A Tableau Calculus for Minimal Modal Model Generation

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(Minimal) Model Generation

Useful for several tasks:

- hardware and software verification
- fault analysis
- commonsense reasoning
- …

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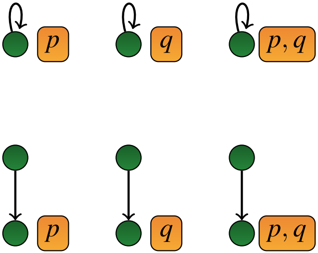
Even though non-classical logics are widely used in Computer Science, minimal model generation for such logics has not been deeply studied

Minimality Criteria

Studying classical logics, we can deduce several minimality criteria:

$\langle has\_father \rangle (p \vee q)$



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- Domain minimality
- Minimization of all predicates
- Minimization of a certain set of predicates

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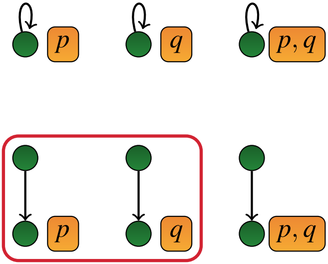

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Models generated by our calculus

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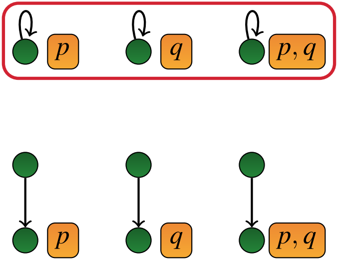

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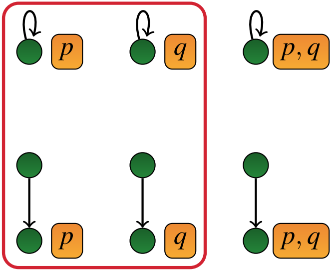

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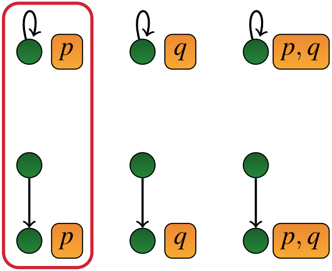

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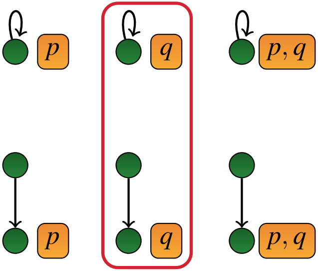

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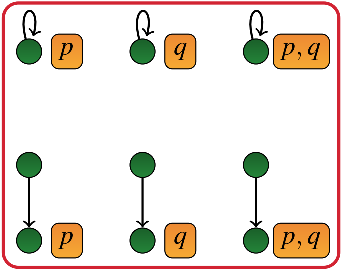

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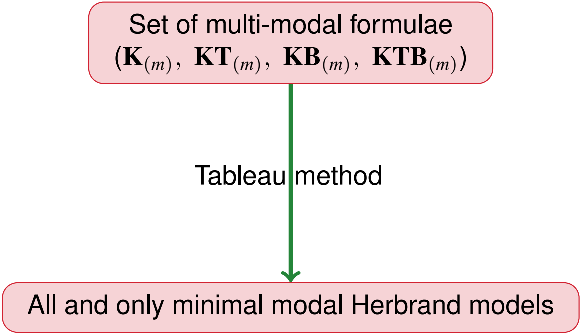
Minimization with respect to has_father

Aim



Set of multi-modal formulae
$(\mathbf{K}_{(m)},\ \mathbf{KT}_{(m)},\ \mathbf{KB}_{(m)},\ \mathbf{KTB}_{(m)})$

Tableau method

All and only minimal modal Herbrand models

Tableau Language: Syntax

A *tableau clause* is defined as follows:

$$\begin{aligned} TC ::= \;& \top \\ \mid\; & \bot \\ \mid\; & u : \phi \\ \mid\; & (u, v) : R_i \\ \mid\; & (u, v) : \neg R_i \\ \mid\; & TC \vee TC \end{aligned}$$

- $u : (p \vee \langle R_1 \rangle q)$
- $u : [R_1](p \vee q) \vee (u, v) : R_2$
- $u : \langle R_2 \rangle \neg p \vee v : (p \wedge q)$

Tableau Language: Semantics

Modal Herbrand universe $(W_{\mathcal{U}})$:

the set of all terms built from a supply of unary function symbols of the form $f_{\langle R_i \rangle \phi_i}$ and $f_{\langle R_i \rangle \sim\phi_i}$, and the terms appearing in N.

$$N = \{\ u{:}\langle R_1 \rangle p,\ v{:}\langle R_1 \rangle p \wedge \neg [R_2] q,\ \ (u, v) : R_1 \}$$

