The Dynamics of Costly Signaling

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Abstract. Costly signaling and screening are two mechanisms through which the honesty of signals can be secured in equilibrium. This paper explores the dynamics of one such signaling game: Spence's model of education. It is found that separating equilibria are unlikely to emerge under either the replicator or best response dynamics, but that partially communicative mixed equilibria are quite important dynamically. After relating these results to traditional refinements, it is suggested that these mixtures may play significant, and underappreciated, roles in the explanation of the emergence and stability of information transfer.

1 Introduction

How can the honesty of communication between two agents be ensured when their interests do not coincide? This is one way of framing the question Spence (1973) posed in his seminal paper on job market signaling. His famous answer was that a particular structure of costly signaling can guarantee honesty in equilibrium. However, a complication typical of signaling models in this tradition is that they often have an infinity of equilibrium outcomes. Such an abundance of equilibria makes equilibrium selection a daunting task. Economists seeking to address this issue have often posited equilibrium refinements with the aim of identifying only a small number believable equilibrium outcomes.

This paper takes a different approach to equilibrium selection in Spence's original model of education. Instead of applying an equilibrium refinement, Spence's game will be embedded into two common game dynamics: the replicator and best response dynamics. These dynamics arise from very different modeling assumptions. The replicator dynamic is a paradigm example of an unsophisticated process of imitation, whereas the best response dynamic is an archetype of myopic rational behavior. Nonetheless, both have been proposed as models of learning in games (Schlag, 1998; Gilboa and Matsui, 1991). In order to study equilibrium selection and maintenance, the dynamic stability of the equilibria in Spence's model and the sizes of the basin of attraction of each attractor will be investigated under both these dynamics. Section 2 reviews Spence's game and section 3 carries out this study into the dynamics. It is found that mixed equilibria, which are largely ignored in the signaling literature, play a very important role in the emergence and stability of information transfer. Since both dynamics are most easily interpreted as models of large populations, these mixtures are naturally interpreted as polymorphic market states. These results are discussed and connected to the literature in section 4. In brief, it is shown that for a wide class of parameter settings, there exists a dynamically stable mixed equilibrium. This equilibrium is equivalent to the mixed sequential equilibrium of the Spence variant from Cho and Kreps (1987). This mixture attracts a large portion of initial conditions under both dynamics studied. It is suggested that this class of mixed equilibria may play significant, and under-appreciated, roles in the explanation of information markets and costly signaling in general.

2 Job market signaling

Spence's (1973) model of education shows how message cost can enable honest communication despite a conflict of interest between sender and receiver. The basic structure is as follows. Nature choose a worker's ability level θ, then the worker observes her ability and sender a message to an employer. Sending a message of intensity or level e incurs a cost $c(\theta, e)$. The employer observes e and offers a wage to the worker. If it is assumed that the value of the worker to the employer is θ and that the employer pays the worker a wage w that is equal to the expectation of θ, then the employer's payoff can be given as $-(w-\theta)^2$. The payoff to the worker is $w - c(\theta, e)$. To simplify analysis, let us assume, as is typical, that workers come in two types: $\theta \in \{\theta_L, \theta_H\}$. Denote the probability of a worker being of these types p_L and p_H (with $p_H = 1 - p_L$). And following Spence's original article, let $c(\theta, e) = \frac{e}{\theta}$, $\theta_L = 1$, and $\theta_H = 2$, so that $c(\theta_L, e) = e$ and $c(\theta_H, e) = \frac{e}{2}$.

It is well known that this game has an infinite number of perfect Bayesian equilibria. Coupled with the appropriate beliefs, they come in three flavors: pooling, separating, and hybrid. In a pooling equilibrium all workers send the same message and are offered the same wage. In a separating equilibrium high quality workers send one message and low quality workers another. Hybrid equilibria are mixtures in which one type of worker chooses one level of education with certainty and the other type randomizes between pooling and separating. In these mixed equilibria, education level carries some information, but information transfer is imperfect. Hybrid equilibria are often ignored in discussions of Spence's signaling game but the dynamic analysis presented in the rest of this paper suggests that they may play an important role in the informational structure of markets.

