Modalities in Knowledge Representation

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ABSTRACT

Standard knowledge representation systems are supposed to be able to represent either common or individual knowledge about the world.
In this paper we propose an extension to such knowledge representation systems which, in a uniform manner, allows to express beliefs of multiple agents as well as knowledge, desire, time and in fact any modality which has a first-order predicate logic possible world semantics.

1 Introduction

Since the mid-seventies a variety of knowledge representation systems in the tradition of semantic networks and terminological logics has been proposed. The most famous example is KL-ONE[3] which first appeared in 1977 in R. J. Brachman's Ph.D. thesis. Other examples are KRYPTON[2], NIKL[12], and $\mathcal{KRIS}$[1].

All these systems can be used to represent common or individual knowledge about the world. Recently, terminological logics have been extended to allow the representation of the knowledge or the beliefs of multiple agents in one knowledge base[4, 7]. In this paper, we investigate such an extension which allows not only to represent either knowledge or beliefs of multiple agents but a huge variety of modal operators for multiple agents.

2 Syntax and Semantics for Mod-$\mathcal{ALC}$

We assume four disjoint alphabets, the *primitive concepts* $\mathcal{C}$, the *primitive roles* $\mathcal{R}$, the set $\mathcal{M}$ of *modal operator names*, and the individual objects $\mathcal{O}$. In particular, there is a distinguished subset $\mathcal{A}$ of the individual objects, called the set of *agents*, containing the special agent *all* which is supposed to denote the union of all the agents in order to be able to express mutual belief. The tuple $(\mathcal{O}, \mathcal{A}, \mathcal{M}, \mathcal{C}, \mathcal{R})$ is called the *signature*, denoted by Σ.

The set of *concept terms* (or just *concepts*) and *role terms* (or just *roles*) is inductively defined as follows. Every primitive concept is a concept term and every primitive role is a role term. Now assume that C, C_1, and C_2 are concepts, R, R_1, and R_2 are roles, m is a modal operator name, a is an agent's name. Then $C_1 \sqcap C_2$, $\neg C$, $\exists\, R.C$, and $\Diamond_{(m,a)}\, C$ are concept terms, and $R_1 \sqcap R_2$, R^{-1}, $R|_C$, $\Box_{(m,a)}\, R$ and $\Diamond_{(m,a)}\, R$ are role terms.

Thus we are now able to describe the sentences of our language Mod-$\mathcal{ALC}$ which are divided into *terminological sentences* and *assertional sentences*. If C_1 and C_2 are concepts and R_1 and R_2 are roles then $C_1 \sqsubseteq C_2$, $R_1 \sqsubseteq R_2$ are terminological sentences. If C is a concept, R is a role, and O, O_1, and O_2 are individual objects then

$O \in C$ and $(O_1, O_2) \in R$ are assertional sentences. Moreover, if Φ is a terminological (respectively assertional) sentence and if m is a modal operator name and a is an agent's name then $\Box_{(m,a)} \Phi$ and $\Diamond_{(m,a)} \Phi$ are terminological (respectively assertional) sentences. A set of terminological and assertional sentences is a *knowledge base*.

So far the syntax of Mod-$\mathcal{ALC}$ has been described. Now we have to provide its semantics.

Definition 1 (Σ-Structures) As usual we define a Σ-*structure* as a pair $(\mathcal{D}, \mathcal{I})$ which consists of a domain $\mathcal{D}$ and an interpretation function $\mathcal{I}$ which maps the individual objects to elements of $\mathcal{D}$, primitive concepts to subsets of $\mathcal{D}$ and the primitive roles to subsets of $\mathcal{D} \times \mathcal{D}$.

Definition 2 (Frames and Interpretations) By a frame $\mathcal{F}$ we understand any pair $(\mathcal{W}, \Re)$ where $\mathcal{W}$ is a non-empty set (of worlds) and $\Re = \biguplus_{m \in \mathcal{M}, a \in \mathcal{A}} \Re^a_m$ where the $\Re^a_m$'s are binary relation on $\mathcal{W}$, the so-called *accessibility relations* between worlds.

