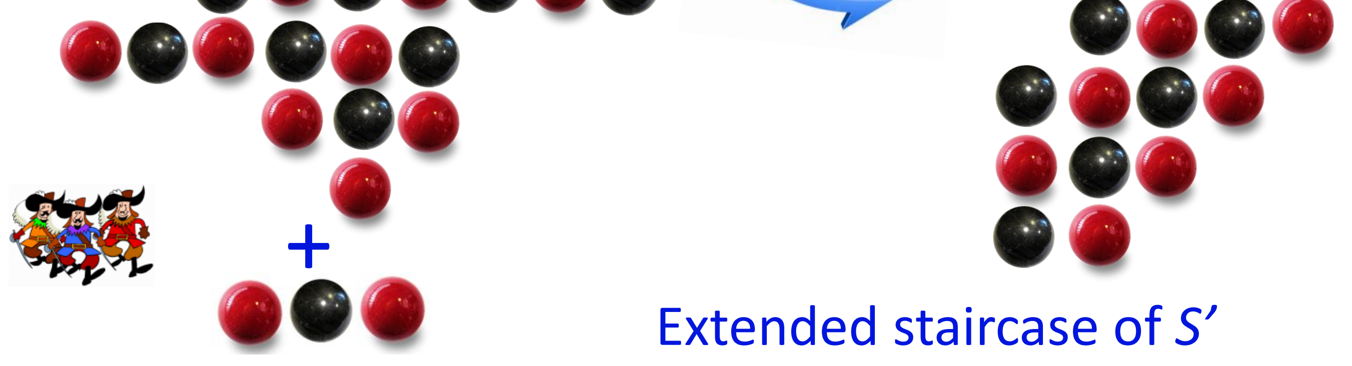
Theorem. Let S and S' be connected color-consistent orthogonal convex shapes. Then there is a connected shape M of 3 nodes (the 3 musketeers) and an attachment of M to the bottom-most row of S, such that $S \cup M$ can reach the configuration S' in $O(n^2)$ moves.





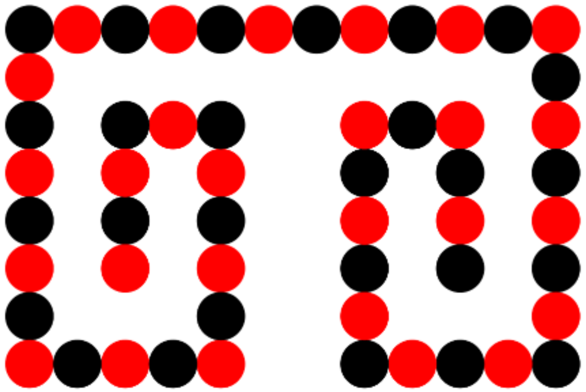
Theorem. Let S and S' be connected color-consistent orthogonal convex shapes. Then there is a connected shape M of 3 nodes (the 3 musketeers) and an attachment of M to the bottom-most row of S, such that $S \cup M$ can reach the configuration S' in $O(n^2)$ moves. □

Summary and open problems

- Seeds can aid the transformation of blocked shapes
- Rot-transformability is universal
- Minimal seed RotC-transformability for nice shapes
- Minimal seed transformations of orthogonal convex shapes
  - Movement of 6/7-node robots around the perimeter - Theorem 1 and Theorem 2
  - Transformation of orthogonal convex into other orthogonal convex by reversibility – Theorem 3

Open problems:

- Decentralising the execution
- Extending the class – universal transformation?
- Double spiral – example of problems of universal transformation




Questions?