Constant symbols: $u,\ v$

Unary function symbols: $f_{\langle R_1 \rangle p}, f_{\langle R_2 \rangle \neg q}$

$$W_{\mathcal{U}} = \{\ u,\ v,\ f_{\langle R_1 \rangle p}(u),\ f_{\langle R_1 \rangle p}(v),\ f_{\langle R_2 \rangle \neg q}(u),\ f_{\langle R_2 \rangle \neg q}(v),\ \ldots\ \}$$

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Tableau Language: Semantics (cont'd)

- A *modal Herbrand interpretation* I is a set of tableau atoms ($u : p_i$ and $(u, v) : R_i$)
- Truth in I:

$I \not\models \bot$	$I \models \top$
$I \not\models u : \bot$	$I \models u : \top$
$I \models u : p_i$	iff $u : p_i \in I$
$I \models (u, v) : R_i$	iff $(u, v) : R_i \in I$
$I \models u : \neg\phi$	iff $I \not\models u : \phi$
$I \models (u, v) : \neg R_i$	iff $I \not\models (u, v) : R_i$
$I \models u : (\phi_1 \vee \phi_2)$	iff $I \models u : \phi_1$ or $I \models u : \phi_2$
$I \models \Delta_1 \vee \Delta_2$	iff $I \models \Delta_1$ or $I \models \Delta_2$
$I \models u : [R_i]\phi$	iff for every v if $(u, v) : R_i \in I$ then $I \models v : \phi$
$I \models u : \langle R_i\rangle\phi$	iff $(u, f_{\langle R_i\rangle\phi}(u)) : R_i \in I$ and $I \models f_{\langle R_i\rangle\phi}(u) : \phi$

- semantics of a diamond formula is expressed in term of the functional symbol assigned to it
- $I \models u : (\phi_1 \vee \phi_2)$ iff $I \models u : \phi_1 \vee u : \phi_2$

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I \models u : (\phi_1 \vee \phi_2) & \text{iff } I \models u : \phi_1 \text{ or } I \models u : \phi_2 \\
I \models \Delta_1 \vee \Delta_2 & \text{iff } I \models \Delta_1 \text{ or } I \models \Delta_2 \\
I \models u : [R_i]\phi & \text{iff for every } v \text{ if } (u, v) : R_i \in I \text{ then } I \models v : \phi \\
I \models u : \langle R_i \rangle\phi & \text{iff } (u, f_{\langle R_i \rangle \phi}(u)) : R_i \in I \text{ and } I \models f_{\langle R_i \rangle \phi}(u) : \phi
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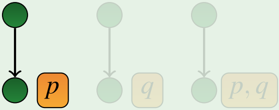

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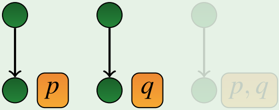

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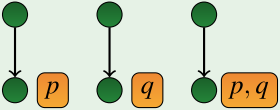

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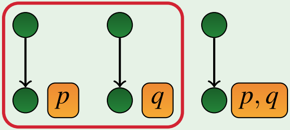

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Minimal Modal Model Generation (3MG) Calculus

The input is obtained by clausal normal form transformation with *box miniscoping* ($\Box(\phi_1 \wedge \phi_2) \Rightarrow \Box\phi_1 \wedge \Box\phi_2$).

Box miniscoping ensures that conjunctions appear only in the scope of diamonds.

The calculus is composed of:

- four expansion rules
- the model constraint propagation rule
- two rules for reflexivity and symmetry

The calculus use a depth-first left-to-right strategy

3MG: Expansion Rules

$$(\Diamond)\ \frac{u : \Diamond(\phi_1 \wedge \ldots \wedge \phi_n)}{\begin{array}{c}(u, f_{\Diamond\phi}(u)) : R \\ f_{\Diamond\phi}(u) : \phi_1 \\ \vdots \\ f_{\Diamond\phi}(u) : \phi_n\end{array}}$$

where $\phi = \phi_1 \wedge \ldots \wedge \phi_n$ and $f_{\langle R_i \rangle \phi}$ is the function symbol uniquely associated with $\langle R_i \rangle \phi$

$$(\vee)_E\ \frac{u : (\phi_1 \vee \ldots \vee \phi_n) \vee \Delta}{u : \phi_1 \vee \ldots \vee u : \phi_n \vee \Delta}$$

