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FIRST-ORDER MODAL LOGIC

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PREFACE

This chapter is divided into two parts. The first (which consists of Sections 1–5) was written by Torben Braüner, and the second (which consists of Sections 6–11) was written by Silvio Ghilardi.

In the first part we give an introduction to first-order modal logic. We discuss a number of logics that make use of constant domain, increasing domain, and varying domain semantics, and also present a first-order intensional logic and a first-order version of hybrid logic. One criterion for selecting these logics has been the availability of sound and complete proof procedures for them, typically axiom systems and/or tableau systems. We compare the first-order modal logics discussed here to fragments of sorted first-order logic via appropriate versions of the standard translation.

In the second part of the chapter, we review both positive and negative results concerning fragment decidability, Kripke completeness and axiomatizability. Modal hyperdoctrines are then introduced, as a unifying tool for analyzing the alternative semantics proposed in the literature. These alternative semantics range from specific semantics for non-classical logics (like metaframes), to interpretations in well-established mathematical frameworks (like topological spaces and toposes). Finally, the strict relationship between topological semantics and D. Lewis's counterpart semantics is investigated in detail and an axiomatization is presented.

1 INTRODUCTION TO PART I

In Part I of this chapter we give an introduction to first-order modal logic. First-order modal logic is a big area with a great number of different logics. This has forced us to make a number of choices. The first choice we made was to concentrate on presenting an appropriate selection of logics rather than trying to be encyclopedic. This has allowed us to give reasonably detailed treatments of each of the selected logics. How did we select the logics in question? We wanted to present logics involving constant domains, increasing domains, and varying domains, and moreover, we wanted to present a first-order intensional logic as well as a first-order version of hybrid logic.

Given these overall requirements, one criteria for selecting particular logics for presentation has been the availability of sound and complete proof procedures, typically axiom systems and/or tableau systems. We have compared the first-order modal logics under consideration to fragments of sorted first-order logic via appropriate versions of the standard translation. The possibility of doing so in a straightforward and simple way has been another criteria for selecting a particular logic for presentation. In fact, we take such a simplicity as a sign of mathematical naturality. We have not included constant symbols in the presented logics, the reason being that constants (having a given sort of semantic values) from a mathematical, model-theoretic point of view are just variables (ranging over the same semantic values) that are not quantified over. In the interest of simplicity, we have not included function symbols either. Counterpart semantics, which can be considered an alternative to first-order intensional logics, is treated in Part II, Section 11 of this chapter.

So, which logics have been chosen? In Section 2 we shall present three first-order modal logics which we call the basic logics. In the basic logics, variables designate individual objects. The three basic logics have respectively constant domains, increasing