Spence showed that message cost structure can allow honest signaling in equilibrium. However, important dynamic questions are left unanswered. For example, how likely is it that a system of senders and receivers ends up at a separating equilibrium instead of at pooling or at a mixture? Also, is it always the case that Spence's model converges to one of the Nash equilibria? Do boundedly rational agents learn to signal honestly?

In order to study the dynamics here in detail, it is necessary to prune the strategy space of Spence's game because there is not a thoroughly developed theory of adaptive dynamics for games, and in particular Bayesian games, with infinite strategy spaces. An obvious way to shrink the strategy space is to consider workers who have a choice to either always send message $e_L = 0$, separate honestly (i.e. type θ_L workers send e_L and type θ_H workers send e^*), or always send message e^*. Call these strategies *Low*, *Sep*, and *High* respectively. Likewise, we can limit the employer's strategies to acting as through the messages are meaningless (i.e. offering the pooling wage regardless of message received) and acting as though the messages correctly identify sender types (i.e., offer 1 is e_L is received and offer 2 if e^* is received). Call the former strategy *Pool* and the latter *Sep*.

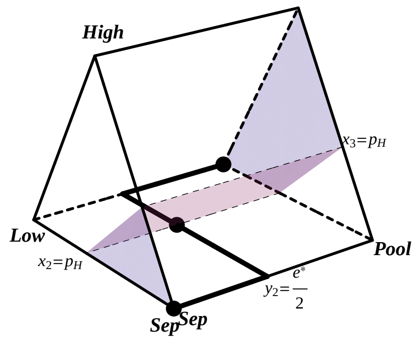
Both the replicator dynamic and the best response dynamic are infinite population models, and payoffs to strategy types are given by the type's expected payoff when matched with a random member of the population. Therefore, we can now focus analysis on the 3×2 normal game shown in Table 1 in which the payoffs are the expectations of payoffs from the extensive form game. Notice that if the receiver plays *Pool*, the sender's unique best response is to play *Low*. Likewise, the receiver's best response to *Low* is to play *Pool*. Thus, the profile (*Low*, *Pool*) is a strict Nash equilibrium. It corresponds to a pooling equilibrium in Spence's original game; workers don't purchase education and employers don't listen to signals.

	Pool	*Sep*
Low	$1 + p_H, -p_L p_H$	$1, -p_H$
Sep	$1 + p_H - \frac{p_H e^*}{2}, -p_L p_H$	$1 + p_H - \frac{p_H e^*}{2}, 0$
High	$1 + p_H - p_L e^* - \frac{p_H e^*}{2}, -p_L p_H$	$2 - p_L e^* - \frac{p_H e^*}{2}, -p_L$

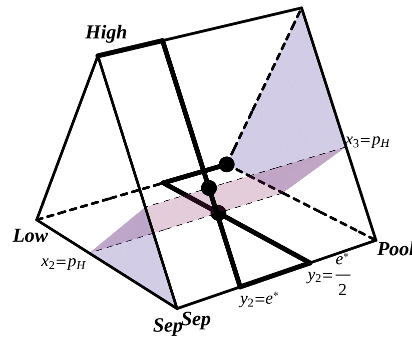
Table 1. The pruned Spence signaling game.

Similarly, the receiver's unique best response to *High* is to play *Pool*. However, the receiver's unique best response to *Sep* is to play *Sep*. All of these best response relationships are independent of e^*. To determine the equilibrium structure of this game, It only remains to determine sender's best response to the receiver's playing the pure strategy *Sep*. *Sep* will be the sender's unique best response just in case $6 - e^* > 4$ and $6 - e^* > 8 - e^*$. These conditions are satisfied if and only if $1 < e^* < 2$. Accordingly, when $1 < e^* < 2$, the profile (*Sep*, *Sep*) corresponds to a separating equilibrium in the full game.