3MG: Expansion Rules (cont'd)

$$(CS)\ \frac{\mathcal{P}_1 \vee \ldots \vee \mathcal{P}_n}{\begin{array}{c|c} \mathcal{P}_1 & \mathcal{P}_2 \vee \ldots \vee \mathcal{P}_n \\ neg(\mathcal{P}_i) & \end{array}}$$

where $neg(\mathcal{P}_i)$ stands for $neg(\mathcal{P}_2)$,. . . , $neg(\mathcal{P}_n)$

$$neg(\mathcal{P}) = \begin{cases} (u, f_{\Diamond\phi}(u)) : \neg R & \text{if } \mathcal{P} = u : \Diamond\phi \\ u : \neg p_i & \text{if } \mathcal{P} = u : p_i \\ (u, v) : \neg R & \text{if } \mathcal{P} = (u, v) : R \end{cases}$$

- applicable only to disjunctions of positive literals ($u : p_i$, $u : \langle R_i \rangle \phi$, $(u, v) : R_i$)
- ensures that no model is generated more than once
- ensures that the first model is a minimal model
- results in a reduction of the search space

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3MG: Expansion Rules - (SBR) rule

- the most complex rule of the calculus
- aims to expand a clause with negative literals ($u : \neg p_i$, $u : [R_i]\phi$, $(u, v) : \neg R_i$) iff it is necessary
- aims to minimize splitting
- can be thought as the composition of the common closure rules and the box expansion rule

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$u : \neg p \vee u : q$ is expanded iff $u : p$ is on the branch

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$$\frac{u : p \quad u : \neg p}{\bot} \qquad \frac{(u, v) : R \quad (u, v) : \neg R}{\bot} \qquad \frac{u : \Box\phi \quad (u, v) : R}{v : \phi}$$

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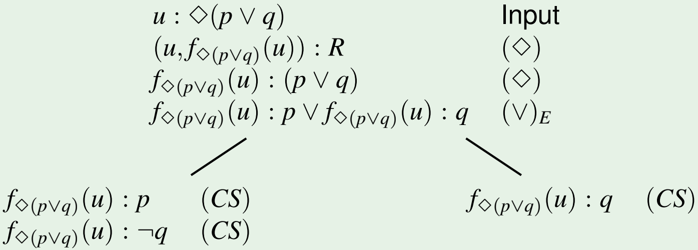
$$(SBR)\ \frac{\begin{array}{c} u_1 : p_1 \quad \dots \quad u_n : p_n \\ (v_1, w_1) : R \quad \dots \quad (v_m, w_m) : R \\ (s_1, t_1) : R \quad \dots \quad (s_j, t_j) : R \\ u_1 : \neg p_1 \vee \dots \vee u_n : \neg p_n \vee v_1 : \Box\phi_1 \vee \dots \vee v_m : \Box\phi_m \\ \vee\ (s_1, t_1) : \neg R \vee \dots \vee (s_j, t_j) : \neg R \vee \Delta^+ \end{array}}{w_1 : \phi_1 \vee \dots \vee w_m : \phi_m \vee \Delta^+}$$

Model Extraction

Once the calculus generates an open and fully expanded branch, the set of tableau atoms appearing on such branch is a modal Herbrand model for the original input.



$u : \Diamond(p \vee q)$ Input

$(u, f_{\Diamond(p\vee q)}(u)) : R$ $(\Diamond)$

$f_{\Diamond(p\vee q)}(u) : (p \vee q)$ $(\Diamond)$

$f_{\Diamond(p\vee q)}(u) : p \vee f_{\Diamond(p\vee q)}(u) : q$ $(\vee)_E$

$f_{\Diamond(p\vee q)}(u) : p$ (CS)

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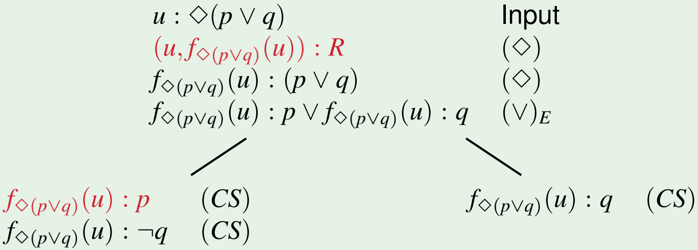

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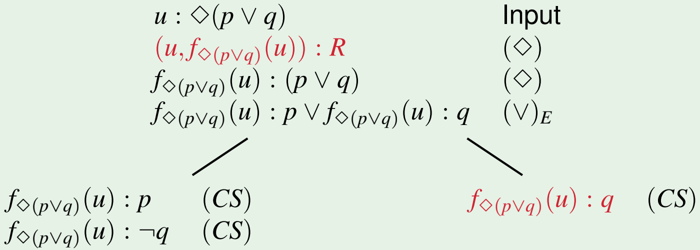
$$I_1 = \{\, f_{\Diamond(p\vee q)}(u) : p,\ (u, f_{\Diamond(p\vee q)}(u)) : R \,\}$$

Model Extraction

Once the calculus generates an open and fully expanded branch, the set of tableau atoms appearing on such branch is a modal Herbrand model for the original input.



