Paul W. Goldberg

\[1\text{Department of Computer Science}\]
\[\text{University of Liverpool, U. K.}\]

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Paul Goldberg

email: P.W.Goldberg@liverpool.ac.uk

http://www.csc.liv.ac.uk/~pwg/COMP218/comp218index.html

office hours: Monday 11-12am, Friday 3-4pm (on my web pages; may be changed...)

I am away (not teaching) during week of April 15th

notes and slides will appear on my COMP218 web pages (above URL)
2 class tests; let me know if you need any special provision to take the tests.

unassessed exercises from time to time; bring paper
Some handout notes, available later

today – general introduction, motivation

**Brief Description**

This module aims to introduce formal concepts of automata, grammars and languages; to introduce ideas of computability and decidability, and to illustrate the importance of automata, formal language theory and general models of computation in Computer Science and Artificial Intelligence.
Languages

Spoken languages and programming languages: in both cases we like to know whether a sequence of symbols belongs to the language.

**Formal Languages** have the property that there is a precise rule that governs what strings belong to the language. Formal languages include programming languages, database query languages, various file formats. (so, in the world of computers, they are everywhere...)
By contrast, English, French etc. are not formal languages, although you can still try to write down rules that work most of the time.
“valid” English sentences?

letters: a, b, c, ..., z — no problem

words: dog, house, therefore, fine, ... — mostly we can agree on what words are real; there are not too many to write down in a list
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But: “Furiously sleep ideas green colorless.” is not a sentence! (see wikipedia)
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Can we write down a specification of a large collection of valid sentences…?
Informal descriptions:
“An arithmetic expression is constructed from variables and numbers, and infix operators +, -, *, /, and subexpressions may possibly be enclosed in parentheses, in which case every ( must have a corresponding ) ...”
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“An arithmetic expression is constructed from variables and numbers, and infix operators +, -, *, /, and subexpressions may possibly be enclosed in parentheses, in which case every ( must have a corresponding ) ...”
“A comment begins with /* and ends with */”
“your password should have 4-8 characters and contain a non-alphabetic character”
By examples:
Let $E$ denote the set of arithmetic expressions

$$E = \{x, 1 + (2 - x), x \times y + ((z)), \ldots\}$$
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$a42$ is a valid variable name; $42a$ is not, because a variable name can’t start with a number.
(The variable-name description is a combination of English and examples.) We need some notation to express these descriptions more precisely!
COMP109: propositional logic. Express facts about the world in formal logic. Why? So we can follow computational procedures to draw inferences from them.
An analogy

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COMP218: notations for representing *formal languages*. These give us

- ways to define them precisely
- ways to build compilers that recognise the languages
- ways to check whether
  - a string of symbols belongs to a language
  - whether two alternative descriptions of languages are actually the same language
Tools to analyse languages

These tool originate in analysis of natural (spoken) language as well as programming languages.

Natural language: want to recognise valid sentence
Programming language: want to recognise valid program

No length limit on sentences, so you can’t list them. A list is infeasible even you limit the length to some “reasonable” amount (e.g. 50 words), also not very enlightening.
Some notations/methods for describing a language (other than explicitly listing it) include:

- java programs\(^1\)
- finite automaton (example on board)
- regular expressions
- Backus-Naur form
- context-free grammar (example on next slide)

We will see that some of the above can describe more languages than others.

\(^1\)but there are problems if we impose no restrictions on the programs...
Grammars

Sets of rules for generating syntactically correct programs/sentences.

A grammar for generating some English sentences:

\[
\begin{align*}
\langle S \rangle & \rightarrow \langle S \rangle \text{ and } \langle S \rangle \\
\langle S \rangle & \rightarrow \langle \text{subject phrase} \rangle \langle \text{Verb} \rangle \langle \text{object phrase} \rangle \\
\langle \text{subject phrase} \rangle & \rightarrow \langle \text{subject pronoun} \rangle \\
\langle \text{subject phrase} \rangle & \rightarrow \langle \text{Article} \rangle \langle \text{Noun} \rangle \\
\langle \text{object phrase} \rangle & \rightarrow \langle \text{object pronoun} \rangle \\
\langle \text{object phrase} \rangle & \rightarrow \langle \text{Article} \rangle \langle \text{Noun} \rangle \\
\langle \text{subject pronoun} \rangle & \rightarrow \text{he} \mid \text{she} \\
\langle \text{object pronoun} \rangle & \rightarrow \text{him} \mid \text{her} \mid \text{me} \\
\langle \text{Article} \rangle & \rightarrow \text{a} \mid \text{the} \\
\langle \text{Noun} \rangle & \rightarrow \text{dog} \mid \text{cat} \mid \text{mouse} \mid \text{house} \\
\langle \text{Verb} \rangle & \rightarrow \text{sees} \mid \text{likes} \mid \text{finds} \mid \text{leaves}
\end{align*}
\]
Observations

