A Modal Tableau Approach for Minimal Model Generation

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Modal Herbrand universe \((U^M)\): The set of all ground terms built from a fixed constant \(w\) and a supply of unary function symbols \(f_{\phi_0}\) uniquely associated with subformulae \(\Diamond \phi\) of a modal formula \(\varphi\).

Minimal Modal Model Generation Calculus

Input: a set of modal clauses

Expansion strategy: depth-first left-to-right strategy. Without this strategy the calculus is no longer minimal model sound and complete.

Two possible outputs:
- the input is unsatisfiable (closed tableau)
- all and only minimal modal Herbrand models, each model exactly once (fully expanded open tableau: open branch \(\sim\) a minimal model)

Expansion rules

\[ (\Diamond) \quad \text{where } \phi = \phi_1 \land \ldots \land \phi_n \text{ and } f_{\phi_0} = \text{function symbol uniquely associated with } \Diamond \phi \]

\[ (\lor_E) \quad \text{if } u_1, p_1; \ldots; u_n, p_n; (v_1, w_1) : R, (s_1, t_1) : R \text{ then } \neg p_1 \lor \ldots \lor \neg p_n \lor v_1 : \Diamond \phi_0 \lor \ldots \lor v_n : \Diamond \phi_n \lor (u_1, v_1) \lor (s_1, t_1) : \neg R \lor \psi \]

\[ (P \lor H R) \quad \text{rule: } \]

- the union of the standard \(\land\) rule and diamond rule
- \(f_{\phi_0}(u)\) is a Skolem term uniquely associated with the premise

\[ (\lor) \quad \text{rule: switches from labelled disjunction to disjunction of label-} \]

\[ (CS) \quad \text{led formulae} \]

\[ (CS) \quad \text{(complement splitting) rule: } \]

- avoids the creation of a model more than once
- ensures that the first model is minimal

Negation of positive tableau literal is defined by the \(\text{neg}\) function:

\[ \text{neg}(P) = \begin{cases} u, \neg p_i & \text{if } P = u : p_i \\ (u, v) : \neg R & \text{if } P = (u, v) : R \\ (u, f_{\phi_0}(u)) : \neg R & \text{if } P = u : \Diamond \phi. \end{cases} \]

Model constraint propagation rule

- is the simultaneous application of the closure rules (for labelled formulae and labelled relations) and the box rule
- expands a disjunction of tableau literals where some of the tableau literals are negative iff it is necessary

Model constraint propagation rule: prevents the generation of non-minimal models