A Modal Tableau Approach for Minimal Model Generation

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Model generation/minimization

Model generation is used in computer science areas like

- system verification and debugging
- validation and debugging of data models
- non-monotonic reasoning

Model minimization can be categorized in:

- minimization of the domain
- minimization of specific predicates
- minimization of all predicates
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- minimization of the domain
- minimization of specific predicates
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Minimal Modal Herbrand Model

Modal Herbrand model ⇔ Herbrand model for the first-order logic translation

Example

Modal Herbrand models of $p_1 \land (\diamond p_2 \lor p_3)$:

$I_1 = \{w : p_1, w : p_3\}$

$I_2 = \{w : p_1, f_{\diamond p_2}(w) : p_2, (w, f_{\diamond p_2}(w)) : R\}$

$I_3 = \{w : p_1, w : p_3, f_{\diamond p_2}(w) : p_2, (w, f_{\diamond p_2}(w)) : R\}$

$I_3$ is not minimal because $I_1 \subseteq I_3$ and $I_2 \subseteq I_3$
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$I_3 = \{ w : p_1, w : p_3, f_\Diamond p_2(w) : p_2, (w, f_\Diamond p_2(w)) : R \}$

$I_3$ is not minimal because $I_1 \subset I_3$ and $I_2 \subset I_3$
Input: a set of modal clauses

Box miniscoping during the conjunctive normal form transformation:

\[ \Box (\phi_1 \land \phi_2) \Rightarrow \Box \phi_1 \land \Box \phi_2 \]

Expansion strategy: depth-first left-to-right strategy

Output:

- the input is unsatisfiable (closed tableau)
- all and only minimal modal Herbrand models, each model exactly once (open branch \( \sim \) minimal modal Herbrand model)
Rules of the 3MG calculus

Four expansion rules:

- $(\Diamond)$: expands diamond formulae
- $(\lor)_E$: prepares the input for the other rules
- $(CS)$ – complement splitting rule:
  - avoids model duplication
  - ensures that the first model is minimal
- $(PUHR)$:
  - expands clauses with negative literals
  - can close a branch

Model constraint propagation rule:
blocks the generation of non-minimal model
A derivation example

Derivation for:
\[ \Diamond p \land q \land (\neg q \lor \Box s) \land (\Diamond p \lor \Diamond s) \]
A derivation example

\[
\begin{align*}
  w & : \Diamond p \\
  w & : q \\
  w & : \neg q \lor \Box s \\
  w & : \Diamond p \lor \Diamond s \\
  w & : \neg q \lor w : \Box s \\
  w & : \Diamond p \lor w : \Diamond s \\
  (w, f_\Diamond p(w)) & : R \\
  f_\Diamond p(w) & : p \\
  f_\Diamond p(w) & : s \\
  w & : \Diamond p \\
  (w, f_\Diamond s(w)) & : \neg R \\
  (w, f_\Diamond p(w)) & : R \\
  f_\Diamond p(w) & : p
\end{align*}
\]

Derivation for:

\[
\Diamond p \land q \land (\neg q \lor \Box s) \land (\Diamond p \lor \Diamond s)
\]

Set of clauses in input
A derivation example

\[ w : \Diamond p \]
\[ w : q \]
\[ w : \neg q \lor \Box s \]
\[ w : \Diamond p \lor \Diamond s \]
\[ w : \neg q \lor w : \Box s \]
\[ w : \Diamond p \lor w : \Diamond s \]
\[ (w, f_{\Diamond p}(w)) : R \]
\[ f_{\Diamond p}(w) : p \]
\[ f_{\Diamond p}(w) : s \]

\[ w : \Diamond p \]
\[ (w, f_{\Diamond s}(w)) : \neg R \]
\[ (w, f_{\Diamond p}(w)) : R \]
\[ f_{\Diamond p}(w) : p \]
\[ w : \Diamond s \]
\[ MC \]
\[ \perp \]

Derivation for:
\[ \Diamond p \land q \land (\neg q \lor \Box s) \land (\Diamond p \lor \Diamond s) \]

\((\lor)_E\) rule
A derivation example

\[
\begin{align*}
\text{Derivation for:} & \quad \lozenge p \land q \land (\neg q \lor \Box s) \land (\lozenge p \lor \lozenge s) \\
(\lor)_E \text{ rule}
\end{align*}
\]
A derivation example

\[ \begin{align*}
  w & : \lozenge p \\
  w & : q \\
  w & : \neg q \lor \Box s \\
  w & : \lozenge p \lor \lozenge s \\
  w & : \neg q \lor w : \Box s \\
  w & : \lozenge p \lor w : \lozenge s \\
  (w, f_{\lozenge p}(w)) & : R \\
  f_{\lozenge p}(w) & : p \\
  f_{\lozenge p}(w) & : s
\end{align*} \]

