Ontology Languages (COMP321)

Solution for Exercise 1

1. Recall the syntax of the Description Logic $\mathcal{EL}$. Assume that A and B are concept names and r and s are role names. For each of the following expressions, state whether

   - it is a $\mathcal{EL}$ concept;
   - a $\mathcal{EL}$ concept definition;
   - a primitive $\mathcal{EL}$ concept definition;
   - $\mathcal{EL}$ concept inclusion;
   - none of the above.

   (a) $A \sqcap B$ is a $\mathcal{EL}$ concept

   (b) $(A \sqcap B) \sqcup A$ is none of the above

   (c) $\neg B$ is none of the above

   (d) $A \sqsubseteq B$ is a primitive $\mathcal{EL}$ concept definition and a $\mathcal{EL}$ concept inclusion

   (e) $\exists r.(A \sqcap B)$ is a $\mathcal{EL}$ concept

   (f) $A \sqcap B \sqsubseteq B$ is a $\mathcal{EL}$ concept inclusion

   (g) $A \equiv A \sqcap B$ is a $\mathcal{EL}$ concept definition and two $\mathcal{EL}$ concept inclusions ($A \sqsubseteq A \sqcap B$ and $A \sqcap B \sqsubseteq A$)

   (h) $\exists A.B$ is none of the above

   (i) $r \sqsubseteq s$ is none of the above

   (j) $A \equiv \exists s.B$ is a $\mathcal{EL}$ concept definition and two $\mathcal{EL}$ concept inclusions ($A \sqsubseteq \exists s.B$ and $\exists s.B \sqsubseteq A$)

   (k) $\bot \sqsubseteq \top$ is none of the above

2. Create an $\mathcal{EL}$ TBox $\mathcal{T}$ that models the following facts:

   (a) Mammals are animals.

$$\mathsf{Mammal} \sqsubseteq \mathsf{Animal}$$

(b) Cats are mammals that are carnivores.

$$\mathsf{Cat} \sqsubseteq \mathsf{Mammal} \sqcap \mathsf{Carnivore}$$

(c) Elephants are mammals that are herbivores.

$$\mathsf{Elephant} \sqsubseteq \mathsf{Mammal} \sqcap \mathsf{Herbivore}$$

(d) Carnivores eat meat.

$$\mathsf{Carnivore} \sqsubseteq \exists \mathsf{eat}.\mathsf{Meat}$$

(e) A vertebrate is any animal that has, amongst other things, a backbone.

$$\mathsf{Vertebrate} \equiv \mathsf{Animal} \sqcap \exists \mathsf{has\_part}.\mathsf{Backbone}$$

3. Is the following $\mathcal{EL}$-TBox an $\mathcal{EL}$-terminology? Explain your answer. Express each concept inclusion in natural language:

$$\begin{aligned}
\mathsf{Fish} &\sqsubseteq \mathsf{Animal} \sqcap \exists \mathsf{lives_in}.\mathsf{Water} \\
\exists \mathsf{eat}.\mathsf{Meat} &\sqsubseteq \mathsf{Carnivore} \\
\mathsf{Bird} &\equiv \mathsf{Vertebrate} \sqcap \exists \mathsf{has_part}.\mathsf{Wing} \\
&\quad \sqcap \exists \mathsf{has_part}.\mathsf{Leg} \sqcap \exists \mathsf{lays}.\mathsf{Egg} \\
\mathsf{Reptile} &\sqsubseteq \mathsf{Vertebrate} \sqcap \exists \mathsf{lays}.\mathsf{Egg}
\end{aligned}$$

Not an $\mathcal{EL}$ terminology because

$$\exists \mathsf{eat}.\mathsf{Meat} \sqsubseteq \mathsf{Carnivore}$$

is not a (primitive) concept definition. In natural language, we have:

- Every fish is an animal that lives in water;
- Something that eats meat is a carnivore;
- A bird is a vertebrate that has wings and legs and lays eggs;

- Every reptile is a vertebrate that lays eggs.

4. Let $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ be an interpretation, where

   - $\Delta^{\mathcal{I}} = \{a, b, c, d, e, f\}$
   - $A^{\mathcal{I}} = \{a, b\}$
   - $B^{\mathcal{I}} = \{c, d, e, f\}$
   - $r^{\mathcal{I}} = \{(a, c), (a, e), (b, f)\}$

   Determine the extension $C^{\mathcal{I}}$ of the following $\mathcal{EL}$-concepts C under $\mathcal{I}$:

   - $A \sqcap B$: $(A \sqcap B)^{\mathcal{I}} = \emptyset$
   - $\exists r.B$: $(\exists r.B)^{\mathcal{I}} = \{a, b\}$
   - $\exists r.(A \sqcap B)$: $(\exists r.(A \sqcap B))^{\mathcal{I}} = \emptyset$
   - $\top$: $\top^{\mathcal{I}} = \Delta^{\mathcal{I}}$
   - $A \sqcap \exists r.B$: $(A \sqcap \exists r.B)^{\mathcal{I}} = \{a, b\}$.

   Which of the following are true:

   - $\mathcal{I} \models A \equiv \exists r.B$ YES
   - $\mathcal{I} \models A \sqcap B \sqsubseteq \top$ YES (always)
   - $\mathcal{I} \models \exists r.A \sqsubseteq A \sqcap B$ YES
   - $\mathcal{I} \models \top \sqsubseteq B$ NO
   - $\mathcal{I} \models B \sqsubseteq \exists r.A$ NO