Heuristics for the Cost-Effective Management of a Temperature Controlled Environment

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***Abstract*—This study investigates the use of linear programming based heuristics for solving particular energy allocation problems. The main objective is to minimize the cost of using a collection of air conditioning units in a residential or commercial building also, keeping the inside temperature within preset comfort levels. Optimal methods do not scale well when the number of appliances or the system time granularity grows past a certain threshold. We find that heuristics based on relaxation and rounding offer a good trade-off between cost and computation time is needed.**

***Index Terms*—Demand-side management, heuristic optimization algorithm, home automation and power management, mixed integer linear programming, smart grid.**

I. Introduction

Residential buildings consume around 54% of the electricity energy in the USA [1], 68% in the European Union (EU) [2], and about 50-70% in the Arab peninsula states [3]. Furthermore, most of this energy is used in air conditioning systems, about 46% in USA[4], 68% in EU [2], and 70% in Saudi Arabia[5].

Many studies investigate different methods for minimizing the cost of electricity in residential buildings, based on electricity price, availability of renewable power, or user preferences. For example [6]–[8] use algorithms that find the optimal cost of electricity, whereas[9]–[12] use heuristic methods that only guarantee suboptimal value. However all of these deal with a limited number of appliances. In [13], the authors introduce an algorithm that investigates fair allocation of limited power resources to set of Air conditioning (AC) units. Additionally, framework [14] studies using an optimization algorithm to minimize the cost and the number of switching the appliances ON/OFF for a set of thermostatic appliances. It also compares three scheduling algorithms. Heating, ventilation, and Air Conditioning (HVAC) systems have been studied before. In [6], [8] various MILP-based algorithms are used to allocate the optimal energy to just one air conditioning unit. Although they add useful knowledge to the field, they have not tackled the computation time problem: the proposed algorithmic solutions become very slow when the number of appliances in the system grows past a certain limit or the scheduling window is very large.

In this paper we focus on energy allocation problems in which energy is needed to keep a given domestic environment within a pre-specified level of comfort. We present a comprehensive and sound combinatorial model, and study four different ways for reducing the electricity cost in such scenarios, all based on a mathematical programming formulation. We investigate the optimal solutions using an exact MILP formulation. Also, to cope with the intractability of the MILP formulation on large inputs, we use various heuristics which provide feasible solutions much more effectively. LP relaxation is a well-known approach to improving the computation time of an MILP formulation. The study [15] uses LP relaxation to decrease the complexity of a quadratic integer programming. On the other hand, Lagrangian relaxation has been utilized in [16], [17]. In this work we present two heuristics based on LP relaxation. In the final part of the paper we provide evidence of the relative quality of the proposed allocation strategies, suggesting that the approaches based on relaxation represent a viable compromise between the need to be efficient and that of delivering a good quality solution.

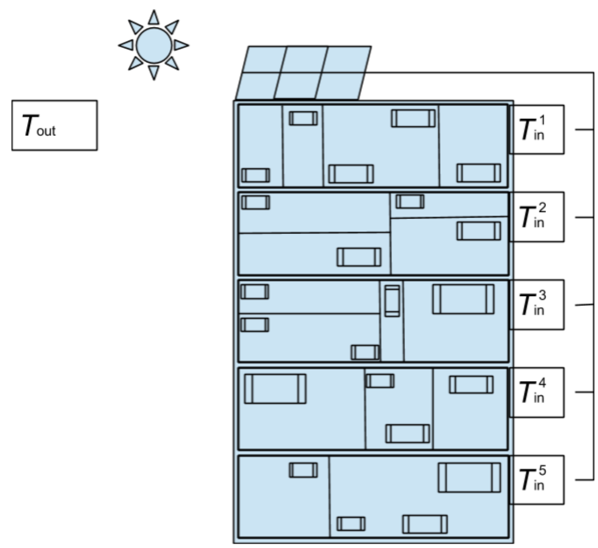


Figure 1: Our model: a building split up in apartments with independent appliances and thermostats

The rest of this paper is organized as follows. The second section states the problem; The third section presents MILP formulation. Section IV describes the strategy we use to design our heustics, and the final section illustrates some empirical results followed by discussions and conclusions.

II. Problem statement

In this section, we present the formalization of the computational problem discussed in this paper.

