Game Theory, Internet and the Web
A new Science?

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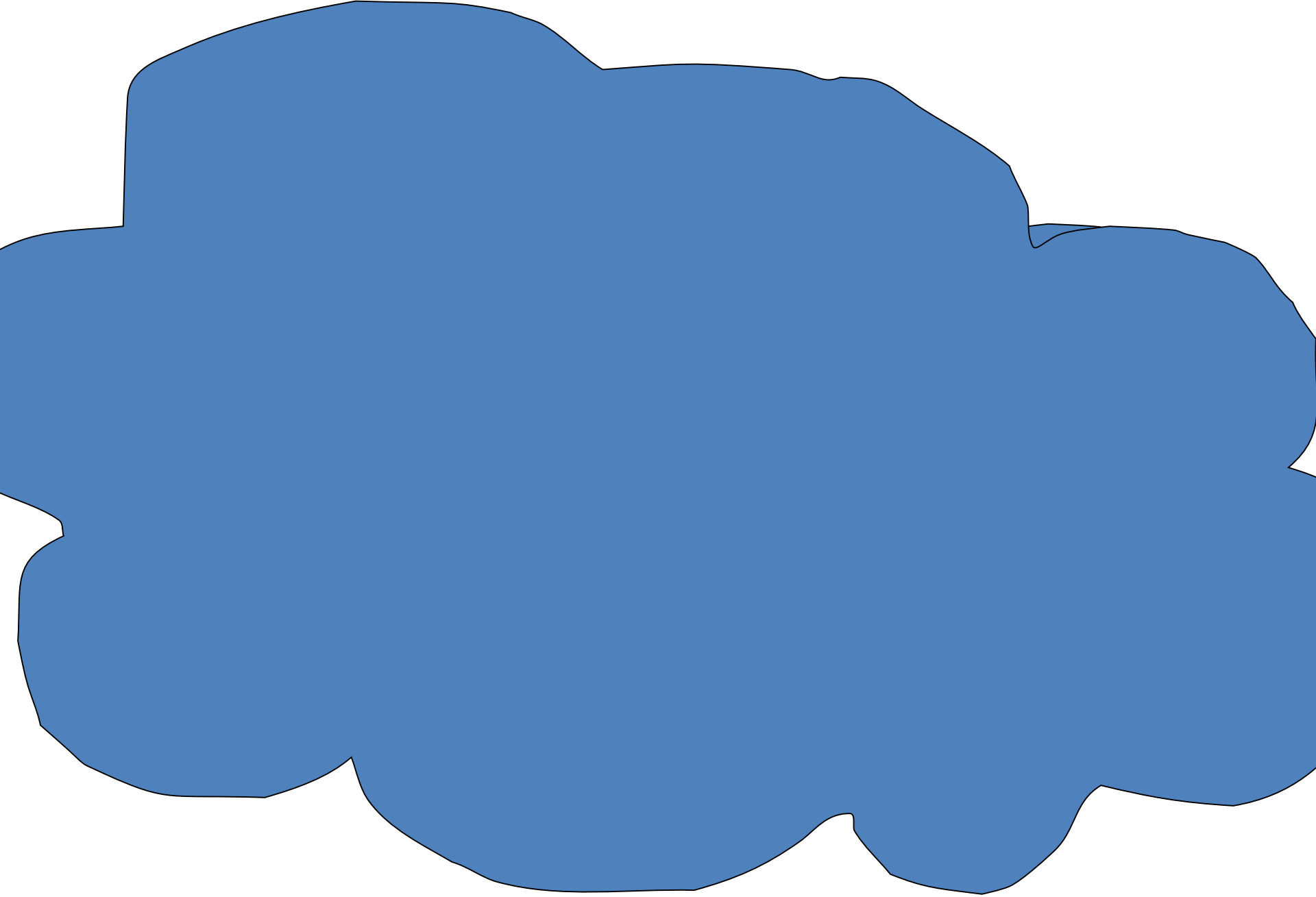
(with help from C. H. Papadimitriou, Berkeley)

- Main Goal of Computer Science (1950-2000):

To investigate the capabilities and limits of the Computing Model of von Neumann – Turing (and its software)

(Math Tools: Logic, Combinatorics, Automata)

- What is the goal of Computer Science for the 21st century?



