COMP108 Algorithmic Foundations

Tutorial 1 (Suggested solution and Feedback) w/c 6th February 2017

1. (a) $(x-2)(x+1)+x+2 = x^2-x-2+x+2 = x^2$

   (b) $\sqrt{x^2+2x+1} = \sqrt{(x+1)^2} = x+1$

   (c) $\frac{x^2-x-2}{x-2} + \frac{x^2+2x+1}{x+1} = \frac{(x-2)(x+1)}{x-2} + \frac{(x+1)^2}{x+1} = x+1+x+1 = 2x+2$
   One way to factorise x^2-x-2: we want to express it as $(x+/-?)(x+/-?)$. To get the constant -2, we have two options, either 2×-1 or -2×1. Then we check that $(x+2)(x-1)$ gives $x^2+2x-x-2 = x^2+x-2$. On the other hand, $(x-2)(x+1)$ gives $x^2-2x+x-2 = x^2-x-2$. Therefore, we know $(x^2-x-2) = (x-2)(x+1)$.

   (d) $2(\frac{x+3}{3} + \frac{x}{6}) = 2(\frac{2x+6}{6} + \frac{x}{6}) = 2(\frac{3x+6}{6}) = 2(\frac{x+2}{2}) = x+2$

   (e) $\log_2 32 + \log_3 9 = 5+2 = 7$
   because 32 is 2^5 and 9 is 3^2.

2. The trace table for $m = 32$.

	x	*count*
Before while loop	1	0
1st iteration	2	1
2nd iteration	4	2
3rd iteration	8	3
4th iteration	16	4
5th iteration	32	5

   The output of the algorithm for $m = 32$ is 5.
   The while loop is executed for $\log_2 m$ times when the input m is a positive power of 2.

3. To solve this problem, we can identify a few sub-problems.

   - We need a counter variable to go from x to y, how do we write this as a skeleton of the loop?
   - We need to accumulate a sum, so we need a variable, what should be its initial value before the loop?
   - In the loop, we need to check if the counter variable is a multiple of 3, how should the if-statement look like?

   There are many ways to write the pseudo code. Some examples are shown below.

```
$sum = 0$
$i = x$
while $i \le y$ do
begin
    if $i\%3 == 0$
    then $sum = sum + i$
    $i = i + 1$
end
output $sum$
```

Alternatively,

```
$sum = 0$
$i = \lceil x/3 \rceil$
**while** $i \leq \lfloor y/3 \rfloor$ **do**
**begin**
    $sum = sum + 3 * i$
    $i = i + 1$
**end**
output $sum$
```

4. The farmer can cross the river as follows. Notations: F-farmer, W-wolf, G-goat, C-cabbages. Remind that W&G and G&C cannot be left alone without F.

	left bank	boat	direction	right bank
initial	W,G,C	—	—	nothing
1st ride	W,C	F+G	→	nothing
2nd ride	W,C	F	←	G
3rd ride	C	F+W	→	G
4th ride	C	F+G	←	W
5th ride	G	F+C	→	W
6th ride	G	F	←	W,C
7th ride	nothing	F+G	→	W,C
final	nothing	—	—	W,G,C