COMP108 Algorithmic Foundations — Tutorial 2

w/c 13th February 2017

Name: _____

Hand in your answer for the question(s) marked "Do this during tutorial" to the demonstrator at the end of the tutorial (even if you haven't finished it). You will get feedback in the next tutorial. Tutorial participation contributes to 5% of overall marks.

1. **[Do this before tutorial]** Alice goes to the sports centre every x days on day x, $2x$, $3x$, $\cdots$ while Bob goes every y days on day y, $2y$, $3y$, $\cdots$. They want to know on which days they will see each other in the sports centre. Write a pseudo code of a while-loop to output all those days for up to 100 days. You can assume that $x > y$. *(Hint: it is related to common multiples.)*

   For example, if $x = 6$ and $y = 4$, then the output should be $12, 24, 36, 48, 60, 72, 84, 96$.

2. **[Do this before tutorial]** Prove by mathematical induction that $2^0 + 2^1 + 2^2 + \cdots + 2^n = 2^{n+1} - 1$, for all integers $n \geq 0$. Remember: we need the base case (when $n = 0$), induction hypothesis (when $n = k$), and induction step (when $n = k + 1$).

3. [**Do this during tutorial**] Prove by mathematical induction about the sum of odd integers: $1 + 3 + 5 + \cdots + (2n - 1) = n^2$, for all integers $n \geq 1$.

4. [**Puzzle for fun**] Nine balls look identical except one is of different weight (can be heavier or lighter). How can you weigh only three times on a balance scale to find out which one is different and whether it is heavier and lighter?

5. [**For those who want more exercises on Mathematical Induction.**]
Prove the following property by mathematical induction.

$$\sum_{i=0}^{n}(1 + 4i) = (n + 1)(2n + 1) \text{ for all integers } n \geq 0.$$

Note that $\sum_{i=0}^{n}(1 + 4i) = 1 + 5 + 9 + \ldots + (1 + 4n)$.