$u : \Diamond(p \vee q)$ Input

$(u, f_{\Diamond(p\vee q)}(u)) : R$ $(\Diamond)$

$f_{\Diamond(p\vee q)}(u) : (p \vee q)$ $(\Diamond)$

$f_{\Diamond(p\vee q)}(u) : p \vee f_{\Diamond(p\vee q)}(u) : q$ $(\vee)_E$

$f_{\Diamond(p\vee q)}(u) : p$ (CS)

$f_{\Diamond(p\vee q)}(u) : \neg q$ (CS)

$f_{\Diamond(p\vee q)}(u) : q$ (CS)

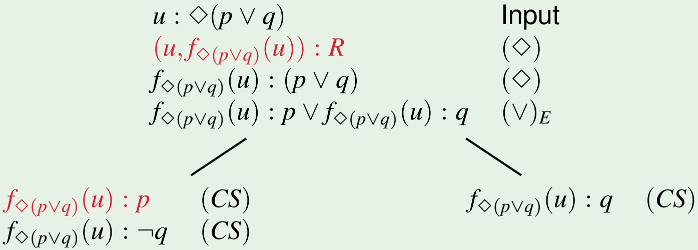
$$I_1 = \{\, f_{\Diamond(p\vee q)}(u) : p,\ (u, f_{\Diamond(p\vee q)}(u)) : R \,\}$$
$$I_2 = \{\, f_{\Diamond(p\vee q)}(u) : q,\ (u, f_{\Diamond(p\vee q)}(u)) : R \,\}$$

Model Constraint Propagation Rule

If $I = \{u_1 : p_1, \ldots, u_n : p_n, (v_1, w_1) : R, \ldots, (v_m, w_m) : R\}$ is the (minimal) modal Herbrand model extracted from a branch $\mathcal{B}$, then

$$u_1 : \neg p_1 \vee \ldots \vee u_n : \neg p_n \vee (v_1, w_1) : \neg R \vee \ldots \vee (v_m, w_m) : \neg R$$

is added to all the branches to the right of $\mathcal{B}$.



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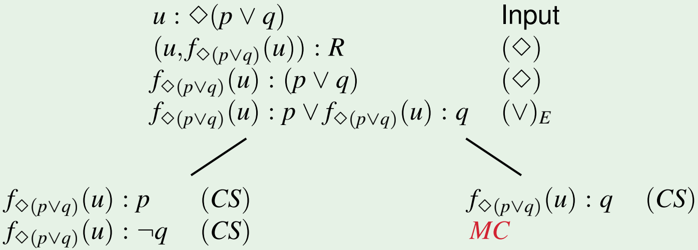

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MC

$$I_1 = \{\, f_{\Diamond(p\vee q)}(u) : p,\ (u, f_{\Diamond(p\vee q)}(u)) : R \,\}$$

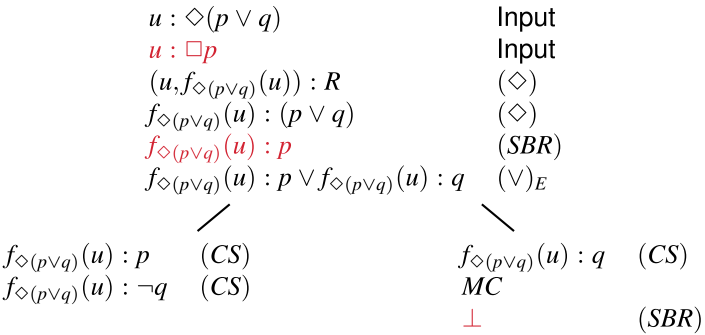
$$MC = f_{\Diamond(p\vee q)}(u) : \neg p \vee (u, f_{\Diamond(p\vee q)}(u)) : \neg R$$

3MG calculus, an example

Derivation for:

$$\{\ u : \Diamond(p \vee q),\ u : \Box p\ \}$$

3MG calculus, an example



$u : \Diamond(p \vee q)$	Input
$u : \Box p$	Input
$(u, f_{\Diamond(p \vee q)}(u)) : R$	$(\Diamond)$
$f_{\Diamond(p \vee q)}(u) : (p \vee q)$	$(\Diamond)$
$f_{\Diamond(p \vee q)}(u) : p$	(SBR)
$f_{\Diamond(p \vee q)}(u) : p \vee f_{\Diamond(p \vee q)}(u) : q$	$(\vee)_E$

$f_{\Diamond(p \vee q)}(u) : p$	(CS)
$f_{\Diamond(p \vee q)}(u) : \neg q$	(CS)

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$\perp$	(SBR)