A. System definition

Our model is well-suited for large buildings (residential, or commercial), see Fig. (1). We assume that the given building consists of a set of apartments, $\mathcal{R}$, each apartment (identified by some label r) is fitted with a set of AC units, $\mathcal{A}_r$, perhaps spread around different rooms, which are capable of cooling down or heating up the environment. Apartment r has n_r appliances, whereas the total number of AC units in the building is $N = \sum_{r \in \mathcal{R}} n_r$. Each AC unit is designed to be switched ON/OFF at any time without disrupting its functionality. Each AC unit has three working modes: it can be "Off", "Cooling" or "Heating". If the appliance is "Off", we may assume it uses no power. However it is "On", without loss of generality, we may assume that it has k_c different ways to cool the place and k_h different ways to produce heat. Let $\alpha_1, \ldots, \alpha_{k_c}$ (resp. $\beta_1, \ldots, \beta_{k_h}$) be the amount of power required by each of the cooling (resp. heating) ways.

The apartment is equipped with a single thermostat that is used to define its internal temperature $T^r_{in}(t)$ at any given time t. We assume that the dwelling's owner may want to be able to specify constrains on the environment's temperature at different times of the day (e.g. "we would like the apartment temperature to be between 20°C and 22°C between 6am and 9am and between 21°C and 24°C between 11am and 3pm"). The building is also equipped with a micro-generation plant. The electricity from such plant can either be used immediately at the property (at a unit cost of $\xi(t)$), or exported to the National Electricity Grid (NEG) and the building is awarded a monetary premium of $\zeta(t)$ pounds (or dollars) per kW. All AC units in the building are controlled by an *energy manager*, whose primary task is to minimize the cost of the electricity used by the AC units in the while keeping each apartment's temperature within pre-specified limits. Such goal is achieved by using the thermostats, weather information (providing readings for the external temperature $T_{out}(t)$) as well as instantaneous information on the electricity unit cost from the NEG $\lambda(t)$ and the eventual export benefit for the locally produced renewable power.

B. Optimization Problem

In this setting, we can associate a cost function Ψ to the building, defined as follows:

$$\Psi = \int \lambda(t) L_g(t) dt + \int \xi(t) L_r(t) dt - \int \zeta(t) E(t) dt, \quad (1)$$

where $L_g(t)$ describes the amount of NEG energy consumed by the AC units, $L_r(t)$ is the amount of renewable power used in the building, and $E(t)$ the amount of renewable power exported to NEG. The problem of allocating energy to set of AC units in a way that satisfies a set of given temperature constraints and is cost-effective for the users (or MINCOSTTEMPCONSTRAINEDALLOC), is equivalent to minimizing Ψ.

III. MILP FORMULATION

The computational problem defined in the previous section lends itself naturally to a simple mathematical programming formulation, provided time is discretized and confined to a window of finite width. From now on we assume that each instance of the given problem is solved over a finite time window, and that the time horizon is subdivided into a finite number of time slots, $\mathcal{T} = \{t_1, t_2, \ldots, t_T\}$, all of length τ.

A. Modeling of Allocated Power

If the ith appliance in apartment r can cool things down (heat things up) in $k^r_{c,i}$ (resp. $k^r_{h,i}$) different ways, then its power consumption at time t can be defined as

$$P^r_i(t) = \sum_{j=1}^{k^r_{c,i}} \alpha^r_i x^r_{i,j}(t) - \sum_{j=1}^{k^r_{h,i}} \beta^r_i y^r_{i,j}(t) \qquad \forall t : t \in \mathcal{T} \quad (2)$$

where

$$x^r_{i,j}(t),\, y^r_{i,j}(t) \in \{0,1\} \quad \forall t : t \in \mathcal{T} \quad (3)$$

and

$$\sum_{j=1}^{k^r_{c,i}} x^r_{i,j}(t) + \sum_{j=1}^{k^r_{h,i}} y^r_{i,j}(t) \leq 1 \qquad \forall t : t \in \mathcal{T} \quad (4)$$

The total power allocated in apartment r at time t is

$$P_r(t) = \sum_{i=1}^{n_r} P^r_i(t) \qquad \forall t : t \in \mathcal{T}. \quad (5)$$

B. Temperature Dynamics

The main task of the AC units in each apartment is to keep the interior temperature within the comfort level specified in b_r time intervals $I^r_1, \ldots, I_{b_r}$ by a lower bound $T^{r,j}_{min}$ and an upper bound $T^{r,j}_{max}$. Following [6] we express relationship between the apartment temperature, the external temperature and the power allocated to the appliance as follows

$$\begin{aligned} T^r_{in}(t) &= \epsilon \cdot T^r_{in}(t-1) + (1-\epsilon)\left[T_{out}(t) - \frac{\eta}{\kappa} P_r(t)\right] \quad (6) \\ T^{r,j}_{min} &\leq T^r_{in}(t) \leq T^{r,j}_{max} \qquad \forall t : t \in I^r_j \end{aligned}$$

where $\epsilon > 0$ is the appliance inertia, $\eta > 0$ is the efficiency of the system, $\kappa > 0$ is the thermal conductivity.