On the other hand, if $0 < e^* < 1$ then this separating profile is not an equilibrium. However, an important mixed equilibrium exists for these values of e^*. The profile in which the sender randomizes between *Sep* and *High* with probabilities p_L and p_H, and the receiver randomizes between *Pool* and *Sep* with probabilities $1 - e^*$ and e^* respectively is a Nash equilibrium when $0 < e^* < 1$. It corresponds to a hybrid equilibrium in the original game in which high productivity workers send message e^* with certainty and low productivity workers randomize between separating from and pooling with the high type. The mixed strategy space and best response correspondences for both cases are



(a)



(b)

Fig. 1. Best response correspondences for the pruned Spence signaling game with (a) $1 < e^* < 2$ and (b) $0 < e^* < 1$. The sender's best reply is shown by the thick line. The receiver's best reply is shown by the translucent surface. x_2 signifies the probability that the sender plays *Sep*, x_3 the probability that the sender plays *High*, and y_2 the probability that the receiver plays *Sep*. Nash equilibria are highlighted by black dots.

drawn in Figure 1. These correspondences are crucial for analyzing the best response dynamic below.

Although this 3×2 game has pruned out a continuum of sending strategies and a continuum of receiving strategies, it still captures the spirit of Spence's model. Pooling is an equilibrium regardless of the value of e^*. And separating can be an equilibrium for e^* set sufficiently high. Thus, just like in Spence's model, a costly education can signal high quality and secure high wages even though education itself may not increase productivity. Now that we have a two player normal form game that retains some of the structure of Spence's original model, it is possible to proceed in analyzing the dynamics of job market signaling.

3 Dynamics

The two adaptive dynamics applied here are the two population replicator and best response dynamics. The first population is the population of workers. They choose from three pure strategies. Denote the proportion that chooses each strategy *Low*, *Sep*, and *High* as x_1, x_2, and x_3. The second population consists of employers. Let y_1 and y_2 be the proportions of the population that play *Pool* and *Sep*. Because $x_1 + x_2 + x_3 = 1$ and $y_1 + y_2 = 1$, the dynamics for this system lives in the three dimensional space $\Delta^3 \times \Delta^2$ where Δ^n is the $n-1$ dimensional simplex $\{(p_1,\ldots,p_n)\,|\,p_i \geq 0, \sum p_i = 1\}$. Coordinates in phase space will be written (x_2,x_3,y_2).[1]

The replicator dynamic for the pruned game is given by the three differential equations

$$\begin{aligned}\dot{x}_2 &= x_2\left[(Ay)_2 - x\cdot Ay\right]\\ \dot{x}_3 &= x_3\left[(Ay)_3 - x\cdot Ay\right]\\ \dot{y}_2 &= y_2\left[(Bx)_2 - y\cdot Bx\right]\end{aligned} \tag{RE}$$

where A is the sender's 3×2 payoff matrix and B is the receiver's 2×3 payoff matrix. Although this dynamic was originally formulated by Taylor and Jonker (1978) to model natural selection in an asexually reproducing population, it also provides a model of cultural learning in economic situations. In this context, the equations give the fluctuations in strategy distributions as agents imitate successful members of their population. In other words, these equations describe large populations of employers and workers in which individual agents, when called on to revise their strategy choice, choose to imitate a more prosperous player.[2]

The best response dynamic for the pruned game is written as

$$\begin{aligned}\dot{x}_2 &= BR(y) - x_2\\ \dot{x}_3 &= BR(y) - x_3\\ \dot{y}_2 &= BR(x) - y_2\end{aligned} \tag{BR}$$

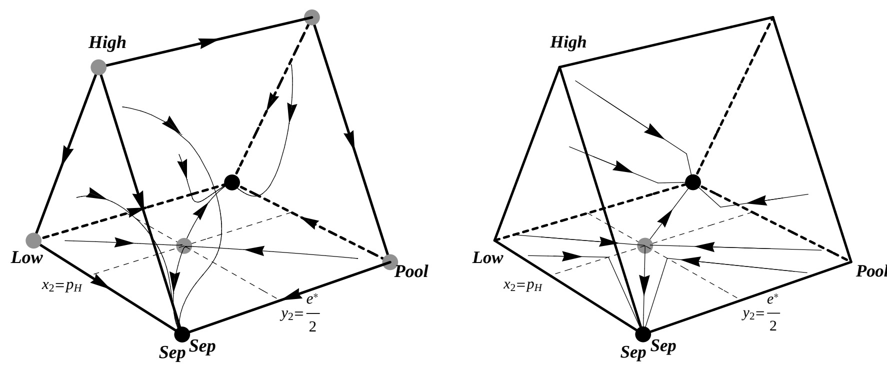
where $BR(y) = \{\hat{x}\in\Delta^3\,|\,\hat{x}\cdot Ay \geq x\cdot Ay$ for all $x\in\Delta^3\}$ and $BR(x)$ is defined similarly. The usual interpretation of this dynamic is that a small fraction of each large population revises their strategy at each time interval. Upon revision, they choose a best reply to the current state.