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Reflexivity and Symmetry

$$(\mathbf{B})^i \ \frac{(u,v) : R_i}{(v,u) : R_i}$$

if R_i is symmetric

$$(\mathbf{T})^i \ \frac{}{(u,u) : R_i}$$

if R_i is reflexive and u appears in a tableau formula of the form $u : \phi$, $(u,v) : R_j$ or $(v,u) : R_j$ on the current branch

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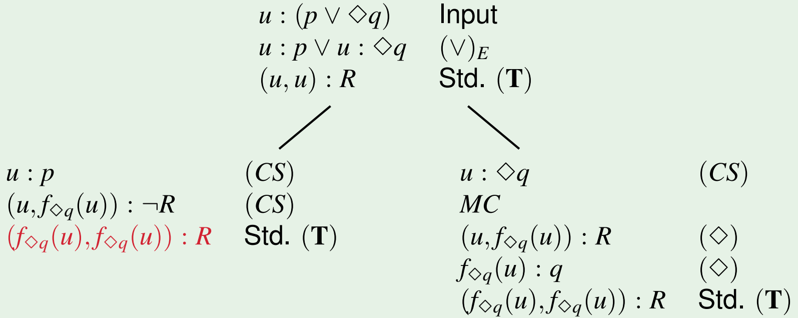


$u : (p \vee \Diamond q)$ Input

$u : p \vee u : \Diamond q$ $(\vee)_E$

$(u,u) : R$ Std. $(\mathbf{T})$

$u : p$ (CS)

$(u, f_{\Diamond q}(u)) : \neg R$ (CS)

$(f_{\Diamond q}(u), f_{\Diamond q}(u)) : R$ Std. $(\mathbf{T})$

$u : \Diamond q$ (CS)

MC

$(u, f_{\Diamond q}(u)) : R$ $(\Diamond)$

$f_{\Diamond q}(u) : q$ $(\Diamond)$

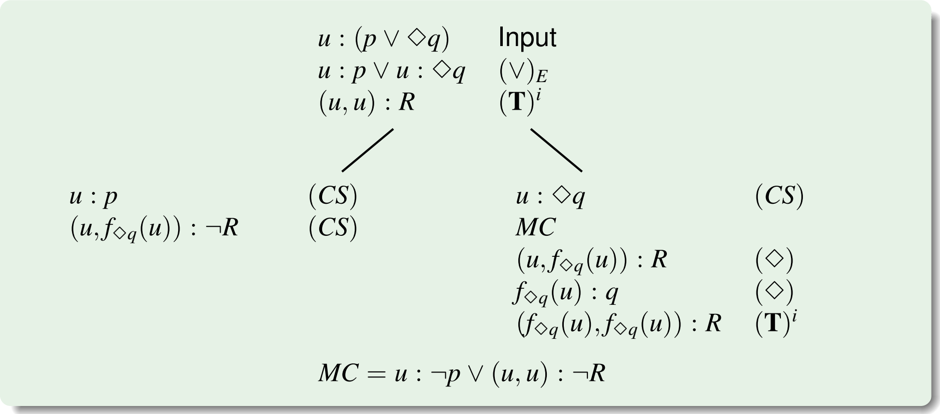
$(f_{\Diamond q}(u), f_{\Diamond q}(u)) : R$ Std. $(\mathbf{T})$

$$MC = u : \neg p \vee (u,u) : \neg R \vee (f_{\Diamond q}(u), f_{\Diamond q}(u)) : \neg R$$

Reflexivity and Symmetry

$$(\mathbf{B})^i \ \dfrac{(u,v) : R_i}{(v,u) : R_i}$$ if R_i is symmetric

$$(\mathbf{T})^i \ \dfrac{}{(u,u) : R_i}$$ if R_i is reflexive and u appears in a tableau formula of the form $u : \phi$, $(u,v) : R_j$ or $(v,u) : R_j$ on the current branch



$u : (p \vee \Diamond q)$ Input

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$(u,u) : R$ $(\mathbf{T})^i$

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MC

$(u, f_{\Diamond q}(u)) : R$ $(\Diamond)$

$f_{\Diamond q}(u) : q$ $(\Diamond)$

$(f_{\Diamond q}(u), f_{\Diamond q}(u)) : R$ $(\mathbf{T})^i$

$$MC = u : \neg p \vee (u,u) : \neg R$$

Minimal Model Soundness and Completeness

Minimal model soundness: only minimal models are generated

Minimal model completeness: all minimal models are generated

Our proof is based on showing a bisimulation between the 3MG calculus and the PUHR approach of Bry and Yahya for first-order logic.