C. Objective Function and Additional Constraints

For the purpose of our experiments we simplify the general model presented in Section II-B. The cost function Ψ in (1) is replaced by the linear function

$$\sum_{t \in \mathcal{T}} \{\lambda(t) \cdot L_g(t) + \xi(t) \cdot L_r(t) - \zeta(t) \cdot E(t)\}, \quad (7)$$

subject to all the constraints defined in this section as well as few more involving functions L_g, L_r and E. Thus, the exported renewable power to NEG and the consumed renewable power at any time must be equal to the predicted renewable power,

$$E(t) + L_r(t) = P_{rew}(t) \;\; \forall t : t \in \mathcal{T}, \quad (8)$$

1: **procedure** CRLP
2: **for** $r \in \mathcal{R}$ **do**
3: **for** $t \in \mathcal{T}$ **do**
4: **if** $P_r(t) \in \Gamma_{N_r}$ **then**
5: $\widetilde{P_r}(t) \leftarrow P_r(t)$
6: **else**
7: $Sum \leftarrow Carry + P_r(t)$
8: Round Sum to closest working level in Γ_{N_r}
9: $\widetilde{P_r}(t) \leftarrow$ Rounded Sum
10: $Carry \leftarrow Sum - \widetilde{P_r}(t)$
11: **end if**
12: CHECK FEASIBILITY of solution
13: **end for**
14: **end for**
15: **end procedure**

where $P_{rew}(t)$ is the renewable power available at time t. The power allocated to the building at any time slot, t, must be equal to building demand,

$$L_g(t) + L_r(t) = \sum_{r \in \mathcal{R}} P_r(t), \quad \forall t : t \in \mathcal{T}. \tag{9}$$

D. Complexity Considerations

The framework presented so far leads to a straightforward implementation of an MILP based algorithm for MINCOSTTEMPCONSTRAINEDALLOC. However there is strong evidence suggesting that the problem may be rather difficult computationally.

In Section V (see results in Table IV) we describe some experiments based on a Java implementation using the Gurobi 6.0 library [18]. The results clearly suggest that the underlying LP solver speed is heavily affected by the number of time slots or appliances in the building. Furthermore the problem is in fact NP-hard [19] even if the building has a single apartment and a single AC unit (with many power levels).

The outcomes of such analysis led us to the study of effective heuristics that can be used to obtain good quality feasible solutions relatively quickly.

IV. RELAXATION AND ROUNDING

Relaxation and rounding is a well-known approach to cope with the computational intractability of an MILP formulation. The relaxation is achieved by removing all constraints restricting the values of some variables to be integer numbers [20]. In the specific of MINCOSTTEMPCONSTRAINEDALLOC this can be done by replacing all constraints described in (3) by

$$0 \leqslant x^r_{i,j}(t) \leqslant 1, \text{ and } 0 \leqslant y^r_{i,j}(t) \leqslant 1. \tag{10}$$

Solving the resulting problem can be done effectively and will lead to a solution that will have cost no larger than that of an optimal solution for the original problem. However, there is no guarantee that all variables forced to take integral values in the initial formulations will do so in the relaxed version. Note that, for MINCOSTTEMPCONSTRAINEDALLOC, this also implies that constraints (4) may not be satisfied. Thus the resulting solution does not immediately translate into a schedule for the building's appliances (e.g. if $x^r_{i,j}(5) = 0.42596$, do we cool appliance i "On" at level j or not?). We now present a *rounding* strategy that can be used to get feasible solutions for MINCOSTTEMPCONSTRAINEDALLOC. Two of the algorithms compared in Section V are based on such strategy.

