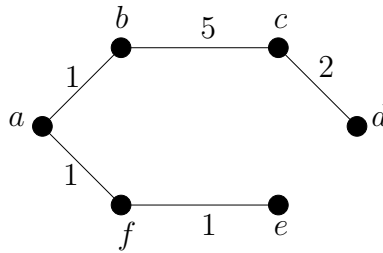


COMP108 Algorithmic Foundations  
 Tutorial 10 (Suggested Solution and Feedback) w/c 1st May 2017

1. (a) Table of edges in ascending order of costs.

edge	$(a, b)$	$(a, f)$	$(b, f)$	$(e, f)$	$(c, d)$	$(a, e)$	$(b, c)$	$(c, f)$	$(d, e)$	$(b, e)$	$(d, f)$
cost	1	1	1	1	2	3	5	6	6	7	8

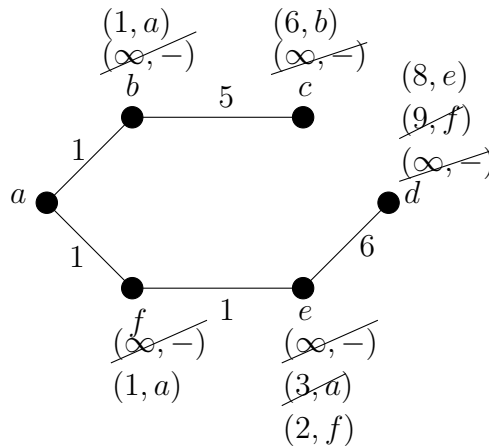
- (b) The MST:



Order of edges selected:  $(a, b)$ ,  $(a, f)$ ,  $(e, f)$ ,  $(c, d)$ ,  $(b, c)$ . (The order of equal-weight edges should follow your table in (a).)

2. In this question, you need to (i) draw the edges that have been chosen, (ii) list the changes of the labels, (iii) list the chosen edges in the order they are chosen.

The shortest paths and changes of labels:



**Order of edges selected:**  $(a, b)$ ,  $(a, f)$ ,  $(f, e)$ ,  $(b, c)$ ,  $(e, d)$

**Note:** This is the only unique solution. Following Dijkstra's algorithm, we will NOT find the path  $a, b, c, d$ .

Alternatively, you can show steps like the following:

Intermediate steps:

edge chosen this round	chosen vertices	vertices not chosen
-	-	$a : (0, a), b : (\infty, -), c : (\infty, -),$ $d : (\infty, -), e : (\infty, -), f : (\infty, -)$
-	$a : (0, a)$	$b : (1, a), c : (\infty, -), d : (\infty, -),$ $e : (3, a), f : (1, a)$
$(a, b)$	$a : (0, a), b : (1, a)$	$c : (6, b), d : (\infty, -),$ $e : (3, a), f : (1, a)$
$(a, f)$	$a : (0, a), b : (2, a), f : (1, a)$	$c : (6, b), d : (9, f), e : (2, f)$
$(f, e)$	$a : (0, a), b : (2, a), f : (1, a)$ $e : (2, f)$	$c : (6, b), d : (8, e)$
$(b, c)$	$a : (0, a), b : (2, a), f : (1, a)$ $e : (2, f), c : (6, b)$	$d : (8, e)$
$(e, d)$	$a : (0, a), b : (2, a), f : (1, a)$ $e : (2, f), c : (6, b), d : (8, e)$	-

Alternatively, you can show steps like the following:

Intermediate steps:

**Step 0:** the label of  $a$  is  $(0, a)$ , other vertices  $(\infty, -)$ .

Vertex  $a$  is the source.

**Step 1:** The label of  $b$  becomes  $(1, a)$ ;  $e$   $(3, a)$ ; and  $f$   $(1, a)$ .

Choose among  $(a, b)$  with total cost 1,  $(a, e)$  with total cost 3,  $(a, f)$  with total cost 1:  
The edge  $(a, b)$  is chosen.

**Step 2:** The label of  $c$  becomes  $(6, b)$ ;  $e$  and  $f$  unchanged.

Choose among  $(a, e)$  with total cost 3,  $(a, f)$  with total cost 1,  $(b, c)$  with total cost 6:  
The edge  $(a, f)$  is chosen.

**Step 3:** The label of  $d$  becomes  $(9, f)$ ;  $e$   $(2, f)$ ;  $c$  unchanged.

Choose among  $(f, e)$  with total cost 2,  $(b, c)$  with total cost 6,  $(f, d)$  with total cost 9:  
The edge  $(f, e)$  is chosen.

**Step 4:** The label of  $d$  becomes  $(8, e)$ .

Choose among  $(b, c)$  with total cost 6,  $(e, d)$  with total cost 8: The edge  $(b, c)$  is chosen.

**Step 5:** No label is changed.

Choose among  $(e, d)$  with total cost 8: The edge  $(e, d)$  is chosen.

3. If the first player takes one petal, the second player takes two in the opposite end to leave two groups of 5 adjacent petals each. If the first player takes two petals, the second player takes one to result in the same two groups of 5 adjacent petals each. After that, no matter what the first player takes, the second player also takes the same in the opposite end such that the number of groups of adjacent petals is always even.