The Price of Selfish Stackelberg Leadership in Network Games

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Abstract
We study a class of games in which a finite number of agents each controls a quantity of flow to be routed through a network, and are able to split their own flow between multiple paths through the network. Recent work on this model has contrasted the social cost of Nash equilibria with the best possible social cost.

Here we show that additional costs are incurred in situations where a selfish "leader" agent allocates his flow, and then commits to that choice so that other agents are compelled to minimize their own cost based on the first agent's choice. We find that even in simple networks, the leader can often improve his own cost at the expense of increased social cost. We give bounds on the social cost of a leader agent, within a widely-studied "network of parallel links" topology.

Introduction
Imagine two firms wish to route their traffic from A to B. Suppose the first road is short but narrow, with the time needed to drive along it increasing sharply with the number of drivers who use it. Further suppose that the time needed to drive using it is 1 hour to drive from A to B, regardless of the number of other drivers on the road. Figure below show this network pictorially.

Assuming that both firms aim to minimize the driving time, we have a good reason to expect that both firms will send one third of their cars to follow the top road, and the rest on the bottom road. So what is the total average driving time for both firms? (53 + 53 = 106 minutes)

Now suppose that the red firm gets to travel first. The blue firm has to wait until the red firm finishes. It is obvious that the red firm benefits from getting to route first. But can this have any effect on the total average driving time? (In fact it does. The new average driving time is 109 minutes.)

What is a network Game?
That is an example of a network game. A network game is the interaction among rational, mutually aware players, in a network, where the decisions of some players impact the payoffs (or cost) of others.

• Each player (selfish and noncooperative) wants to efficiently route from a source node to a destination node.
• The delay (also called latency) on each edge depends on the number of players using that edge.

What is Selfish Routing?
Each user chooses a strategy to minimize his own cost without cooperation with others, and without regard to the consequences his choice has for others. Under this assumption, we can view users as independent agents participating in a noncooperative game and expect the routes chosen by users to form a Nash equilibrium.

What is Nash Equilibrium?
A Nash equilibrium, named after John Nash, is a set of choices (strategies), one for each player, such that no player has incentive to unilaterally change his action.

So why big firm?
In reality, people don’t act alone. In many situations, they do collude: contracts are made, businesses agree to cooperate, and corporations merge. In many cases, agents will form small coalitions to improve their collective well-being.

What is Stackelberg Leadership?
Stackelberg leadership refers to a situation where one player (the "leader") selects his action first, and commits to it. The other player(s) then choose their own action based on the choice made by the leader.

What is Selfish Stackelberg Leadership?
Selfish Stackelberg leadership is a game where one player (the leader) allocates his flow, and then commits to that choice so that other agents are compelled to minimize their own cost based on the leader's choice. We find that even in simple networks, the leader can often improve his own cost at the expense of increased social cost. We give bounds on the social cost of a leader agent, within a widely-studied "network of parallel links" topology.

Our focus
• atomic game: We are interested in a game with a finite number of players.
• splittable flow: players can split flow between paths. This is equivalent to a big firm with many employees.
• symmetric network first, asymmetric later:
• Most of this works focus on linear cost (latency) function, but we also present some experimental result for higher degree cost function.

Our ultimate goal is to find (lower bound and upper bound for) the maximum ratio between a social cost (total cost) of a game that has selfish Stackelberg Leader and a social cost of a normal simultaneous game. This is what we call The price of selfish Stackelberg Leadership.

What has been done?
A large body of recent work (initiated mainly by Roughgarden and Tardos (4; 3)) has studied from a game-theory perspective, how selfishness can degrade the overall performance of a system that has multiple (selfish) users (the price of anarchy). Much of this work has focused on congestion games, where users have access to shared resources, and the cost of using a resource increases as the resource attracts more usage.

Hayrapetyan et al. (1) study the cost of selfish behaviour in this model. Flow is split into two or more "coalitions". A coalition is a fraction of the total flow demand whose total cost is shared, and is essentially the same thing as the kind of player with splittable flow that we study here. The cost to (members of) a coalition is the average cost of flow belonging to that coalition. For games that are less restrictive than symmetric parallel links, the price of anarchy can be higher in the presence of these coalitions, than in the Wardrop setting (where flow is controlled by "infinitesimal users").

Recent work on Stackelberg scheduling (e.g. (2; 5)), has studied it as a tool to mitigate the performance degradation due to selfish users. The flow that is controlled by the leader is routed so as to minimize social cost in the presence of followers who minimize their own costs. In contrast, here we consider what happens when the leading flow is controlled by another selfish agent. We show here that the price of decentralized behaviour goes up even further in the presence of a Stackelberg leader.

What have we found?
• We have found that the price of selfish Stackelberg Leadership for a symmetric game where links have no fixed cost is 1 (cost function is in the form of "a*f" where a is a constant and f is a traffic function).
• For 2 players, m parallel links in which one of its cost function is a linear function with fixed cost, the price of selfish Stackelberg Leadership is between 1.057 and 8. The price of selfish Stackelberg Leadership in a two-parallel-links network is 1.169 when one of the cost function is f^4 and the other link cost function is fixed at 5.67. (Lower bound)
• There is the price of selfish Stackelberg Leadership in asymmetric network even links do not have fixed cost.

What's next?
• Improve the upper bound of 8 that we have for a linear cost function.
• Get some results on n player and m links network.
• Consider a more general cost function like concave or convex function.
• Get some upper bound for asymmetric network game

References