Algorithm CRLP (pseudo-code above) works on the solution produced by the LP relaxation and generates (in polynomial time) a feasible solution for the initial MILP problem. Different apartments are treated independently. Let us assume that Γ_r is the set of all permissible power values for apartment r. The rationale behind algorithm CRLP is to loop through all time steps t and check whether $P_r(t)$ is permissible in apartment r. If that is the case we set $\widetilde{P_r}(t)$, the rounded power as $P_r(t)$ and the apartment controlling variables $x^r_{i,j}$ and $y^r_{i,j}$ are set according to the assignment giving the value in Γ_r. In the opposite case ($P_r(t)$ is NOT permissible in apartment r) we round $P_r(t)$ to the closest value in Γ_r, and we use such value to set the controlling variables. The rounding process described so far does not guarantees that the rounded solution satisfies the temperature constraints (6). Step 12 in CRLP (described by the additional pseudo-code below) explains how we fix this.

procedure CHECK FEASIBILITY
for each time slot, $t \in I^r_1 \cup, \ldots, I^r_{b^r_i}$ **do**
Calculate $\widetilde{T^r_{in}}(t)$ using $\widetilde{P_r}(t)$.
if $\widetilde{T^r_{in}}(t) > T_{max}$ **then**
Adjust $\widetilde{P_r}(t)$, $\widetilde{P_r}(t) \leftarrow \widetilde{P_r}(t) + \alpha$.
end if
if $\widetilde{T^r_{in}}(t) < T_{min}$ **then**
Adjust $\widetilde{P_r}(t)$, $\widetilde{P_r}(t) \leftarrow \widetilde{P_r}(t) - \alpha$.
end if
end for
Update All dependent variables.
end procedure

V. EMPIRICAL EVALUATION

All the experiments in this paper have been done on a PC with an Intel(R) Core(TM) i7-2600 CPU @ 3.4 GHZ, RAM is 16 GB, 64-bit Operating System (Windows 7). Also, Gurobi has been used to solve LP and MILP problems, whereas Java was the main tools to build our model. In the rest of this section we will compare four different ways of finding feasible solutions for instances of MINCOSTTEMPCONSTRAINEDALLOC: using the exact MILP formulation, using a truncated version of the same process MILP H, using algorithm CRLP, or a slightly faster version of the same process, named CRLP V, which omits step 12 in procedure CRLP.

Table I: TWO COMFORTABLE PERIOD IN THE FLAT WHERE INSIDE TEMPERATURE SHOULD BE IN COMFORABLE RANGE

	First period		**Second period**	
Room	*Start*	*Finish*	*Start*	*Finish*
$r = 1$	05:00:00	10:00:00	17:00:00	18:00:00
$r = 2$	05:00:00	13:00:00	14:00:00	23:00:00
$r = 3$	09:00:00	11:00:00	16:00:00	20:00:00

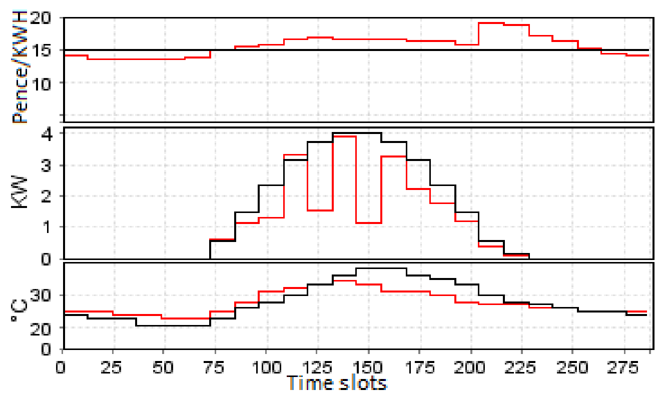


Figure 2: First chart shows electricity prices, second one shows the predicted renewable power, Day 2 in Red and day 3 in Black, and the last one shows the outside temperature, day 1 and day 2 in Red and Day 3 in Black

Two case studies will be demonstrated both based on the following scenario. We assume to be working on a small building including $r = 3$ studio flats and that we need to keep the temperature in comfortable level in each of them. The system includes $N = 6$ identical AC units: $n_1 = 3$, $n_2 = 2$ and $n_3 = 1$. Thus the possible allocated power sets are $\Gamma_1 = \{0, 2.3, 4.6, 6.9\}$, $\Gamma_2 = \{0, 2.3, 4.6\}$, and $\Gamma_3 = \{0, 2.3\}$, respectively. Each flat has a thermostat, measuring the inside temperature, and the thermal parameters have the following values: $\epsilon_1 = \epsilon_2 = \epsilon_3 = 0.96$, $\eta_1 = 10$, $\eta_2 = 20$, $\eta_3 = 30$, and $\kappa_1 = \kappa_2 = \kappa_3 = 0.98$ KW/°C, respectively. Comfort intervals for the three apartments are described in Table I. T^r_{min}= 18.0 and T^r_{max}= 22.0 °C in both comfort intervals, in all apartments.