The Internet and the Web

- **Built, operated and used by a variety of entities with diverse interests.**
- Not yet understood deeply

"The Web is a huge arena of competition and cooperation between many logical entities with selfish interests" (C.H. Papadimitriou)

New Tool: Math Foundations of Economics, Game Theory

Game Theory

Game = Any interaction among rational and logical entities each of which may have different motives and goals.

Game $\Gamma = (N, \{S_i\}, \{u_i\})$

N = Set of "players"

S_i = Set of pure strategics of player i

$u_i: XS_i \rightarrow R$ = The utility function of player i

(Expected Utility Theorem of Von Neumann & Morgenstern)

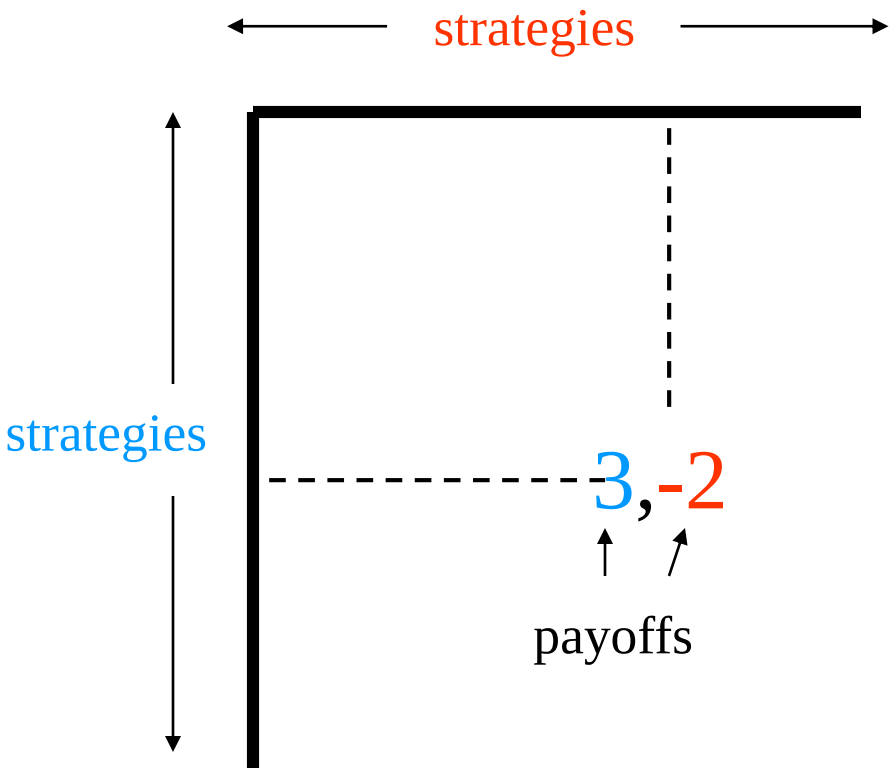
- A game is a **system** of rational and logical entities in interaction

- **Selfish entities**: Each of them has a possibly different utility function (and wants to maximize it)

  "People are expected utility maximizers"

- Such systems are very different from the "usual"

Game Theory



Similarly for many players

e.g.

This for that

1,-1	-1,1
-1,1	1,-1

Prisoner's dilemma

3,3	0,4
4,0	1,1

Rational Behaviour

- Dominant Strategies (but they do not always exist)
- Nash Equilibria (mutual best response)

  Each player will not benefit if she deviates unilaterally
- Mixed Strategies allowed (i.e. prob. distributions on the pure strategies of each player).

John Forbes Nash, Jr.

(A beautiful mind)

Theorem [Nash, 1952]

Every finite game has at least one Nash Equilibrium

The beauty of Mathematics

Discrete Math (Graphs)

$\downarrow$

Sperner Lemma (Combinatorics)

$\downarrow$

Fixpoint Theorem of Brower (Analyis)

$\downarrow$

Kakutani's Theorem $\Rightarrow$ Market Equilibria

$\downarrow$

Nash's Theorem

$\uparrow$?