This indirect proof has the advantage of:

- giving us useful insight about the relation of the two calculi
- giving us the opportunity to study how structural transformation affects model generation for first-order clauses
- allowing us to compare our calculus with techniques for fragments of first-order logic (e.g. Georgieva - Hustadt - Schmidt, Niemelä)

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Minimal Model Soundness and Completeness (cont'd)

Modal Formulae

PUHR

3MG

Minimal Model Soundness and Completeness (cont'd)

New Translation

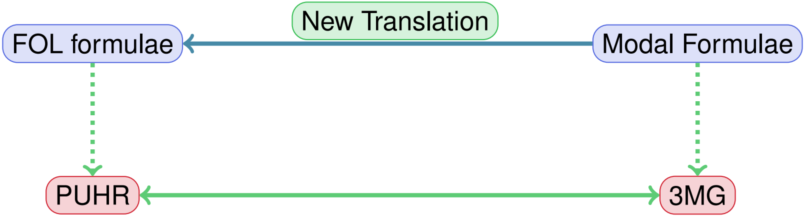
FOL formulae

Modal Formulae

PUHR

3MG

Minimal Model Soundness and Completeness (cont'd)



New Translation

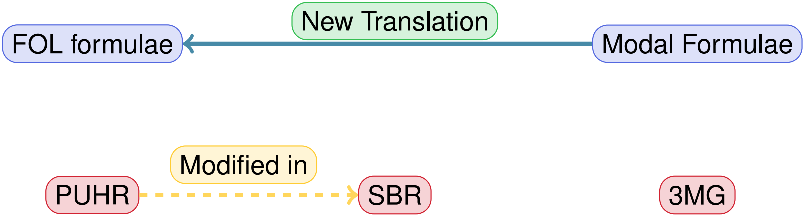
FOL formulae

Modal Formulae

PUHR

3MG

Minimal Model Soundness and Completeness (cont'd)



New Translation

FOL formulae

Modal Formulae

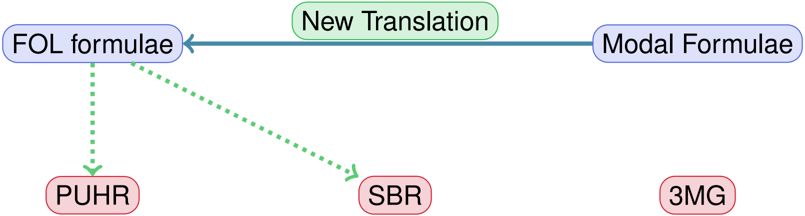
Modified in

PUHR

SBR

3MG

Minimal Model Soundness and Completeness (cont'd)



New Translation

FOL formulae

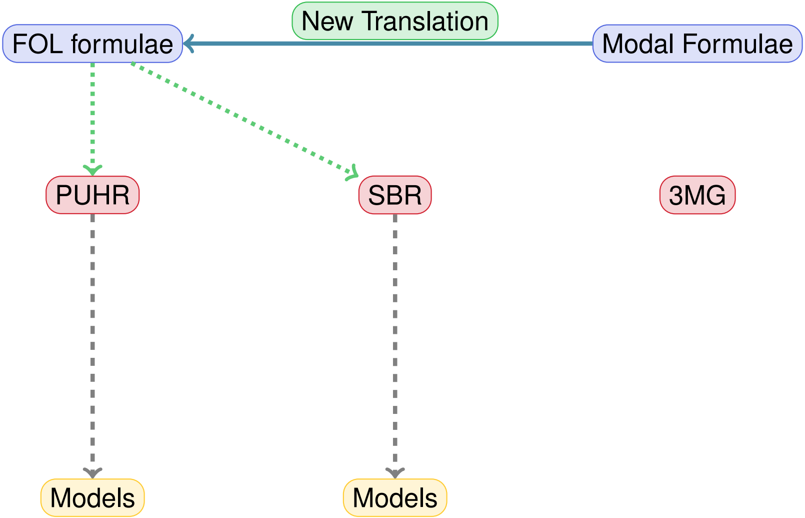
Modal Formulae

PUHR

SBR

3MG

Minimal Model Soundness and Completeness (cont'd)