We assume that the building is equipped with a domestic microgeneration plant, say a PV solar panel arrays. These PV arrays generate a maximum amount of 4.1 KWH of solar power, three shapes of renewable power are used, zero (cloudy day), bell shape (sunny day), and intermittent form (partly cloudy day), see the second chart in Fig. 2. Locally generated renewable energy costs nothing ($\xi = 0.0$ p/KWH), and the building benefits of an export tariff $\zeta = 5.0$ p/KWH. Two pricing schemes will be used in our empirical study for the NEG electricity: a "Fixed" and a "Dynamic" pricing scheme, as described in Fig. (2).

A. First case study

The main purpose of this study is to investigate the performance of the four processes in terms of cost and the effect of input data on solution cost. Input data is as described above. The time horizon is split into T= 288 time slots, $\tau = 5$ minutes.

Six scenarios will be illustrated to investigate the effect of input data on maximum saving (11). We will use input data for three different days, in each day we will use 2 pricing scheme which give us six different scenarios.

1) Findings: Table II shows the maximum saving (11) using MILP exact algorithm. To get an idea of the quality of our algorithmic solutions, in our experiments we compare the cost values of the various heuristics (column **Min**) with a

Table II: OPTIMAL COST AND MAXIMUM SAVING

	Price	Max	Min	Runtime	Saving
Day 1	*Fixed*	3.88	3.88	76 Sec	00.0 %
	Dynamic	5.24	4.23	147 Sec	19.1 %
Day 2	*Fixed*	3.16	1.50	4 h,34m	52.5 %
	Dynamic	3.66	1.69	19 h,47m	53.8 %
Day 3	*Fixed*	3.16	1.10	13 h,22m	65.2 %
	Dynamic	3.73	1.19	27 h,02m	68.1 %

quantity we call **Max**. This is defined as the cost obtained by solving the maximization version of MINCOSTTEMPCONSTRAINEDALLOC with the extra constraint that the total amount of energy used by the solution must match the one corresponding to the optimal solution of MINCOSTTEMPCONSTRAINEDALLOC. The right-most column in the table is computed as follows:

$$\text{Saving} = \frac{|\text{Max} - \text{Min}|}{|\text{Max}|}, \qquad (11)$$

Table II also shows that the run-time of the exact solver becomes very large when τ is small.

Table III compares the three heuristic algorithms CRLP , CRLP V and MILP H. This should be read against Table II to get a feeling for the differences in run-time and cost between the exact and the heuristic algorithms. The table shows that there is no saving when the electricity price is fixed and there is no domestic renewable resources. By contrast, the best saving is achieved when the electricity price is dynamic and there is renewable power. The table also suggests that the heuristic results are close to optimal, especially the results of MILP H and the results of CRLP V.

Fig. 3a and 3b give an even more detailed picture. The show allocated power and inside temperature in room one using CRLP and CRLP V, respectively. Based on this picture we may argue that although CRLP V is NOT guaranteed feasibility in practice the algorithm never goes astray, and in fact returns reasonably cheap solutions.

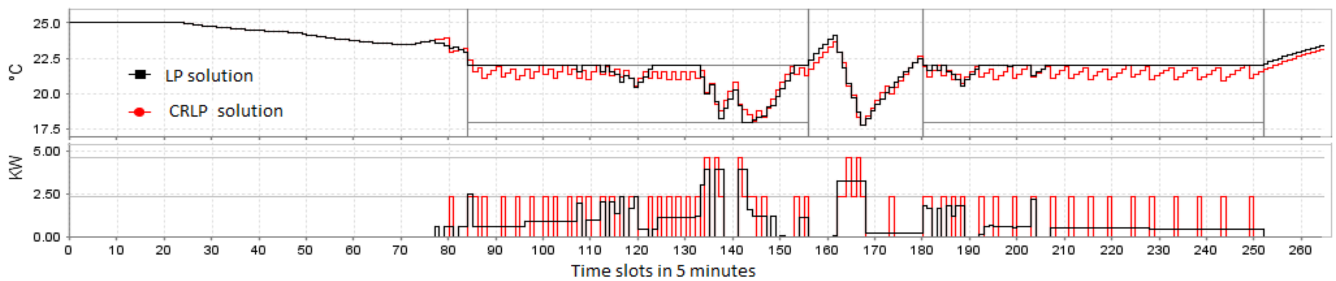
B. Second case study

The main purpose of this case study is to do scalability test. In other words, the main goal is to investigate the performance of the various heuristics in terms of computation time when there is a large number of AC units and high time resolution (τ is small). We will use almost the same input data in first case study, we will just vary τ and N.