By a Σ-interpretation $\Im$ based on $\mathcal{F}$ we understand any tuple $(\mathcal{D}, \mathcal{F}, \Im_{\text{loc}}, \epsilon)$ where $\mathcal{D}$ denotes the common domain of all Σ-structures in the range of $\Im_{\text{loc}}$, $\mathcal{F}$ is a frame, and $\Im_{\text{loc}}$ maps worlds to Σ-structures with common domain $\mathcal{D}$ which interpret agents' names equally.

Definition 3 (Interpretation of Terms) Let $\Im = (\mathcal{D}, \mathcal{F}, \Im_{\text{loc}}, \epsilon)$ be a Σ-interpretation and let $\Im_{\text{loc}}(\epsilon) = (\mathcal{D}, \mathcal{I})$. We define the interpretation of terms inductively over their structure:

$$
\begin{array}{lcll}
\Im(C) & = & \mathcal{I}(C) & \text{if } C \text{ is a primitive concept} \\
\Im(R) & = & \mathcal{I}(R) & \text{if } R \text{ is a primitive role} \\
\Im(C_1 \sqcap C_2) & = & \Im(C_1) \cap \Im(C_2) & \\
\Im(\neg C) & = & \mathcal{D} \setminus \Im(C) & \\
\Im(\exists R.C) & = & \{d \in \mathcal{D} \mid e \in \Im(C) \text{ for some } e \text{ with } (d,e) \in \Im(R)\} & \\
\Im(\Diamond_{(m,a)} C) & = & \{d \in \mathcal{D} \mid d \in \Im[\chi](C) \text{ for some } \chi \text{ with } \Re^a_m(\epsilon, \chi)\} & \\
\Im(R_1 \sqcap R_2) & = & \Im(R_1) \cap \Im(R_2) & \\
\Im(R^{-1}) & = & \{(x,y) \in \mathcal{D} \times \mathcal{D} \mid (y,x) \in \Im(R)\} & \\
\Im(R|_C) & = & \{(x,y) \in \Im(R) \mid y \in \Im(C)\} & \\
\Im(\Box_{(m,a)} R) & = & \{(x,y) \mid (x,y) \in \Im[\chi](R) \text{ for all } \chi \text{ with } \Re^a_m(\epsilon, \chi)\} & \\
\Im(\Diamond_{(m,a)} R) & = & \{(x,y) \mid (x,y) \in \Im[\chi](R) \text{ for some } \chi \text{ with } \Re^a_m(\epsilon, \chi)\} &
\end{array}
$$

where $\Im[\chi] = (\mathcal{D}, \mathcal{F}, \Im_{\text{loc}}, \chi)$.

Definition 4 (Satisfiability) Let $\Im = (\mathcal{D}, \mathcal{F}, \Im_{\text{loc}}, \epsilon)$ be a Σ-interpretation. We define the satisfiability relation $\models$ inductively over the structure of Mod-$\mathcal{ALC}$ sentences:

$$
\begin{array}{lll}
\Im \models C_1 \sqsubseteq C_2 & \text{iff} & \Im(C_1) \subseteq \Im(C_2) \\
\Im \models R_1 \sqsubseteq R_2 & \text{iff} & \Im(R_1) \subseteq \Im(R_2) \\
\Im \models x \in C & \text{iff} & \mathcal{I}(x) \in \Im(C) \\
\Im \models (x,y) \in R & \text{iff} & (\mathcal{I}(x), \mathcal{I}(y)) \in \Im(R) \\
\Im \models \Box_{(m,a)} \Phi & \text{iff} & \Im[\chi] \models \Phi \text{ for every } \chi \text{ with } \Re^a_m(\epsilon, \chi) \\
\Im \models \Diamond_{(m,a)} \Phi & \text{iff} & \Im[\chi] \models \Phi \text{ for some } \chi \text{ with } \Re^a_m(\epsilon, \chi)
\end{array}
$$

Definition 5 Let $\Im$ be an interpretation and let Φ be a Mod-$\mathcal{ALC}$ sentence with $\Im \models \Phi$. Then we call Φ *satisfiable* and we call $\Im$ a *model* for Φ. If all interpretations are models for Φ then we call Φ a *theorem.* Any sentence for which no model exists is called *unsatisfiable.* Thus, Φ is a theorem iff its negation is unsatisfiable.

