Coalitional Manipulation Under Realistic Assumptions
(based on joint work with Shaun White)

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A Comparison of Basic Assumptions

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• to come last
• to know how others voted
Perils of Coalitional Manipulability

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- a manipulating coalition must be somehow formed. Given its size, the process must be complex with a lot of private communication. Opinion polls tell you that there are your potential coalition partners but they do not tell you who they are.
- this group must include a coordination centre who calculates who should submit which linear order and then privately communicates those to coalition members.
- all the coalition members must obey the instructions of the centre but there does not seem to be obvious ways to reinforce the discipline.
A New Framework

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If the value of the social choice function may not drop below the status quo, then we say that such call is safe.
Example 1

Suppose the Borda rule is used.

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</table>

Then \( \text{Sc}(A) = 96 \), \( \text{Sc}(B) = 99 \), \( \text{Sc}(C) = 87 \). So

\[
F(R) = B.
\]

This profile is not manipulable from GS Theorem point of view but incentives to vote strategically exist.
Example 1 continued

*ACB* types are unhappy.

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</tr>
</tbody>
</table>

\[
\begin{bmatrix}
A \\
C \\
B
\end{bmatrix}
\xrightarrow{13}
\begin{bmatrix}
C \\
A \\
B
\end{bmatrix}
\]

makes $\text{Sc}(A) = 83$, $\text{Sc}(B) = 99$, $\text{Sc}(C) = 100$. So

\[
F(R') = C.
\]

If a smaller number of *ACB* types switch, nothing happens. The call is safe.
Example 1 continued

*ABC* types are not completely happy.

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<td>C</td>
<td>A</td>
<td>B</td>
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</tr>
</tbody>
</table>

\[
\begin{bmatrix}
A \\
B \\
C
\end{bmatrix} \xrightarrow{4-8} \begin{bmatrix}
A \\
C \\
B
\end{bmatrix}
\]

makes \( F(R') = A \).

But

\[
\begin{bmatrix}
A \\
B \\
C
\end{bmatrix} \xrightarrow{>8} \begin{bmatrix}
A \\
C \\
B
\end{bmatrix}
\]

makes \( F(R'') = C \).

The call is unsafe.
The Geometry of Example 1

Given weights \( w_1 \geq w_2 \geq \ldots \geq w_m = 0 \) and a profile \( R = (R_1, \ldots, R_n) \), every alternative \( a \) gets a positional score \( sc(a) \).

Then the normalised positional score of the alternative \( a \) is given by:

\[
scn(a) = \frac{sc(a)}{sc(a_1) + \ldots + sc(a_m)}.
\]

After this normalisation we have

\[
scn(a_1) + scn(a_2) + \ldots + scn(a_m) = 1.
\]
Geometric representation of scores

A normalised vector of scores $scn(a)$ can be represented as a point $x$ of the $m$-dimensional simplex $S^{m-1}$:

$$x = (x_1, \ldots, x_m), \quad x_1 + \ldots + x_m = 1,$$

where $x_i = scn(a_i)$ is the normalised score of the $i$th alternative. We treat $x_1, \ldots, x_n$ as the homogeneous barycentric coordinates of $x$. 
Winning Areas

The simplex $S^{m-1}$ is divided into three zones: where the candidates $A$, $B$ and $C$ win, respectively.

The green arrow is the safe manipulation and the red arrow is the unsafe one.
Example 2

Suppose the \((3, 1, 0)\) scoring rule is used.

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<tr>
<th></th>
<th>30</th>
<th>0</th>
<th>20</th>
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<tbody>
<tr>
<td>A</td>
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\(F(R) = B\) since \(Sc(A) = 110\), \(Sc(B) = 120\), \(Sc(C) = 90\).

But

\[
\begin{bmatrix}
A \\
B \\
C
\end{bmatrix}
\quad \xrightarrow{10 < k < 20} \quad 
\begin{bmatrix}
A \\
C \\
B
\end{bmatrix}
\]  

makes \(F(R') = A\),

\[
\begin{bmatrix}
A \\
B \\
C
\end{bmatrix}
\quad \xrightarrow{k > 20} \quad 
\begin{bmatrix}
A \\
C \\
B
\end{bmatrix}
\]  

makes \(F(R''') = C\).

Only unsafe strategic votes exist!
Theorem
Suppose that the number of alternatives is at least three. Let $F$ be any onto and non-dictatorial social choice function. Then there is a profile $R$ at which a voter can make a safe strategic call.
Main Results

Theorem
Suppose that the number of alternatives is at least three. Let $F$ be any onto and non-dictatorial social choice function. Then there is a profile $R$ at which a voter can make a safe strategic call.

Theorem (Extension of the GS Theorem)
Suppose that the number of alternatives is at least three. Then any onto and non-dictatorial social choice rule is safely manipulable by a single voter.
Sample Questions

1. How to evaluate the real complexity of forming a coalition of manipulators?
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2. What is the complexity of deciding if it possible for someone to make a safe strategic call?