3.1 When separating is an equilibrium

When $1 < e^* < 2$ the dynamics of the pruned game shown in Table 1 are straightforward. There are two asymptotically stable states which correspond to the two strict Nash equilibria. So, depending on the initial conditions of the system, the dynamics carry it to either the pooling or the separating equilibrium. Figure 2 shows phase portraits for both the replicator and best response dynamics. The only perhaps unexpected features of these systems are the potentially very small basins of attraction for the separating equilibria. For the replicator dynamic it is necessary to use numerical integration to estimate the proportion of phase space that converges to each of the attractors. But, for at least some values of p_H that make the geometry relatively simple, the size of the basin of attraction for separating under the best response dynamic can be found analytically. The basin

[1] It is convenient here to work directly with x_2, x_3, and y_2 instead of x_1 or y_1.

[2] See Weibull (1997) for a survey of imitative dynamics and their relationship with the replicator dynamic.



(a) (b)

Fig. 2. Phase portraits showing the dynamics of the pruned Spence signaling game with $1 < e^* < 2$ for (a) the replicator dynamic and (b) the best response dynamic. Black and grey dots indicate stable and unstable rest points respectively.

of attraction for separating is the portion of phase space contained within the two two-dimensional separatrixes that lead directly to the unstable rest point. A chart of the proportion of phase space that leads to separating under both dynamics is shown in Figure 3.

3.2 When separating is not an equilibrium

When $0 < e^* < 1$ the dynamics become more complex. $(Low, Pool)$ remains a strict Nash equilibrium and hence asymptotically stable. There are also two other rest points, each mixtures. One, at coordinates $(p_H, 0, \frac{e^*}{2})$ is unstable. The other rest point lies at $H = (p_L, p_H, e^*)$. This rest point corresponds to a hybrid equilibrium in which the high quality workers always send e^* and the low quality workers flip a biased coin to determine whether to send e^* or 0.

Theorem 1 *The hybrid equilibrium* $H = (1 - p_H, p_H, e^*)$ *is neutrally stable under the replicator dynamic when* $0 < e^* < 1$.

Proof. Omitted for brevity in this extended abstract.

Initial conditions near H quickly spiral toward the $x_1 = 0$ boundary face. Then, once on this face, they cycle endlessly in closed periodic orbits centered on H. Thus, the hybrid equilibrium H is neutrally stable. Figure 4 shows a phase portrait for this system. Unfortunately, since H is not a hyperbolic rest point, nothing can be concluded about the stability of H under all uniformly monotone selection dynamics. A perturbation to the dynamic will change the