Let T be a set of Mod-$\mathcal{ALC}$ sentences. We say T *entails* Φ, written $T \models \Phi$, iff every model of T is a model of Φ.

So far we did not consider any special properties of the given modal operators. Typical axiom schemata which reflect such potential additional properties are listed below.

Axiom Schema	Property
$\Box_{(m,a)}\,\Phi \Rightarrow \Diamond_{(m,a)}\,\Phi$	$\forall x \exists y\ \Re^a_m(x,y)$
$\Box_{(m,a)}\,\Phi \Rightarrow \Phi$	$\forall x\ \Re^a_m(x,x)$
$\Phi \Rightarrow \Box_{(m,a)}\Diamond_{(m,a)}\,\Phi$	$\forall x,y\ \Re^a_m(x,y) \Rightarrow \Re^a_m(y,x)$
$\Box_{(m,a)}\,\Phi \Rightarrow \Box_{(m,a)}\Box_{(m,a)}\,\Phi$	$\forall x,y,z\ \Re^a_m(x,y) \wedge \Re^a_m(y,z) \Rightarrow \Re^a_m(x,z)$
$\Diamond_{(m,a)}\,\Phi \Rightarrow \Box_{(m,a)}\Diamond_{(m,a)}\,\Phi$	$\forall x,y,z\ \Re^a_m(x,y) \wedge \Re^a_m(x,z) \Rightarrow \Re^a_m(y,z)$

Table 1: Properties of the accessibility relation

Historically, these properties are called D, T, B, 4, and 5 respectively.

If we consider modal operators for belief, i.e. $\Box_{(believe,a)}$ and $\Diamond_{(believe,a)}$, then 4 and 5 are axioms of introspection. Intuitively, they say that the agent a has complete insight what his own beliefs are concerned. Property T is one of the major characteristics of modal operators for knowledge, i.e. $\Box_{(knows,a)}$ and $\Diamond_{(knows,a)}$, because it enables us to deduce a fact about the real world from an agent's knowledge about the world.

Definition 6 Let $\mathcal{R}$ be a set of properties of the accessibility relation. An interpretation $\Im$ is called a *$\mathcal{R}$-interpretation* if the accessibility relation $\Re$ of the underlying frame $\mathcal{F}$ satisfies all properties in $\mathcal{R}$. We say a set of Mod-$\mathcal{ALC}$ sentences T *entails* Φ *in all $\mathcal{R}$-interpretations*, written $T \models_{\mathcal{R}} \Phi$, if all $\mathcal{R}$-interpretations which are models of T are also models of Φ.

What is remarkable for the properties from above is that they all are first-order properties. Therefore it is not too surprising that a translation of modal terminological and modal assertional axioms into first-order predicate logic can be done very easily.

3 Translating Mod-$\mathcal{ALC}$ into Classical Logic

There have been a lot of proposals for correct and complete calculi for nonclassical logics — temporal logics[11], epistemic logics[8], etc. — and terminological logics[3, 5].

Unfortunately, these calculi require implementations for their respective theorem proving system with hardly a chance to apply results and techniques of the traditional work on automated theorem proving. Therefore, we follow the approach of Ohlbach[10] to eliminate modal operators in a way that we get standard first-order predicate logic formulae that still represent the modal semantics.

