A fair payoff distribution for myopic rational

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Summary

- How to partition a population of agents? (e.g. making multiple teams from a pool of players, groups of students, etc.)
- Each agent has a valuation for a partition
- Preference of agents conflicts
  → there may not exist any stable partition.
- Which partition to form?
- How to make it stable?

Proposed Solution

Form a partition $s^*$ that maximizes utilitarian social welfare (efficiency of the population)

Use side payments to stabilize population

Agents have incentive to follow our mechanism.
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Population $N$ of $n$ agents.

**Definition (Coalition)**

A **coalition** $\mathcal{C}$ is a set of agents: $\mathcal{C} \in 2^N$.

$\mathcal{C}$ is the set of all coalitions.

**Definition (Coalition structure)**

A **coalition structure** $s$ is partition of agents into coalitions: $s = \{C_1, \ldots, C_k\}$ where $\bigcup_{i \in {1..k}} C_i = N$ and $i \neq j \Rightarrow C_i \cap C_j = \emptyset$

$\mathcal{S}$ is the set of all coalition structures.

$s(i)$ denotes the coalition of agent $i$ in the coalition structure $s$. 
Valuation function $v : N \times \mathcal{I} \mapsto \mathbb{R}$

→ private valuation (hedonic coalition formation flavor)

→ valuation may depend on other coalition in the population (externalities, endogeneous coalition formation)

→ Preference order over CSs $\succeq_i$
Hypothesis

- Self interested agents: agents maximize expected private utility
- Myopic agents: agents only care about immediate reward and do/can not analyze future implication of their actions.
  - no coordinated change of coalition (only individual actions)
  - one agent at a time can change coalition
  - a coalition’s member can veto the arrival of a new agent in the coalition (individually stable)

Fairness & efficiency

- Agents should feel that the payoff they obtain corresponds to their abilities
- The coalition chosen should maximize social welfare
Definition ($\succsim_i$ denotes preferences over coalitions)

A coalition structure $s$ is **core stable** iff $\not\exists C \subset N | \forall i \in C, C \succ_i s(i)$.

A coalition structure $s$ is **Nash stable**

$$(\forall i \in N) \ (\forall C \in s \cup \{\emptyset\}) \ s(i) \succsim_i C \cup \{i\}$$

A coalition structure $s$ is **individually stable** iff

$$(\not\exists i \in N) \ (\not\exists C \in s \cup \{\emptyset\}) \ | \ (C \cup \{i\} \succ_i s(i)) \text{ and } (\forall j \in C, C \cup \{i\} \succsim_j C)$$

A coalition structure $s$ is **contractually individually stable** iff

$$(\not\exists i \in N) \ (\not\exists C \in s \cup \{\emptyset\}) \ | \ (C \cup \{i\} \succ_i s(i)) \text{ and } (\forall j \in C, C \cup \{i\} \succsim_j C) \text{ and } (\forall j \in s(i) \setminus \{i\}, s(i) \setminus \{i\} \succsim_j s(i))$$
Additional criteria

**Individual rationality:** \( \forall i \in N, u(i) \geq v(\{i\}) \)
agent obtains at least its self-value as payoff.

**Pareto Optimal:** \( \nexists y | \exists i \in N | y_i > u_i \) and \( \forall j \neq i, y_j \geq u_j. \)
no agent can improve its payoff without lowering the payoff of another agent.
Example of a transition function

\[
\begin{align*}
\{1, 2, 3\} & \quad \frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{3} \\
\{1, 2\} \{3\} & \quad \frac{11}{32} \quad \frac{13}{32} \quad \frac{1}{4} \\
\{1, 3\} \{2\} & \quad \frac{13}{32} \quad \frac{11}{32} \quad \frac{1}{4} \\
\{2, 3\} \{1\} & \quad \frac{11}{32} \quad \frac{13}{32} \quad \frac{1}{4} \\
\{1\} \{2\} \{3\} & \quad \frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{3}
\end{align*}
\]
**Markov chains**

**Transient states**: states the chain will eventually leave to never visit again  

**Ergodic states**: states the chain will keep coming back to  

**Communication class**: set of ergodic states where the chain is trapped (sink equilibrium). Which communication class is reached depends on 1) initial state 2) transient states visited
Proposed approach

Provide an incentive to form a social welfare maximizing coalition structure

1. Compute the expected utility of each agent $i$, $E(v_i)$, when agents are acting as myopic rational agents (exact computation requires the analysis of a Markov chain)

2. Share the value of the social maximizing coalition structure proportionally to the expected value.

$$u_i = \frac{E(v_i)}{\sum_{j \in N} E(v_j)} v(s^*)$$
Guarantees a payoff that is at least the expected utility:

\[ u_i = \sum_{j \in N} \frac{E(v_i)}{E(v_j)} v(s^*) \geq E(v_i), \]

i.e., the payoff of an agent is at least as good as the expected utility that an agent would get on average if the agents are myopically rational.
Properties

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- Size of the share is “Fair” in the sense that, on average, assuming equal probability of the initial state, an agent gets \( E(v_i) \).
Experimental results

Average payoff over all CSs, expected value, weight and protocol payoff for each agent for a random valuation function in $D$

<table>
<thead>
<tr>
<th>agent</th>
<th>avg</th>
<th>$\bar{V}_i$</th>
<th>$w_i$</th>
<th>$u_i$</th>
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<tr>
<td>0</td>
<td>0.50</td>
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<td>0.17</td>
<td>0.96</td>
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<td>0.54</td>
<td>0.15</td>
<td>0.85</td>
</tr>
<tr>
<td>5</td>
<td>0.50</td>
<td>0.58</td>
<td>0.16</td>
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<tr>
<td>total</td>
<td>3.06</td>
<td>3.60</td>
<td>1.00</td>
<td>5.63</td>
</tr>
</tbody>
</table>
Experimental results - Approximation

Dynamics of the error of the estimated payoff averaged over 50 instances of the ART problem

![Graph showing dynamics of the error](image)

Dynamics of the error

\[ \frac{\| \text{estimate} - \text{true value} \|}{\text{mean(score)}} \]

5 agents
6 agents
7 agents
8 agents

Airiau, Sen (UvA, TU)
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Discussion and Conclusion

1. it is possible that, for each coalition $C \in s^*$, $\sum_{i \in C} u_i \neq \sum_{i \in C} v_i(s^*)$. (unbalanced inter coalition side payments)

2. given the valuation function, the agents have the choice between signing a bidding contract and receive $u_i$, or go on with a coalition formation process.

3. a rational agent should choose our protocol.

4. problem: expensive, requires revelation of $v_i$ or $\succ_i$

Future Work

- Analysis of approximations
- Analysis of manipulation
- Complete protocols
contacts

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