COMP108 Algorithmic Foundations — Tutorial 3

w/c 20th February 2017

Name: ____________________________________

Hand in your answer for the question(s) marked "Do this during tutorial" to the demonstrator at the end of the tutorial (even if you haven't finished it). You will get feedback in the next tutorial. Tutorial participation contributes to 5% of overall marks.

All logarithms are to the base 2.

1. **[Do this before tutorial]** State (without proof) the order of growth of the following functions in big-O notation.

   (a) $10 + 3n + 2n^2 + n^4 + 5n^3$ $\qquad O(\_\_\_\_\_)$

   (b) $5 + 3n^3 + 2n^2 \log n + 5n + 2n^2$ $\qquad \_\_\_\_\_\_\_\_$

   (c) $\sqrt{n^5} + n^3$ $\qquad \_\_\_\_\_\_\_\_$

2. **[Do this before tutorial]** *Prove* that the function $f(n) = 2n^2 + 3n + 4$ is $O(n^2)$.
   (That is, show that there exists constants c and n_0 such that $2n^2 + 3n + 4 \leq cn^2$ for all $n \geq n_0$.)

$$\begin{array}{rcll} 2n^2 & = & 2n^2 & \forall n \\ 3n & \leq & \_\_\_\_\_\_ & \forall n \geq \_\_\_\_\_\_ \\ 4 & \leq & \_\_\_\_\_\_ & \forall n \geq \_\_\_\_\_\_ \\ \therefore \quad 2n^2 + 3n + 4 & \leq & \_\_\_\_\_\_ & \forall n \geq \_\_\_\_\_\_ \end{array}$$

3. **[Do this during tutorial]** *Prove* that the function $f(n) = 2n^3 + 3n^2 + n \log n$ is $O(n^3)$.
   (That is, show that there exists constants c and n_0 such that $f(n) \leq cn^3$ for all $n \geq n_0$.)

$$\begin{array}{rcll} 2n^3 & = & 2n^3 & \forall n \\ 3n^2 & \leq & \_\_\_\_\_\_ & \forall n \geq \_\_\_\_\_\_ \\ n \log n & \leq & \_\_\_\_\_\_ & \forall n \geq \_\_\_\_\_\_ \\ \therefore \quad 2n^3 + 3n^2 + n \log n & \leq & \_\_\_\_\_\_ & \forall n \geq \_\_\_\_\_\_ \end{array}$$

4. **[Do this during tutorial]** A *prime number* is a number that can be divisible by 1 and itself only. Write a pseudo code of an algorithm to determine if x is a prime number or not. What is the *time complexity* (in Big-O notation) of this algorithm in terms of x?

   Hint: (1) We can use a loop to check for each integer i smaller than x whether x is divisible by i. (2) If we want to make it quicker, we can stop earlier, the question is when should we stop the loop.

5. **[Puzzle for fun]** Assuming an infinite supply of the numbers 1, 3 and 7. Can you pick 10 numbers to obtain a sum of 39? If yes, write down the 10 numbers; if not, explain why. [For example, if you choose 1, 1, 1, 1, 1, 3, 3, 3, 3, 7, then the sum is 24.]