However, if we take a closer look at the results of a straightforward translation of terminological and assertional sentences into first-order logic we can see quite a big problem. Already simple knowledge bases result in rather huge clause sets where many of the clauses merely express certain informations about accessibilities and role correlations. But we can exploit a property of the language Mod-$\mathcal{ALC}$ which is typical for most modal and terminological logics, namely, each accessibility relation symbol or role symbol R occurs either in the form $\exists x\, R(\ldots,x) \wedge \Phi(x)$ or in the form $\forall x\, R(\ldots,x) \Rightarrow \Phi(x)$.

First of all let us define what we mean by a *functional simulator.*

Definition 7 (Functional Simulators) Let $\Re$ be an n-ary relation. A set $\mathtt{F}_\Re$ of functions is called a *functional simulator for* $\Re$ if for any $x_1,\ldots,x_n$

$$\Re(x_1,\ldots,x_n) \text{ iff there exists a } f \in \mathtt{F}_\Re \text{ with } f(x_1,\ldots,x_{n-1}) = x_n$$

In particular, if $\Re$ is total, i.e. for any $x_1 \ldots x_{n-1}$ there is a x_n such that $\Re(x_1,\ldots,x_n)$ then each function in $\mathtt{F}_\Re$ is total, too.

I.e. a functional simulator for some relation R is a set of functions which is able to take over the responsibilities of R. What a functional simulator is on the meta-level are the simulator axioms on the object level.

Definition 8 (Simulator Axioms for Total Predicates) According to Definition 7 we call the two axioms

$$\begin{aligned} \mathrm{Sim}_1^R &= \forall u_i, v\; R(\ldots,u_i,\ldots,v) \Rightarrow \exists f{:}\mathtt{F}_R\; f(\ldots,u_i,\ldots) = v \\ \mathrm{Sim}_2^R &= \forall u_i \forall f{:}\mathtt{F}_R\; R(\ldots,u_i,\ldots,f(\ldots,u_i,\ldots)) \end{aligned}$$

the *simulator axioms* for the total predicate R. Note that in the case of non-total predicates the definition of Sim_2^R needs to be changed because in this case we have to be able to deal with partial functions instead of total functions. The collection of all axioms Sim_1^R, respectively Sim_2^R, is called Sim_1, respectively Sim_2.

Thus, for any total (serial) accessibility relation or role we can introduce functional simulators. This allows the following approach for the translation of knowledge bases into first-order predicate logic.

First, we transform all concepts and roles which occur in a given knowledge base into *negation normal form*, i.e. in a form where all negation signs occur solely in front of the primitive concepts. Now we translate the terminological and assertional sentences into first-order logic formulae using the function $[\![\ \_\ ]\!]$.

Axiom	Translation
$[\![\Box_{(m,a)} M_2 \ldots M_n \Phi]\!]_U$	$\forall V : \Re_m^a(U,V) \Rightarrow [\![M_2 \ldots M_n \Phi]\!]_V$
$[\![\Diamond_{(m,a)} M_2 \ldots M_n \Phi]\!]_U$	$\exists V{:}\mathtt{F}_{\Re_m^a}\ [\![M_2 \ldots M_n \Phi]\!]_{V(U)}$
$[\![C_1 \sqsubseteq C_2]\!]_U$	$\forall X : [\![C_1]\!]_{U,X} \Rightarrow [\![C_2]\!]_{U,X}$
$[\![R_1 \sqsubseteq R_2]\!]_U$	$\forall X,Y : [\![R_1]\!]_{U,(X,Y)} \Rightarrow [\![R_2]\!]_{U,(X,Y)}$
$[\![a \in A]\!]_U$	$[\![A]\!]_{U,a}$
$[\![(a,b) \in P]\!]_U$	$[\![P]\!]_{U,(a,b)}$

where the translation of concepts and roles is defined by:

