1 INTRODUCTION

That logic and language are closely related is almost true by definition. Logic is concerned with the study of valid inferences in arguments, and these are most commonly defined in terms of truth in models. Symbolic logic studies formal languages (logics) as models of certain aspects of natural languages, such as quantification, while abstracting away from certain other aspects of natural languages, such as ambiguity, as models typically do. Linguistics studies the structure of natural languages as well as the relation of language to other areas of cognitive science. The roles that logic in general, and modal logic in particular, play in linguistics are quite varied, as we shall see.

In linguistic semantics, logic is used to formalize, or interpret, an object language. We take as given that we want to study the semantics of some natural language, and in this
chapter the language that we shall deal with is English. Above all else, we would like
to directly interpret English sentences in some formally specified model. So even at this
point we can see some connection to the Kripke semantics of modal operators: just as
all of the other phenomena in this “applied” section of the handbook have been modeled
with mathematical structures involving possible worlds, so too have these been used in
semantic applications. For example, all manner of linguistic phenomena involving time
have led to proposals for using the models from temporal logic. More generally, the main
models of all types of intensional phenomena are closely related to the models in modal
logic.

But so far we have only considered the matter of interpreting natural language directly.
Usually, this is difficult or even impossible. (For example, consider the famous quanti-
ﬁer scope ambiguities in sentences like every handbook has a famous editor. The ambiguity
is neatly expressed in logical notation as ∃∀ vs ∀∃: is there one person, let’s call him
Dov, who edits all the handbooks, or is it merely that every handbook has some editor
or other? One lesson to take from such ambiguities is that it is impossible to associate a
function from (English × models) to truth values in a way that respects our intuitions.)
So one way or another, we translate natural language to some artiﬁcial language and then
interpret that other language, in such a way that ambiguous sentences will be translated
into multiple logical formulas. And here is a second place modal logic comes in: the
language of higher-order modal logic has been used extensively to drive this translation
process, as we shall see when we discuss Montague semantics.

We next turn to syntax, a ﬁeld in which one ﬁnds several different uses of logic.
There are syntactic frameworks which are heavily proof-theoretic, so the question of
whether a given string is a sentence or not boils down to whether a related (formal)
sentence is a theorem in some logical system. This proof-theoretic move is especially
prominent in categorial grammar. Another quite different use of logic is as a meta
language in which one formalizes a linguistic formalism declaratively. This is the move
of model theoretic syntax, a research program we consider in depth in the second half
our chapter. This application relates logic to linguistics in the same way that logic can
be applied to formalize theories of other sciences, like set theory. However, the aims of
this formalization are somewhat different from those of other areas, since model theoretic
syntax is particularly interested in using decidable logics for this formalization so that
matters can be implemented. This is of course one of the reasons why modal logics are
attractive in this context, although much of the focus has been on monadic second-order
logic of trees, which is decidable as well.

Applications of logic in linguistics have traditionally not been too concerned with
meta-results. The main uses of modal logic in semantics are independent from the main
corns of modal logicians: completeness and correspondence. We are not aware of any
serious application of the basic theory of modal logic in semantics, let alone the advanced
theory that is showcased in various chapters of this handbook. The only exception is
definability theory, interest in which is motivated by trying to ﬁnd a logic for linguist-
ic applications that has the right kind of “expressiveness.” For example, the fact that
most A are B is not ﬁrst-order deﬁnable is of some importance for semantics. On the
other side, the application of logic in syntax has led to more applications of sophisticated
meta-results, for example proof theoretical results like cut-elimination or normalization
in categorial grammar. It is interesting to note that deﬁnability is also of importance
in model theoretic syntax, due to its relation to descriptive complexity theory. A re-
lated point: because so many current syntactic frameworks are designed with a hope of implementation, sharper theoretical results about them are called for.

In this chapter, we only survey applications of modal logic to the syntax and semantics of natural languages. We concentrate on these two applications because of the historical importance of modal logic in the development of natural language semantics and because of the significance of model theoretic syntax in current research in mathematical linguistics. There are many areas of applications of logic in linguistics that we do not mention, some of which are surveyed in the Handbook of Logic & Language [4].

2 SEMANTICS

Linguistic semantics studies meaning in natural languages. The central assumption of current semantic theory is that meaning should be studied model theoretically, in the same way that semantics of logics are studied. Thus, the study of meaning is tied to the concept of truth. Of course, there are other ways to pursue the project of understanding meaning, most notably to tie it to action in some way. As it happens, for some purposes possible worlds semantics is even better for this second purpose than for the first; see, for example, [69].

The interpretation of logical formulas usually involves the interpretation of subformulas in some systematic fashion. For instance, in propositional logic we have interpretational clauses like

\[[\varphi \land \psi] = [\land]([\varphi], [\psi])\]

where \([\land]\) is the boolean and function. The methodological principle that stipulates that all interpretations of complex expressions should involve the interpretations of its parts is called the “principle of compositionality,” and it plays a central role in linguistic semantics. Whether that principle is in fact a meaningful restriction on semantic theory or whether it is vacuous is a point of ongoing debate. For one source that discusses the matter at length, see Janssen [41].

Since natural language semantics applies model theoretic methods, the role of modal logic in this context involves the application of possible worlds semantics to natural languages, mainly to model intensional phenomena. However, in order to follow the principle of compositionality uniformly, the meanings of some expressions are modeled using higher-order logic. Thus, the most influential, systematic application of modal logic to linguistic semantics, usually referred to as Montague semantics after its founder, involved higher-order intensional logic. Although Montague’s application of higher-order intensional logic to natural language semantics yielded many important results, almost all of the contemporary research is concerned with finding suitable alternatives to this framework. Many of these are surveyed in the Handbook of Logic and Language [4]. Another handbook in this area, with a more empirical and linguistic, as opposed to theoretical and logical, slant is the Handbook of Contemporary Semantic Theory [57]. There are many introductory textbooks in linguistic semantics, including [15, 22, 36].

2.1 Possible worlds in semantics

The major use of modal logic in semantics stems from possible worlds semantics. Indeed, this is the only kind of application we are considering in this chapter.