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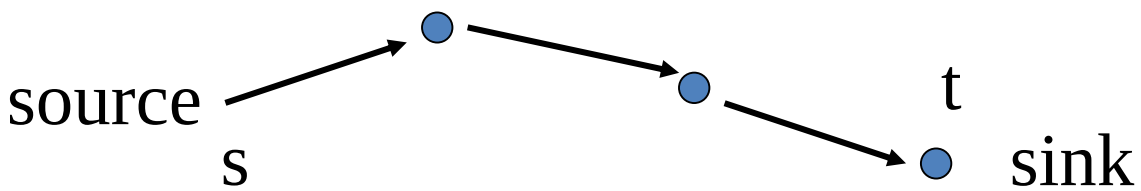
zero sum games

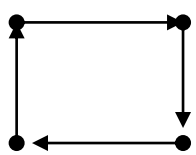
$\downarrow \in \mathrm{P}$

duality, linear programming

Discrete Mathematics

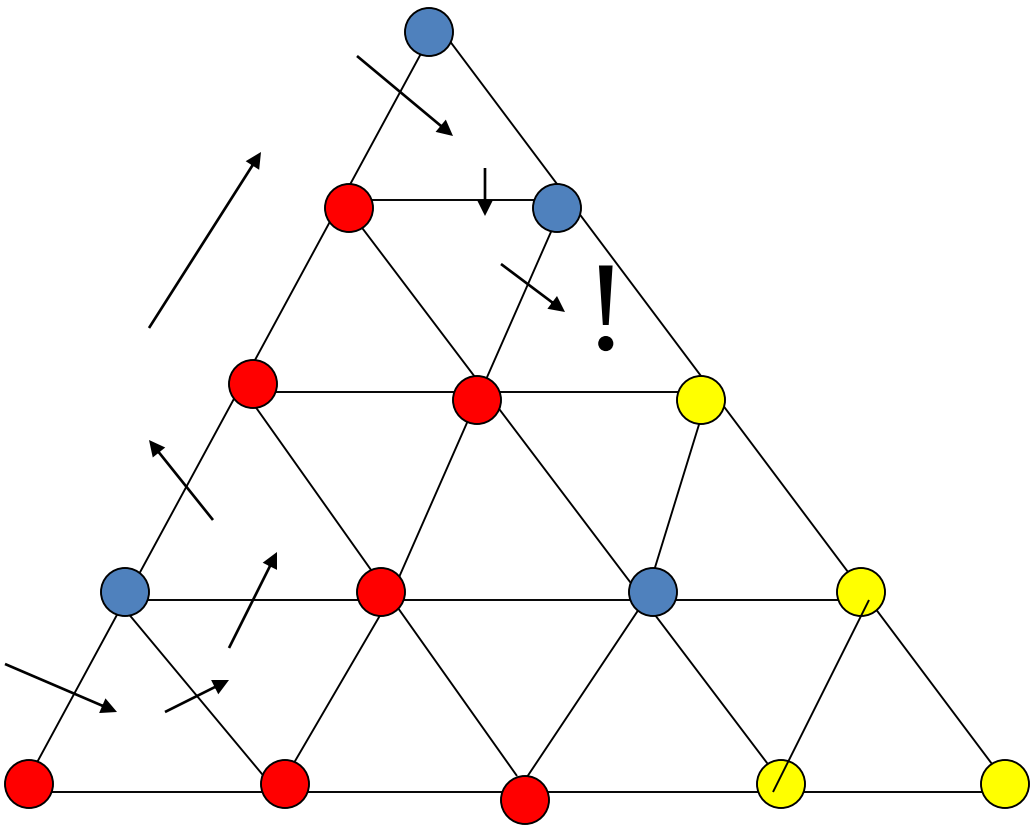
«Any directed graph with indegrees and outdegrees at most 1, if it has a **source** then it has a **sink**»





Sperner's Lemma: Any legal coloring of a triangulated polytope contains a trichromatic triangle.

Proof:



Sperner $\Rightarrow$ Brower

Brower's Thm:: Any continuous function from a polytope to itself has a fix point.

Proof

Triangulate the polytope. Color the vertices according to the direction indicated by the function.

Sperner $\Rightarrow$ There exist a triangle with "no exit"

Now make the triangulation dense

The subsequence of the centers of the Sperner triangles converges

QED

Brower $\Rightarrow$ Nash

For each pair of mixed strategies x, y let:

$\varphi(x,y) = (x', y')$, where x' maximizes

$\quad off_1(x',y) - |x - x'|^2$,

(off_1 = expected payoff of player 1)

Similarly for y'.