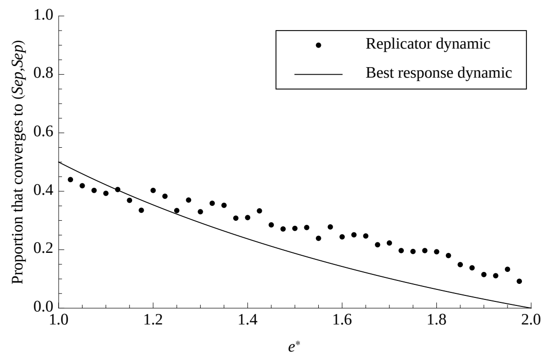
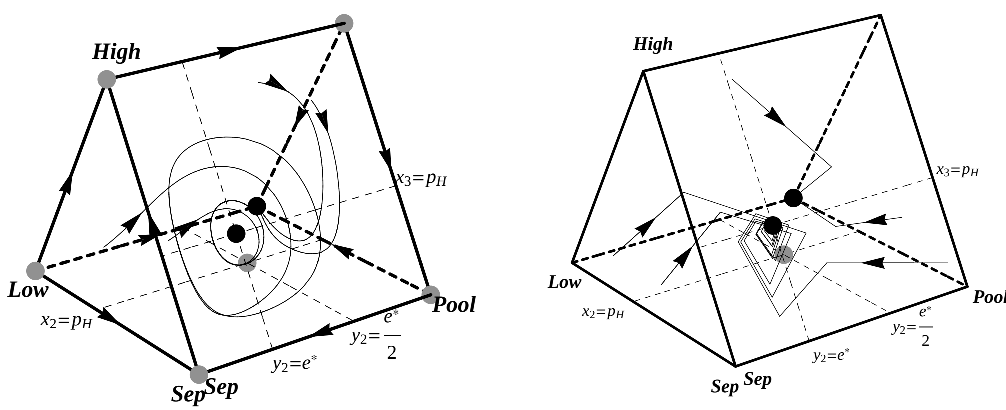


Fig. 3. The proportion of phase space that converges to separating under both dynamics with $p_H = .5$. Each data point for the replicator dynamic is the average of $1,000$ randomly chosen initial conditions. The volume of space that leads to separating under the best response dynamic when $p_H = .5$ is $\frac{(2-e^*)(p_H-1)(p_H e^{*2} - e^*(1+2p_H) - 3p_H)}{3e^*}$.



(a) (b)

Fig. 4. Phase portraits showing the dynamics of the pruned Spence signaling game with $0 < e^* < 1$ for (a) the replicator dynamic and (b) the best response dynamic. Black and grey dots indicate stable and unstable rest points respectively.

system's qualitative behavior, sending orbits, for instance, either spiraling into or away from H.

The stability of H under the best response dynamic is not as delicate as under the replicator, however it is also not straightforward to demonstrate. But, as might be expected by analogy to matching pennies, H is indeed asymptotically stable under the best response dynamic.

Theorem 2 *The hybrid equilibrium $H = (1-p_H, p_H, e^*)$ is asymptotically stable under the best response dynamic.*

Proof. Omitted for brevity in this extended abstract.

The system's phase portraits are shown in Figure 4. Since H is not an attractor under the replicator dynamic, we cannot ask how much of phase space is attracted to H. But, since the linearization of H does have one negative eigenvalue, it is possible to investigate how much of phase space is attracted onto the $x_1 = 0$ boundary face. And, once again, it is possible to solve for the exact proportion of phase space that is attracted to H under the best response dynamic (at lease for some values of p_H for which the geometry is not too complex). Figure 5 shows the sizes of these basins of attraction for H under both dynamics. Notice that, for all values of e^*, a greater fraction of phase space ends in either at H or in oscillations centered on H than ended at the separating equilibrium above.

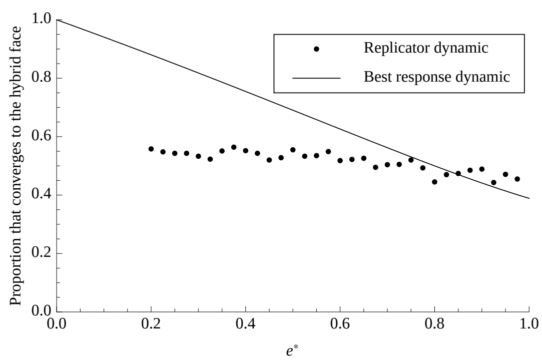


Fig. 5. The proportion of phase space that converges to the $x_1 = 0$ boundary face under both dynamics with $p_H = .5$. Each data point for the replicator dynamic is the average of $1,000$ randomly chosen initial conditions. The volume of space that leads to H under the best response dynamic when $\frac{48-64e^*+23e^{*2}}{48-36e^*+6e^{*2}}$.