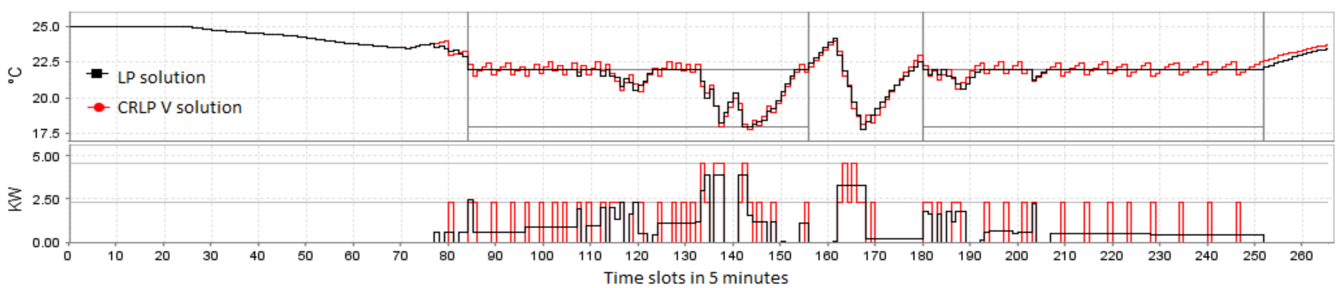
1) Findings: As expected we found that the optimal algorithm can not find a feasible optimal solution in a large problem where the number of appliances or time resolution is large, see Table IV. By contrast, the heuristic algorithms find feasible solution relatively quickly. The time provided in Table V is achieved by CRLP V only. MILP H can not beat CRLP V or CRLP in term of calculation time. Note also that CRLP is slower than CRLP V by just a few milliseconds, as it uses these milliseconds to check and guarantee that no other constraints are violated by rounding the allocated power.

Table III: PERFORMANCE COMPARISON (MILP HEURISTIC IS STOPPED AFTER 600 SEC).

		CRLP				CRLP V			MILP H		
		Max	*Min*	*Runtime*	*Saving*	*Min*	*Run time*	*Saving*	*Min*	*Runtime*	*Saving*
Day 1	*Fixed*	£3.88	£3.88	0.044	00.00 %	£3.59	0.038	07.40 %	£3.88	600	00.00 %
	Dynamic	£5.24	£4.27	0.051	18.51 %	£3.96	0.050	24.42 %	£4.25	600	18.89 %
Day 2	*Fixed*	£3.16	£1.80	0.048	43.10 %	£1.44	0.047	54.43 %	£1.51	600	52.22 %
	Dynamic	£3.66	£2.02	0.064	44.80 %	£1.68	0.045	54.10 %	£1.70	600	53.60 %
Day 3	*Fixed*	£3.16	£1.50	0.076	52.53 %	£1.07	0.059	66.13 %	£1.12	600	64.87 %
	Dynamic	£3.73	£1.79	0.079	52.01 %	£1.34	0.049	64.07 %	£1.32	600	67.56 %



(a) The red line presents solution of CRLP, black curve presents solution of impractical LP solution befor rounding.



(b) The red line presents solution of CRLP V, black curve presents solution of impractical solution before rounding.

Figure 3: The allocated power and room temprature of 3rd room

Table IV: THE AVERAGE COMPUTATION TIME, IN SECONDS, OF EXACT ALGORITHM.

	Time slots, τ, in minutes					
N	30	20	15	10	5	1
1	0.698	0.705	0.735	0.945	2375.87	∞
5	5.905	31.677	3451	∞	∞	∞
10	2151.74	4586.41	∞	∞	∞	∞
50	∞	∞	∞	∞	∞	∞

Table V: THE AVERAGE COMPUTATION TIME, IN SECONDS, OF HEURISTIC CRLP V.