- The grammar can generate sentences like “the dog sees the cat and the mouse leaves the house”, which may be nonsense e.g. “the house sees me”.
- Arbitrarily long sentences can be generated (which you can’t do by enumeration!)
- Semantics can be given with reference to grammar, e.g. logical conjunction of subsentences formed by word “and”
- The grammar could be extended to handle subordinate clauses, adverbs etc.
- You can’t define natural language sentences completely this way, but you can for programming languages
The deriv tree:

```
S
  /   |
S   S
 |   /|
subj. phrase  obj. phrase
 |  |  |  |
article  the  noun  dog
  |  |  |  |  |
verb  sees
  |  |  |  |  |  |
article  the
  |  |  |  |  |  |  |
noun  cat
  |  |  |  |  |  |  |  |
and
  |  |  |  |  |  |  |  |  |  |
article  the
  |  |  |  |  |  |  |  |  |  |  |  |
noun  mouse
  |  |  |  |  |  |  |  |  |  |  |  |  |  |
verb  leaves
  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
article  the
  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
noun  house
```
Extending the grammar

Suppose we add these rules:
\[
\begin{align*}
\langle S \rangle & \rightarrow \text{either } \langle S \rangle \text{ or } \langle S \rangle \\
\langle S \rangle & \rightarrow \text{if } \langle S \rangle \text{ then } \langle S \rangle
\end{align*}
\]

Do we run into problems?
Stages of compilation:

1. lexical analysis: divide sequence of characters into tokens, such as variable names, operators, labels. In a natural language tokens are strings of consecutive letters (easy to recognise!)

2. parsing: identify relationships between tokens

3. code generation: generate object code

4. code optimisation
Lexical Analysis

\[ \text{pay} = \text{salary} + (\text{overtimerate} \times \text{overtime}) \]

Break into tokens as follows:

```plaintext
pay  =  salary + ( ovetimerate * overtime )
;  
```
Definitions and notation

- An *alphabet* is a finite set of symbols.
- A *word* over alphabet $A$ is a string of symbols belonging to $A$.
- The *empty word* will be denoted $\epsilon$.
- $A^*$ denotes the set of *all* words over $A$.
- $A^+$ denotes the set of all non-empty words over $A$. 
The *concatenation* (a.k.a. "product") of two words is obtained by appending them together to form one long word. Concatenation of words $w_1$ and $w_2$ can be written $w_1 w_2$. For any word $w$, note that $w \epsilon = \epsilon w = w$. Concatenation is associative.

$w^n$ denotes the concatenation of $n$ copies of $w$.

$|w|$ denotes the length (number of letters) of $w$.

$|w_1 w_2| = |w_1| + |w_2|$
For $w \in A^*$, the reverse of $w$ is denoted $w^R$ and consists of $w$’s letters in reverse order.  
A palindrome is a word $w$ satisfying $w = w^R$.

If $u, v, w$ are words and $w = uv$ then $u$ is a prefix of $w$ and $v$ is a suffix of $w$. A proper prefix of $w$ is a prefix that is not equal to $\epsilon$ or $w$. (Similarly for proper suffix)
A language (or formal language) over alphabet $A$ is a subset of $A^*$. We can express new languages in terms of other languages using concatenation and closure.

$$L_1 L_2 = \{w_1 w_2 : w_1 \in L_1 \text{ and } w_2 \in L_2\}.$$

$$L^* = \{w_1 w_2 \ldots w_n : n \geq 0 \text{ and } w_1, w_2, \ldots w_n \in L\}$$