3.3 Separating vs. hybrid equilibria

So, hybrid equilibria are stable under both dynamics and have a large influence on the emergence of costly signaling. But so far this equilibrium type has only been pitted against pooling. Do hybrid equilibria outperform separating? To approach this question it is possible to study a slightly larger strategic form game in which pooling, separating, and hybrid equilibria all coexist in phase space. This enlarged game is shown in Table 2. For $0 < e_1^* < 1 < e_2^* < 2$, there are three stable rest points corresponding to the types of equilibria. The pooling profile $(Low, Pool)$ and the separating profile $(Sep_{e_2^*}, Sep_{e_2^*})$ are strict Nash equilibria and hence asymptotically stable states. There is also a hybrid equilibrium in which the sender randomizes between $Sep_{e_1^*}$ and $High_{e_1^*}$ and the receiver randomizes between Low and $Sep_{e_1^*}$. This equilibrium is neutrally stable under the replicator dynamic and asymptotically stable under the best response dynamic. In fact, the replicator dynamics from sections 3.1 and 3.2 above are recaptured on boundary faces of the replicator dynamic for the expanded game in Table 2.

	$Pool$	$Sep_{e_1^*}$	$Sep_{e_2^*}$
Low	$\frac{3}{2}, -\frac{1}{4}$	$1, -\frac{1}{2}$	$1, -\frac{1}{2}$
$Sep_{e_1^*}$	$\frac{3}{2} - \frac{e_1^*}{4}, -\frac{1}{4}$	$\frac{3}{2} - \frac{e_1^*}{4}, 0$	$1 - \frac{e_1^*}{4}, -\frac{1}{2}$
$High_{e_1^*}$	$\frac{3}{2} - \frac{3e_1^*}{4}, -\frac{1}{4}$	$2 - \frac{3e_1^*}{4}, -\frac{1}{2}$	$1 - \frac{3e_1^*}{4}, -\frac{1}{2}$
$Sep_{e_2^*}$	$\frac{3}{2} - \frac{e_2^*}{4}, -\frac{1}{4}$	$\frac{3}{2} - \frac{e_2^*}{4}, -\frac{1}{2}$	$\frac{3}{2} - \frac{e_2^*}{4}, 0$

Table 2. The expanded pruned Spence signaling game with $p_H = \frac{1}{2}$.

It is convenient to use the logit dynamic to estimate the proportion of the space attracted to each rest point under the best response dynamic. For small values of η, this dynamic approximates the best response dynamic (Fudenberg and Levine, 1998). Figure 6 shows the number of randomly chosen initial conditions that converged to the pooling, separating, and hybrid equilibria. Notice that for all values of e_1^*, the hybrid face attracts a larger portion of phase space than the separating equilibrium. Indeed, even under the best of conditions, perfect communication seems a relatively unlikely outcome of the dynamic process. Most initial states lead to partial information transfer or to no communication at all.

4 Discussion

The previous section showed that the ordinary predictions of equilibrium refinement theory are not validated by dynamic analysis in two respects. First, contrary to influential refinements such as the Intuitive Criterion, pooling is a likely result of both adaptive processes investigated above. As much as optimists may hope to rule out such uninspired states in which no information

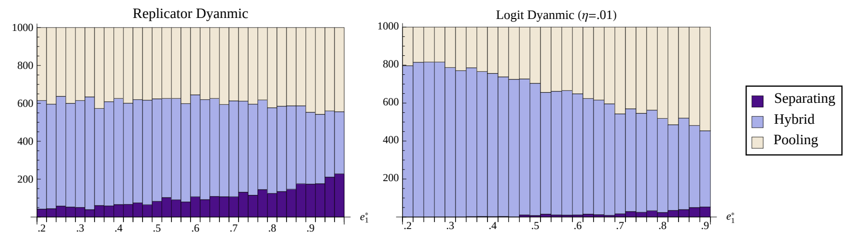


Fig. 6. The number of randomly chosen initial conditions that converged to pooling, separating, and the hybrid face under both the replicator dynamic and the logic dynamic with $\eta = .01$. e_2^* is held fixed at 1.025 while e_1^* is varied.

is conveyed, these non-communicative states may be the norm rather than the exception. Second, mixtures may be more likely than previously thought. Refinements often exclude hybrid equilibria from being considered rational solutions to Spence's game. However, the results above demonstrate that mixtures are likely outcomes of dynamic processes. Although the hybrid is not asymptotically stable under the replicator dynamic, its boundary face can attract a large proportion of initial conditions. And even more strikingly, under the best response dynamic, the hybrid is asymptotically stable and, for small values of e^*, attracts almost all of phase space.