	Time slots, τ, in minutes					
N	30	20	15	10	5	1
1	0.002	0.003	0.004	0.005	0.007	0.061
10	0.007	0.011	0.013	0.021	0.033	0.548
100	0.027	0.049	0.073	0.157	0.461	3.111
300	0.113	0.237	0.298	0.549	0.992	9.044

The runtime to find a solution using CRLP and CRLP V is minuscule, but their solution is not always as good as the solution provided by MILP heuristic in a small and medium problem. In particular scenarios the user may like to use either MILP exact, or MILP heuristic.

Table VI shows a comparison between MILP H (deadline is 10 minutes), and CRLP V algorithms in terms of cost. The results illustrate that when the problem is large CRLP V gives a better solution in terms of cost and runtime and vice versa. The results may change slightly if we changed the input, but, in general, this is the general pattern of their results, whereas Table VII compares between CRLP and MILP H.

Fig. 4 compares between CRLP, CRLP V and MILP-H algorithms for building has 200 AC units and time resolution $\tau = 1$ minute. The results show when MILP H can find a solution that is better than CRLP and CRLP V. According to our finding, MILP H can not beat CRLP and CRLP V in a large problem in reasonable time.

VI. DISCUSSIONS

Regarding first case study, the findings in Table II illustrate that an exact algorithm can be used for small problems (buildings with a handful of AC units). Moreover, the runtime varied considerably (from 76 seconds to 27 hours) for the same problem just by changing the electricity price and predicted renewable power, this behavior is common in MILP. Also,

Table VI: COMPARISON BETWEEN MILP HEURISTIC VS CRLP V IN TERMS OF COST

N/τ	30	20	10	5	1
1	MILP H	MILP H	MILP H	MILP H	CRLP V
10	MILP H	MILP H	CRLP V	MILP H	CRLP V
50	MILP H	MILP H	MILP H	CRLP V	CRLP V
100	MILP H	CRLP V	MILP H	CRLP V	CRLP V
200	MILP H	MILP H	CRLP V	CRLP V	CRLP V
300	CRLP V	CRLP V	CRLP V	CRLP V	CRLP V

Table VII: COMPARISON BETWEEN MILP HEURISTIC VS CRLP IN TERMS OF COST

N/τ	30	20	10	5	1
1	MILP H	MILP H	MILP H	MILP H	CRLP
10	MILP H	MILP H	MILP H	MILP H	CRLP
50	MILP H	MILP H	MILP H	MILP H	CRLP
100	MILP H	MILP H	MILP H	CRLP	CRLP
200	MILP H	MILP H	MILP H	CRLP	CRLP
300	MILP H	MILP H	MILP H	CRLP	CRLP

Table III demonstrates that the maximum saving provided by any of the three heuristics is close to the optimal. These algorithms can be used in large and medium problems (of course it is possible to combine various heuristics, even run all of them and pick the best solution. Additionally, CRLP V can find a cheaper solution than the optimal solution of MILP that is because CRLP V violate temperature constraint (6) which mean that it could allocate less power to building than MILP.

Of course the effective use of our system hinges on reliable weather forecasts, and the accuracy of this data depends on the country or the area where this model will be used. For instance, the weather in Mediterranean and Middle Eastern countries is more stable than in North Europe, especially in the summer which makes the system more reliable. The error in weather forecasting and the uncertainty of electricity pricing are outside of the scope of this framework, and more investigations are needed to tackle these issues.

Minimizing the number of times that we switch the AC unit On/Off could be possible subject of further investigation.

VII. Conclusion

In the conclusion, this paper has examined the performance of various heuristic algorithms that is developed for solving a particular type of energy management problem in terms of computation time and cost in residential or commercial building. Some of the algorithms we presented may be applied to very large problem (building with many AC units) in a matter of seconds, and return good quality feasible solutions, others are appropriate for small problem instances such as small flats and houses.

References

[1] Zhichuan Huang, Ting Zhu, Yu Gu, David Irwin, Aditya Mishra, and Prashant Shenoy. Minimizing electricity costs by sharing energy in sustainable microgrids. In *Proceedings of the 1st ACM Conference on Embedded Systems for Energy-Efficient Buildings*. ACM, 2014.