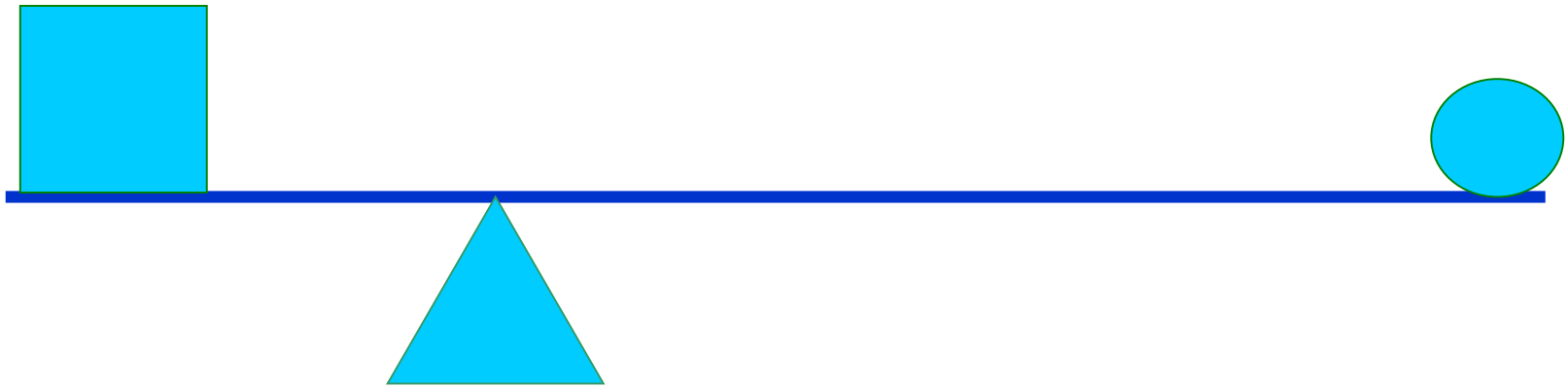
Now any Brower fixpoint is a Nash Equilibrium

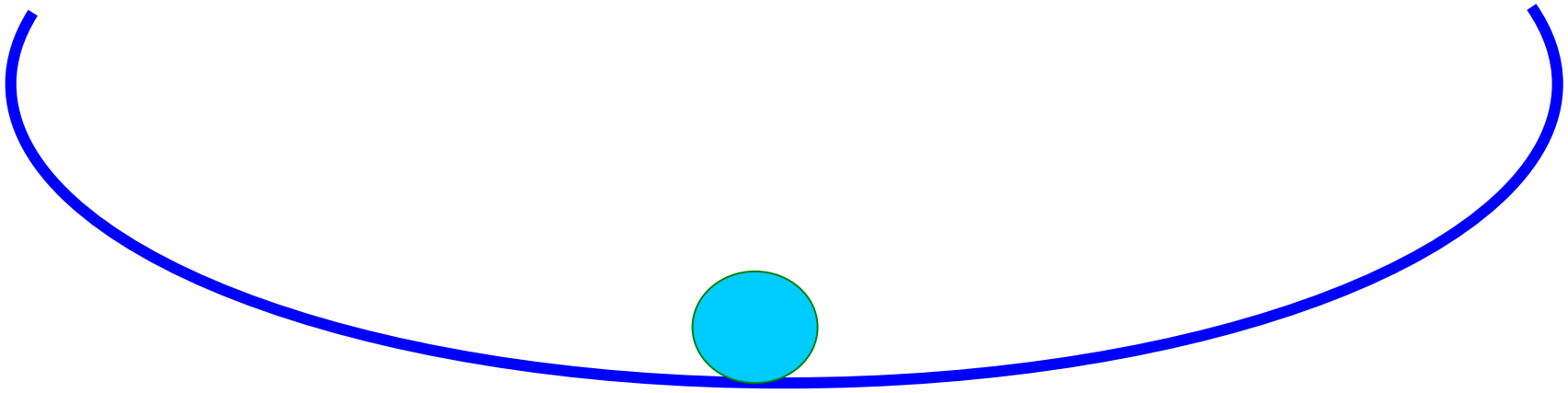
QED

Nash $\Rightarrow$ von Neumann

If the game is zero – sum (constant sum) them the mutual best responses are the same as a max-min pair (and due to duality, the solution of a Linear Program).

