COMP108 Algorithmic Foundations — Tutorial 6

w/c 13th March 2017

Name: ____

Hand in your answer for the question(s) marked "Do this during tutorial" to the demonstrator at the end of the tutorial (even if you haven't finished it). You will get feedback in the next tutorial. Tutorial participation contributes to 5% of overall marks.

1. **[Do this before tutorial]** Consider the following recurrence.

$$T(n) = \begin{cases} 3 & \text{if } n = 1 \\ 4 \times T(\frac{n}{4}) + 3 & \text{if } n > 1 \end{cases}$$

Prove that $T(n)$ is $O(n)$ by the *substitution method*, i.e., use mathematical induction.

We are going to prove that $T(n) = 4 \times n - 1$ for all $n \geq 1$.

Base case:

When $n = 1$, L.H.S. $= T(1) =$ ______________ R.H.S. $=$ ________________________

Therefore, L.H.S. = R.H.S.

Induction hypothesis: Assume that the property holds for all integers $n' < n$, i.e., assume

$$T\left(\frac{n}{4}\right) = 4 \times \frac{n}{4} - 1$$

Induction step:

We want to prove $\boldsymbol{T(n) = 4 \times n - 1}$.

[The induction step can be proved by first using the recurrence to express $T(n)$ in terms of $T(\frac{n}{4})$, and then use the hypothesis.]

L.H.S. $= T(n) =$ $\leftarrow$ use the recurrence

$=$ $\leftarrow$ use induction hypothesis

$=$ $\leftarrow$ arithmetic

Therefore, L.H.S. = R.H.S. and the property holds for n.

Conclusion: $T(n) = 4 \times n - 1$ for all positive integers n and therefore, $T(n)$ is $O(n)$.

2. [**Do this during tutorial**] Consider the following recurrence.

$$T(n) = \begin{cases} 1 & \text{if } n = 1 \\ 4 \times T(\frac{n}{4}) + n & \text{if } n > 1 \end{cases}$$

Prove that $T(n)$ is $O(n \log n)$ by the *substitution method*, i.e., use mathematical induction.

We are going to prove that $T(n) \leq 4 \times n \times \log n$ for all $n \geq 4$.

Base case:

When $n = 4$, L.H.S. $= T(4) =$ ____________________________ R.H.S. $=$ ____________________

Therefore, L.H.S. $\leq$ R.H.S.

Induction hypothesis: Assume that the property holds for all integers $n' < n$, i.e., assume

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Induction step:

We want to prove $\boldsymbol{T(n) \leq 4 \times n \times \log n}$.

L.H.S. $= T(n) \quad =$ $\qquad\qquad$ $\leftarrow$ use the recurrence

$\leq$ $\qquad\qquad$ $\leftarrow$ use induction hypothesis

$=$ $\qquad\qquad$ $\leftarrow$ arithmetic

3. [**Do this during tutorial**] Describe or write pseudo code for a divide-and-conquer algorithm to find the product of the numbers in an array $A[\,]$ with n integers $A[1]..A[n]$.

For simplicity, you can assume that n is a power of 2.