New Translation

FOL formulae

Modal Formulae

PUHR

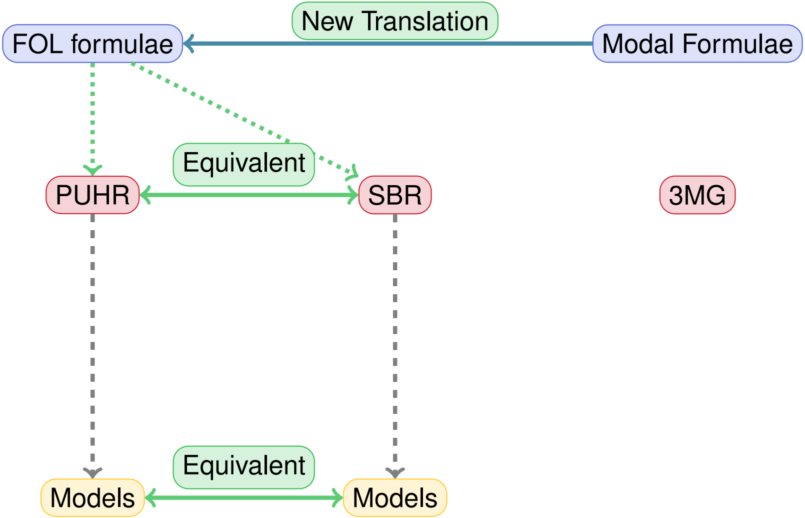
SBR

3MG

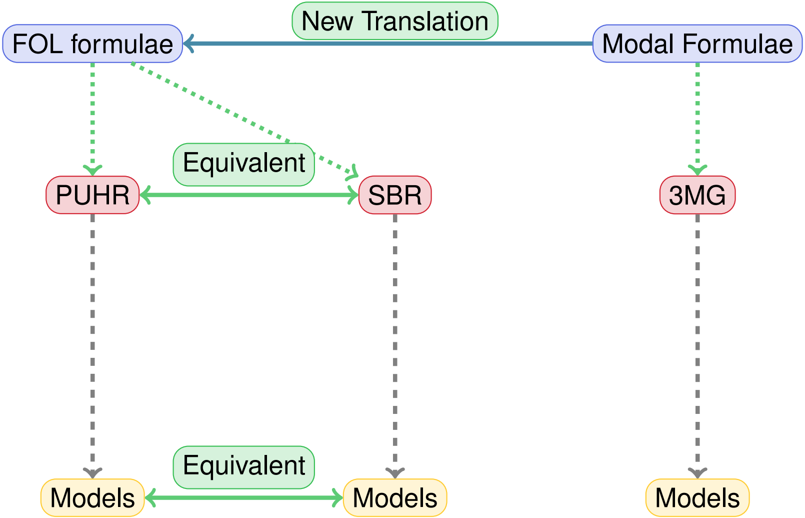
Models

Models

Minimal Model Soundness and Completeness (cont'd)



Minimal Model Soundness and Completeness (cont'd)



New Translation

FOL formulae

Modal Formulae

Equivalent

PUHR

SBR

3MG

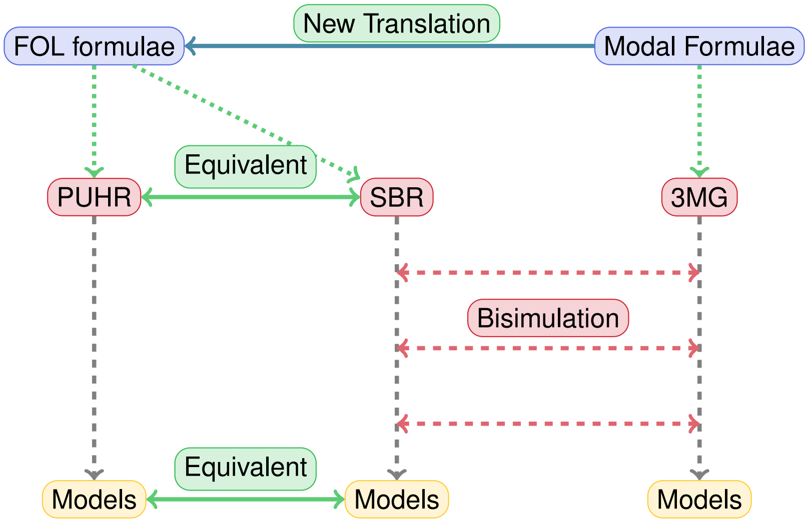
Equivalent

Models

Models

Models

Minimal Model Soundness and Completeness (cont'd)