The notion of Equilibrium is basic in many Sciences





Some Questions

- How logical is the probabilistic play? (poker bluffs, tax evasion)
- Can we “learn” (or compute) an equilibrium;
- What is the best (worst) Equilibrium;

Approximate Equilibria

- **ε-Nash**: Each player stays at equilibrium decision, even if she may gain at most "epsilon" by unilaterally deviating

"We don't change our mate for a slightly better"

- Can we compute ε-Nash equilibria efficiently?

BEST Poly-time result: $\varepsilon = 0.34$
[Tsaknakis, Spirakis, 07]
Sub exponential methods (Lipton, Markakis, Mehta, 03)
(Tsaknakis, Spirakis, 10)

- Still open to go below "1/3"

The battlefield

- The “system”
- The Web
- The terrain
- Society

SOCIAL COST (Function of Social happiness)

$$SC : XC_i \rightarrow R$$

The function measures the social cost, given the choices (strategies) y_i of each player i.

Examples of Social Cost

- Energy spent
- Max delay in streets
- Political cost for the country / EU given the decisions of the leaders.

Altrouist: A player whose utility function "agrees" with the social cost function

If God would order everybody how to decide then we would get an Optimal Social Cost, OPT

- But, actually, the “system” reaches an equilibrium P

- How far is SC(p) from OPT?

(Usually OPT is not even an equilibrium!)

The Price of Anarchy (PoA)

$$R = \max \frac{SC(p)}{OPT} \geq 1$$

(max over all NE p).

[Koutsoupias, Papadimitriou, 1999]
Coordination Ratio

[Mavronicolas, Spirakis, 2001]

[Roughgarden, Tardos, 2001]

But also

The Price of Stability (PoS)

$$T = \min \frac{SC(p)}{OPT}$$

(min over all NE p)

[Schulz, Stier Moses, 2003]
[Anshelevich et al, 2004]

- Lots of results for PoA, PoS for congestion games, network creation games etc.

How to Control Anarchy

- **Mechanisms** design
- **A set of rules and options put by game's designers**. Does not affect the **free will** of players. But appeals to their selfishess (e.g. payments, punishments, ads). Aims in "moving the game" to "good equilibria" (desirable by the designer)
- New challenges in algorithms!
- Auctions
- Lies and truthfullness
- Stackelberg's games (Leader plays first)

Dynamics

- How can a **Selfish System** (e.g. the markets, Society, the Web) **approach** an Equilibrium?

- **Dynamics**

Players interact, learn and do selfish choices, and the "state" of the System changes with time

- Many, repeated, concurrent games all the time.

The world is not perfect

- Players may be illogical and not so rational
- Players may have limited information about the game (s), or limited knowledge.
- Errors are human / also for Computers ("Trembling Hand")
- Other factors (enemies of the System, "free-riders", strange behaviour, …)

But, fortunately:

- Players can learn, adapt, evolve
- Biology and “Self-regulation” [Self-stabilization) [Dijkstra] [S. Dolev, E. Schiller]
- Equilibria in animal, plants (microbes) communities in antagonism or cooperation
- John Maynard Smith (1974) (Evolutionary Games).

Yet another Science

- Mathematical Ecology
  (Alfred Lotka, Vito Volterra, 1920)

(dynamics of moskitos, also of hunter-prey fish in Adriatic Sea).

- Ancestor of Evolutionary Game Theory
- Evolutionary Methods in Economics

  [Robert Axelrod, 1984]

Relevant Math.

- Nonlinear dynamical systems
- Differential Equations
- Attractors, oscillations, Equilibria
- Chaotic Behaviour!

(and, again, fixpoints!)

Dynamics of Selfish Systems

- **Norms** (Contracts, Social Rules)

- **"Internal" causes for change**:
  - players' selfish behaviour
  - learning, adaptation

- **Externalities**

- **"Final" result** (equilibrium, stability, but also complex behaviour, chaos)

A New Science

- Deep and elegant
- Different
- Strong interaction with Foundations of CS
- Emerges everywhere (Research, Education, funds)
  (also **new Industry**: e-commerce, ads, Social Nets …)
- A new light in Complexity
- Isaac Asimov's "psychohistory"?

MANY THANKS
FOR LISTENING TO ME.