Term	Translation
$[\![A_1]\!]_{U,X}$	$A_1(U,X)$
$[\![\neg A_1]\!]_{U,X}$	$\neg A_1(U,X)$
$[\![C_1 \sqcap \ldots \sqcap C_k]\!]_{U,X}$	$[\![C_1]\!]_{U,X} \wedge \ldots \wedge [\![C_k]\!]_{U,X}$
$[\![\exists R.C]\!]_{U,X}$	$\exists Y : \mathtt{F}_R [\![C]\!]_{U,Y(X)}$
$[\![\Diamond_{(m,a)} C]\!]_{U,X}$	$\exists V : \mathtt{F}_{\Re^a_m} [\![C]\!]_{V(U),X}$
$[\![P]\!]_{U,(X,Y)}$	$P(U,X,Y)$
$[\![R_1 \sqcap \ldots \sqcap R_l]\!]_{U,(X,Y)}$	$[\![R_1]\!]_{U,(X,Y)} \wedge \ldots \wedge [\![R_l]\!]_{U,(X,Y)}$
$[\![R^{-1}]\!]_{U,(X,Y)}$	$[\![R]\!]_{U,(Y,X)}$
$[\![R\vert C]\!]_{U,(X,Y)}$	$[\![R]\!]_{U,(X,Y)} \wedge [\![C]\!]_{U,Y}$
$[\![\Box_{(m,a)} R]\!]_{U,(X,Y)}$	$\forall V : \Re^a_m(U,V) \Rightarrow [\![R]\!]_{V,(X,Y)}$
$[\![\Diamond_{(m,a)} R]\!]_{U,(X,Y)}$	$\exists V : \mathtt{F}_{\Re^a_m} [\![R]\!]_{V(U),(X,Y)}$

Thus, given any Mod-$\mathcal{ALC}$-formula Φ, the result of the above translation, $[\![\Phi]\!]_\epsilon$ is a first-order predicate logic formula which can easily be transformed into clause normal form with the help of the well-known standard techniques.

Note that the clause form of such a translation does not contain any positive occurrence of a role or accessibility relation symbol. This fact can be exploited in a very interesting manner. Since any role or accessibility relation symbol can only occur in the additional axioms we can examine these independently from the translated formula. Nonnengart[9] presents extremely good simplifications for various modal logics. So, for instance, the whole theory for the modal logic KD45 can be simplified to the almost trivial unit clause $R(U, V(W))$.

In the following theorem we suppose that we have some simplification Sim which can be applied to a set $\mathcal{R}$ of formulae expressing the properties of the accessibility relation such that

$$\mathcal{R} \cup \{\mathrm{Sim}_2\} \models [\![\Phi]\!]_\epsilon \text{ iff } Sim(\mathcal{R}) \cup \{\mathrm{Sim}_2\} \models [\![\Phi]\!]_\epsilon$$

Theorem 3.1 *Let Φ be a Mod-$\mathcal{ALC}$ sentence and T a set of Mod-$\mathcal{ALC}$ sentences. Let $\mathcal{R}$ be a set of formulae which express the properties of the underlying accessibility relation. Then*

$$T \models_{\mathcal{R}} \phi$$

iff $\bigcup_{\Psi \in T} [\![\Psi]\!]_\epsilon \cup Sim(\mathcal{R}) \cup \{\mathrm{Sim}_2\} \models [\![\Phi]\!]_\epsilon$

iff $[\![\Phi]\!]_\epsilon$ *is provable from* $\bigcup_{\Psi \in T} [\![\Psi]\!]_\epsilon \cup Sim(\mathcal{R}) \cup \{\mathrm{Sim}_2\}$ *using some theorem prover which is correct and complete for first-order logic with equality.*

4 Conclusion and Further Work

We presented an approach which allows to reason within modal terminological logics by utilizing an appropriate translation technique into first-order predicate logic. The main idea behind this method is the *functional simulation* of accessibility relations and roles by suitable sets of functions. This turns out to have two main advantages: it decreases the size and the number of clauses and it allows significant simplifications on the set of theory clauses.

Until now, we have only worked out the optimization for those modalities which have a simple possible world semantics. If the interaction of multiple modalities requires the more complex neighbourhood semantics, we have to rely on the straightforward translation which will result in a rather large number of clauses. Looking for optimizations for this case is part of future work.

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