FOL formulae

New Translation

Modal Formulae

Equivalent

PUHR

SBR

3MG

Bisimulation

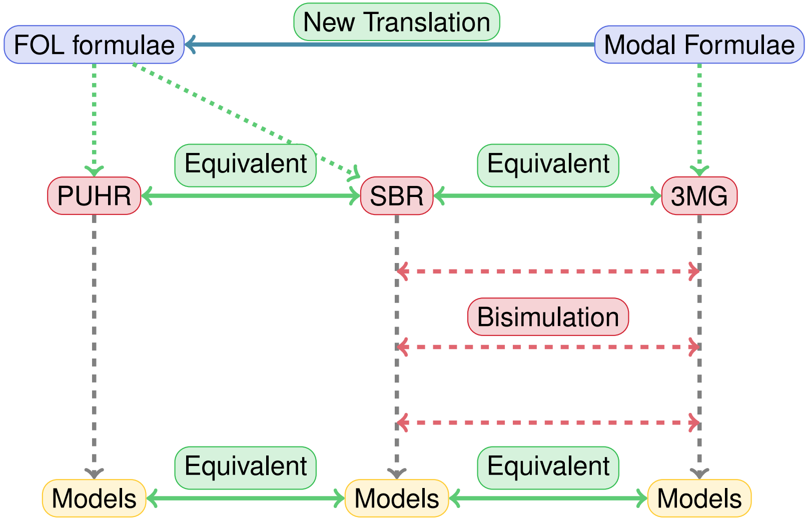
Equivalent

Models

Models

Models

Minimal Model Soundness and Completeness (cont'd)



New Translation

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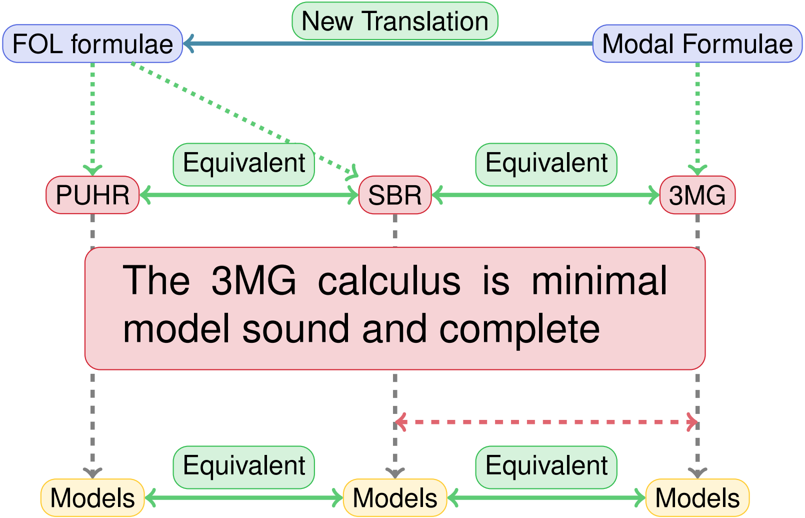
Minimal Model Soundness and Completeness (cont'd)



New Translation

FOL formulae

Modal Formulae

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3MG

The 3MG calculus is minimal model sound and complete

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Models

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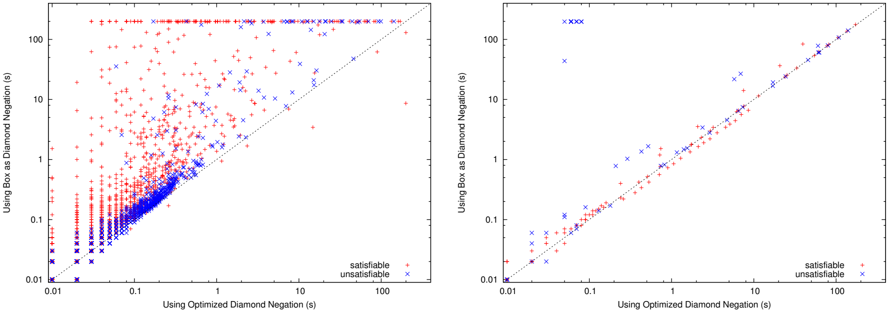
Evaluation of (CS) with neg

- for standard tableaux
- in our implementation complement splitting is applied only to diamond formulae



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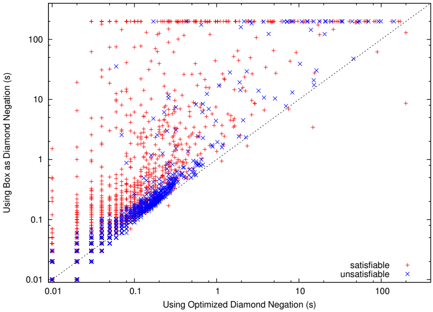

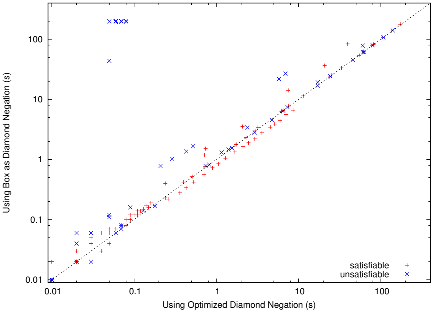

The neg function is applicable due to the uniquely assigned functional symbols

Conclusion and Further Work

The 3MG calculus

- is the only calculus generating minimal modal Herbrand models
- works directly on modal formulae
- is minimal model sound and complete
- each model is generated exactly once
- terminates

Several extensions are possible:

- avoiding the clausal normal form transformation
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Thank You!