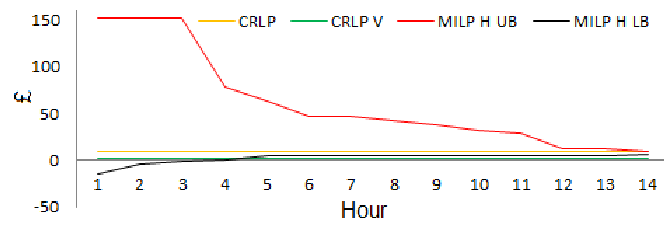


Figure 4: The cost of MILP H, CRLP, and CRLP V over time for 200 AC units, when $\tau = 1$ minutes. MILP H UB present the best known solution, whereas MILP H LB is the best known bound.

[2] Anastasios I Dounis and Christos Caraiscos. Advanced control systems engineering for energy and comfort management in a building environment a review. *Renewable and Sustainable Energy Reviews*, 13(6), 2009.

[3] Y Alyousef and M Abu-Ebid. *Energy efficiency initiatives for Saudi Arabia on supply and demand sides*. INTECH Open Access Publisher, 2012.

[4] Jan F Kreider, Peter S Curtiss, and Ari Rabl. *Heating and cooling of buildings: design for efficiency*. CRC Press, 2013.

[5] Hanan M Taleb and Steve Sharples. Developing sustainable residential buildings in saudi arabia: A case study. *Applied Energy*, 88(1), 2011.

[6] Tanguy Hubert and Santiago Grijalva. Modeling for residential electricity optimization in dynamic pricing environments. *IEEE TRANSACTIONS ON SMART GRID*, 2012.

[7] Zhi Chen, Lei Wu, and Yong Fu. Real time price based demand response management for residential appliances via stochastic optimization and robust optimization. *IEEE TRANSACTIONS ON SMART GRID*, 2012.

[8] KM Tsui and SC Chan. Demand response optimization for smart home scheduling under real-time pricing. *IEEE TRANSACTIONS ON SMART GRID*, 2012.

[9] E Matallanas, M Castillo-Cagigal, A Gutiérrez, F Monasterio-Huelin, E Caamaño-Martín, D Masa, and J Jiménez-Leube. Neural network controller for active demand-side management with pv energy in the residential sector. *Applied Energy*, 91(1):90–97, 2012.

[10] Thillainathan Logenthiran, Dipti Srinivasan, and Tan Zong Shun. Demand side management in smart grid using heuristic optimization. *IEEE Transactions on Smart Grid*, 3(3):1244–1252, 2012.

[11] S. Zhou, X.-P. Zhang, and X. Yang. Design of demand management system for household heating amp; cooling. In *2012 3rd IEEE PES International Conference and Exhibition on Innovative Smart Grid Technologies (ISGT Europe)*, pages 1–6, Oct 2012.

[12] Federica Mangiatordi, Emiliano Pallotti, Paolo Del Vecchio, and Fabio Leccese. Power consumption scheduling for residential buildings. In *2012 11th International Conference on Environment and Electrical Engineering (EEEIC)*, pages 926–930. IEEE, 2012.

[13] Y-W Chen, Xiuxing Chen, and Nicholas Maxemchuk. The fair allocation of power to air conditioners on a smart grid. *IEEE Transactions on Smart Grid*, 3(4):2188–2195, 2012.

[14] G. Karmakar, A. Kabra, and K. Ramamritham. Coordinated scheduling of thermostatically controlled real-time systems under peak power constraint. In *Real-Time and Embedded Technology and Applications Symposium (RTAS), 2013 IEEE 19th*, pages 33–42, April 2013.

[15] Bo Chai, Zaiyue Yang, and Jiming Chen. Optimal residential load scheduling in smart grid: A comprehensive approach. In *Control Conference (ASCC), 2013 9th Asian*, pages 1–6, June 2013.

[16] Seung-Jun Kim and G.B. Giannakis. Efficient and scalable demand response for the smart power grid. In *Computational Advances in Multi-Sensor Adaptive Processing (CAMSAP), 2011 4th IEEE International Workshop on*, pages 109–112, Dec 2011.

[17] Xiaodao Chen, Tongquan Wei, and Shiyan Hu. Uncertainty-aware household appliance scheduling considering dynamic electricity pricing in smart home. *IEEE Transactions on Smart Grid*, 4(2):932–941, June 2013.

[18] Inc. Gurobi Optimization. Gurobi optimizer reference manual, 2014.

[19] T. Karaboghossian and M. Zito. Complexity of renewable energy allocation problems. submitted for publication, 2015.

[20] Wayne L Winston. *Introduction to mathematical programming:applications and algorithms*. International Thomson publishing, 2nd edition, 1995.