Ontology Languages (COMP321)

Solution for Exercise 1

1. Recall the syntax of the Description Logic $\mathcal{EL}$. Assume that $A$ and $B$ are concept names and $r$ and $s$ are role names. For each of the following expressions, state whether

- it is a $\mathcal{EL}$ concept;
- a $\mathcal{EL}$ concept definition;
- a primitive $\mathcal{EL}$ concept definition;
- $\mathcal{EL}$ concept inclusion;
- none of the above.

(a) $A \cap B$ is a $\mathcal{EL}$ concept
(b) $(A \cap B) \sqcup A$ is none of the above
(c) $\neg B$ is none of the above
(d) $A \sqsubseteq B$ is a primitive $\mathcal{EL}$ concept definition and a $\mathcal{EL}$ concept inclusion
(e) $\exists r.(A \cap B)$ is a $\mathcal{EL}$ concept
(f) $A \cap B \sqsubseteq B$ is a $\mathcal{EL}$ concept inclusion
(g) $A \equiv A \cap B$ is a $\mathcal{EL}$ concept definition and two $\mathcal{EL}$ concept inclusions ($A \sqsubseteq A \cap B$ and $A \cap B \sqsubseteq A$)
(h) $\exists A.B$ is none of the above
(i) $r \sqsubseteq s$ is none of the above
(j) $A \equiv \exists s.B$ is a $\mathcal{EL}$ concept definition and two $\mathcal{EL}$ concept inclusions ($A \sqsubseteq \exists s.B$ and $\exists s.B \sqsubseteq A$)
(k) $\bot \sqsubseteq \top$ is none of the above

2. Create an $\mathcal{EL}$ TBox $\mathcal{T}$ that models the following facts:

(a) Mammals are animals.

$$
\text{Mammal} \sqsubseteq \text{Animal}
$$
(b) Cats are mammals that are carnivores.

\[ \text{Cat} \subseteq \text{Mammal} \cap \text{Carnivore} \]

(c) Elephants are mammals that are herbivores.

\[ \text{Elephant} \subseteq \text{Mammal} \cap \text{Herbivore} \]

(d) Carnivores eat meat.

\[ \text{Carnivore} \sqsubseteq \exists \text{eat. Meat} \]

(e) A vertebrate is any animal that has, amongst other things, a backbone.

\[ \text{Vertebrate} \equiv \text{Animal} \cap \exists \text{has.part. Backbone} \]

3. Is the following \( \mathcal{EL} \)-TBox an \( \mathcal{EL} \)-terminology? Explain your answer. Express each concept inclusion in natural language:

\[
\begin{align*}
\text{Fish} & \subseteq \text{Animal} \cap \exists \text{lives.in. Water} \\
\exists \text{eat. Meat} & \subseteq \text{Carnivore} \\
\text{Bird} & \equiv \text{Vertebrate} \cap \exists \text{has.part. Wing} \\
& \cap \exists \text{has.part. Leg} \cap \exists \text{lays. Egg} \\
\text{Reptile} & \subseteq \text{Vertebrate} \cap \exists \text{lays. Egg}
\end{align*}
\]

Not an \( \mathcal{EL} \) terminology because

\[ \exists \text{eat. Meat} \sqsubseteq \text{Carnivore} \]

is not a (primitive) concept definition. In natural language, we have:

- Every fish is an animal that lives in water;
- Something that eats meat is a carnivore;
- A bird is a vertebrate that has wings and legs and lays eggs;
• Every reptile is a vertebrate that lays eggs.

4. Let $\mathcal{I} = (\Delta^\mathcal{I}, \cdot^\mathcal{I})$ be an interpretation, where

- $\Delta^\mathcal{I} = \{a, b, c, d, e, f\}$
- $A^\mathcal{I} = \{a, b\}$
- $B^\mathcal{I} = \{c, d, e, f\}$
- $r^\mathcal{I} = \{(a, c), (a, e), (b, f)\}$

Determine the extension $C^\mathcal{I}$ of the following $\mathcal{E}_\mathcal{L}$-concepts $C'$ under $\mathcal{I}$:

- $A \cap B$: $(A \cap B)^\mathcal{I} = \emptyset$
- $\exists r.B$: $(\exists r.B)^\mathcal{I} = \{a, b\}$
- $\exists r.(A \cap B)$: $(\exists r.(A \cap B))^\mathcal{I} = \emptyset$
- $\top$: $\top^\mathcal{I} = \Delta^\mathcal{I}$
- $A \cap \exists r.B$: $(A \cap \exists r.B)^\mathcal{I} = \{a, b\}$.

Which of the following are true:

- $\mathcal{I} \models A \equiv \exists r.B$ YES
- $\mathcal{I} \models A \cap B \sqsubseteq \top$ YES (always)
- $\mathcal{I} \models \exists r.A \sqsubseteq A \cap B$ YES
- $\mathcal{I} \models \top \sqsubseteq B$ NO
- $\mathcal{I} \models B \sqsubseteq \exists r.A$ NO