COMP108 Algorithmic Foundations
Tutorial 7 (Suggested solution and Feedback) w/c 20th March 2017

1. Using the iterative method:

$$\begin{aligned}
T(n) &= T(n-1)+1 && \leftarrow \text{ use the recurrence for } T(n)\\
&= (T(n-2)+1)+1 && \leftarrow \text{ use recurrence for } T(n-1)\\
&= T(n-2)+2 && \leftarrow \text{ expand the bracket}\\
&= (T(n-3)+1)+2 && \leftarrow \text{ use recurrence for } T(n-2)\\
&= T(n-3)+3 && \leftarrow \text{ expand the bracket}\\
&= (T(n-4)+1)+3 && \leftarrow \text{ use recurrence for } T(n-3)\\
&= T(n-4)+4 && \leftarrow \text{ expand the bracket}\\
&\ \ \vdots\\
&= T(1)+(n-1) && \leftarrow \text{ in terms of } T(1)\\
&= 1+n-1\\
&= n
\end{aligned}$$

2. Refer to the solution in Tutorial 6.

3. Iterative method:

$$\begin{aligned}
T(n) &= 2\times T(\tfrac{n}{2})+1 && \leftarrow \text{ use the recurrence for } T(n)\\
&= 2\times(2\times T(\tfrac{n}{2^2})+1)+1 && \leftarrow \text{ use recurrence for } T(\tfrac{n}{2})\\
&= 2^2\times T(\tfrac{n}{2^2})+2^1+1 && \leftarrow \text{ expand the bracket}\\
&= 2^2\times(2\times T(\tfrac{n}{2^3})+1)+2^1+1 && \leftarrow \text{ use recurrence for } T(\tfrac{n}{2^2})\\
&= 2^3\times T(\tfrac{n}{2^3})+2^2+2^1+1 && \leftarrow \text{ expand the bracket}\\
&= 2^4\times T(\tfrac{n}{2^4})+2^3+2^2+2^1+1 && \leftarrow \text{ use recurrence for } T(\tfrac{n}{2^3}) \text{ and expand}\\
&\ \ \vdots\\
&= 2^{\log n}\times T(\tfrac{n}{2^{\log n}})+2^{\log n-1}+\cdots+2^3+2^2+2^1+1 && \leftarrow \text{ in terms of } T\big(\tfrac{n}{2^{\log n}}\big)\text{, i.e., } T(1)\\
&= 2^{\log n}\times T(1)+2^{\log n-1}+\cdots+2^3+2^2+2^1+1 && \leftarrow \text{ using Hint (i)}\\
&= 2^{\log n}+2^{\log n-1}+\cdots+2^3+2^2+2^1+1 && \leftarrow \text{ because } T(1)=1\\
&= 2^{\log n+1}-1 && \leftarrow \text{ using Hint (ii)}\\
&= 2\times 2^{\log n}-1\\
&= 2n-1 && \leftarrow \text{ using Hint (i)}
\end{aligned}$$

4. See lecture notes "Divide-and-conquer methods"

5. This problem is the classic Mutilated Chessboard Problem discussed by Martin Gardner.

   The answer is NO. Both removed squares have identical color! So their removal leaves us with 30 squares of one color and 32 squares of the other color. Since a domino always covers one black and one white squares, the 62 remaining squares cannot be tiled.