Derivation for:
\[ \lozenge p \land q \land (\neg q \lor \Box s) \land (\lozenge p \lor \lozenge s) \]

(\lozenge) rule
A derivation example

\[ w : \Diamond p \]
\[ w : q \]
\[ w : \neg q \lor \Box s \]
\[ w : \Diamond p \lor \Diamond s \]
\[ w : \neg q \lor w : \Box s \]
\[ w : \Diamond p \lor w : \Diamond s \]

\((w, f_{\Diamond p}(w)) : R \)
\[ f_{\Diamond p}(w) : p \]
\[ f_{\Diamond p}(w) : s \]

\[ w : \Diamond p \]
\[ (w, f_{\Diamond s}(w)) : \neg R \]
\[ (w, f_{\Diamond p}(w)) : R \]
\[ f_{\Diamond p}(w) : p \]
\[ w : \Diamond s \]

Derivation for:
\[ \Diamond p \land q \land (\neg q \lor \Box s) \land (\Diamond p \lor \Diamond s) \]

\((PUHR)\) rule
A derivation example

\[
\begin{align*}
w &: \Diamond p \\
w &: q \\
w &: \neg q \lor \Box s \\
w &: \Diamond p \lor \Diamond s \\
w &: \neg q \lor w &: \Box s \\
w &: \Diamond p \lor w &: \Diamond s \\
(w,f_{\Diamond p}(w)) &: R \\
f_{\Diamond p}(w) &: p \\
f_{\Diamond p}(w) &: s \\
\end{align*}
\]

Derivation for:
\[
\Diamond p \land q \land (\neg q \lor \Box s) \land (\Diamond p \lor \Diamond s)
\]

\((CS)\) rule
A derivation example

\[ w : \Diamond p \]
\[ w : q \]
\[ w : \neg q \lor \Box s \]
\[ w : \Diamond p \lor \Diamond s \]
\[ w : \neg q \lor w : \Box s \]
\[ w : \Diamond p \lor w : \Diamond s \]
\[ (w, f_{\Diamond p}(w)) : R \]
\[ f_{\Diamond p}(w) : p \]
\[ f_{\Diamond p}(w) : s \]

Derivation for:
\[ \Diamond p \land q \land (\neg q \lor \Box s) \land (\Diamond p \lor \Diamond s) \]

(\(\Diamond\)) rule
A derivation example

Derivation for:
\( \Diamond p \land q \land (\neg q \lor \Box s) \land (\Diamond p \lor \Diamond s) \)

fully expanded
open branch

\[ (w, f_{\Diamond p}(w)) : R \]
\[ f_{\Diamond p}(w) : p \]
\[ f_{\Diamond p}(w) : s \]

Minimal Model:
\[ \{ w : q, f_{\Diamond p}(w) : p, f_{\Diamond p}(w) : s, (w, f_{\Diamond p}(w)) : R \} \]
A derivation example

Derivation for:
\( \lozenge p \land q \land (\neg q \lor \Box s) \land (\lozenge p \lor \lozenge s) \)

Model constraint

Minimal Model:
\[
\{ w : q, f_{\diamond p}(w) : p, \\
\quad f_{\diamond p}(w) : s, (w, f_{\diamond p}(w)) : R \}
\]

Constraint (MC): \( w : \neg q \lor f_{\diamond p}(w) : \neg p \lor f_{\diamond p}(w) : \neg s \lor (w, f_{\diamond p}(w)) : \neg R \)
A derivation example

\[ w : \Diamond p \]
\[ w : q \]
\[ w : \neg q \lor \Box s \]
\[ w : \Diamond p \lor \Diamond s \]
\[ w : \neg q \lor w : \Box s \]
\[ w : \Diamond p \lor w : \Diamond s \]
\[ (w, f_{\Diamond p}(w)) : R \]
\[ f_{\Diamond p}(w) : p \]
\[ f_{\Diamond p}(w) : s \]

Derivation for:
\[ \Diamond p \land q \land (\neg q \lor \Box s) \land (\Diamond p \lor \Diamond s) \]

\((PUHR)\) rule

Minimal Model:
\[ \{ w : q, f_{\Diamond p}(w) : p, f_{\Diamond p}(w) : s, (w, f_{\Diamond p}(w)) : R \} \]

Constraint \((MC)\):
\[ w : \neg q \lor f_{\Diamond p}(w) : \neg p \lor f_{\Diamond p}(w) : \neg s \lor (w, f_{\Diamond p}(w)) : \neg R \]
Conclusion

The presented calculus

- is minimal model sound and complete
- terminates
- does not create model duplicate

Most of these properties can be proved by a translation proof using the PUHR approach for the first-order logic