As for applicability of these results to actual social interactions, it is not implausible that hybrid equilibria play a more important role than they are credited for. The study of costly signaling goes back at least to Veblen (1899), who brought attention to the American leisure class's predilection to flaunt wealth through ornate silver utensils, flamboyant homes, and other methods of conspicuous consumption. Veblen's famous thesis held that these members of the upper crust were investing in costly signals to demonstrate their prestige. But status signals are not always honest. The system Veblen described, no less than the system of consumerism and status signaling today, was not one at a separating equilibrium. It seems that a better description is that it was out of equilibrium (perhaps in oscillations centered on a mixture) or in equilibrium, but a partially reliable hybrid rather than one of perfect communication. Similarly, it is unrealistic to maintain that education as a job market signal is perfectly communicative; it is not always the case that only the most productive individuals invest in education. More realistic is the out-of-equilibrium cycling or hybrid picture. The dynamic analysis above shows how it is possible for such real-life market states to be reached. If the goal of costly signaling research is to explain how information is transfered through competitive markets, then it is reasonable to suspect that it is the hybrid equilibrium type that likely does the explanatory work. Refinements have been too quick to exclude such mixtures.

The relevance of this moral may extend beyond economics. Spence's model of job market signaling bears a remarkable resemblance to the structure of costly signaling models from biology. Zahavi (1975) proposed that extravagant characteristics, such as the peacock's tail, evolved because they honestly signal quality to prospective mates. According to this theory of sexual signaling, the peacock can be thought of as investing – at a potentially high cost – in the production of a gaudy tail in order to win access to peahens. Thus, the peacock's tail plays the role of education in Spence's model. Grafen (1990) spelled out Zahavi's proposal with a mathematical framework that is structurally similar to Spence's game. However, Zahavi's costly signaling hypothesis has recently been questioned. Arguments leveled against it include the charge that signaling equilibria leave all participants worse off (Bergstrom and Lachmann, 1997) and the observation that signal cost is not necessary in equilibrium if there are costs imposed on out of equilibrium play (Lachmann et al., 2001).

In one respect, the above analysis suggests another criticism of the costly signaling hypothesis. Perhaps it is the case that, from a dynamic point of view, costly signaling is just a very unlikely outcome of the evolutionary process. Of course, this research does not immediately transfer to biological models, but studying the dynamics of such systems would an interesting next step. However, in another respect the preceding analysis might vindicate one aspect of the costly signaling hypothesis. Game theoretic modeling in biology is centered around static analysis and the evolutionary stable strategy (ESS) refinement in particular (Maynard Smith and Price, 1973). The ESS concept is unambiguous in single population models, but in multi-population models – like those that naturally arise in studies of signaling – there is some debate about how to proceed. A common position maintains that the correct interpretation of a multi-population ESS is simply a strict Nash equilibrium (Weibull, 1997). Hybrid equilibria, however, are mixtures and thus not strict Nash. Consequently, biologists have not payed much attention to polymorphic outcomes of multi-population models. But, due to the deep similarities between Spence and Grafen's games, it is likely that Grafen's may admit mixed equilibria and that, dynamically, these mixtures may be crucial for understanding out-of-equilibrium behavior and perhaps even likely outcomes of the evolutionary process. Before biologists discount the costly signaling hypothesis too much, perhaps it would be wise to investigate such possible mixtures in which messages can be low cost and information transmission is partial.

5 Conclusion

I began by asking how the honesty of communication can be guaranteed when agents' interests diverge. This paper has sought to address this question by investigating how honest communication can emerge through a dynamic process. It turns out that perfectly honest communication is not too likely an outcome, even if it is a possible equilibrium result. If communication does happen to emerge from an out-of-equilibrium market, the model predicts that such communication will likely be partial.

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