Answer all questions.

Marks for this test account for 10% of the total credit for COMP218.

Write your answers straight onto the question paper in the space provided below each question. In the unlikely event that you require additional space to answer any question, write on the reverse side of the paper, indicating clearly that you have done this. If you want, you may do rough work on the reverse of the question pages.

Enter your name, University ID Number and degree program and year of study in the spaces below.

Name __________________________ University ID Number __________________________
Degree program and year of study _______________________________________

Do not write below this line
1. Consider the following context-free grammar, having starting symbol $S$, additional variable symbols $A$, $B$, and $C$, and alphabet \{a, b, c\}:

$$
\begin{align*}
S & \rightarrow AB \mid Cb \\
A & \rightarrow \epsilon \mid aaC \\
B & \rightarrow \epsilon \mid acCSa \\
C & \rightarrow aB \mid c
\end{align*}
$$

(a) Write down a derivation of the string $acccba$. (Hint: to get started, notice that using the rule $S \rightarrow Cb$ will generate a word that ends with $b$.) [5 marks]

(b) Is your answer to part (a) a leftmost derivation? Explain why (or why not, as the case may be). [2 marks]
(c) Recall that “valid” sequences of parentheses can be described using the grammar

\[
S \rightarrow SS \\
S \rightarrow (S) \\
S \rightarrow \epsilon
\]

Show that the grammar is ambiguous by drawing two derivation trees for the string \( ()() \). [5 marks]
(d) The following is an alternative grammar for the language of part (c):

\[
S \rightarrow (S)S \\
S \rightarrow \epsilon
\]

Is this grammar also ambiguous? Justify your answer. (You may find it useful to try constructing derivations for various words in the language.) [6 marks]
2. A pushdown automaton has states $i$, $s$ and $t$, where $i$ is the initial state and $t$ is the only accepting state. The input alphabet is $\{a, b, c\}$, the memory alphabet is the single symbol $X$, and transitions are

\[
\begin{align*}
\delta(i, a, \epsilon) &= \{(i, X), (i, \epsilon), (s, \epsilon)\} \\
\delta(s, b, X) &= \{(s, \epsilon), (t, \epsilon)\} \\
\delta(t, c, \epsilon) &= \{(t, \epsilon)\}
\end{align*}
\]

(a) Explain how the pushdown automaton accepts the input $aaabc$. [8 marks]
2(b) What sort of words are accepted by this pushdown automaton? (You may give either an English-language description, or a context-free grammar.) [2 marks]

| (c) Suggest a way to modify the automaton so that it only accepts those words in its original language that contain at least three b’s. [2 marks] |