Ontology Languages (COMP321)

Solution to Exercise 6

Consider the database instance $\mathcal{D}_{Nemo}$ given by

\[
\begin{align*}
\text{Clownfish}(Nemo), & \quad \text{Clownfish}(Karl) \\
\text{Surgeonfish}(Dory), & \quad \text{has\_friend}(Nemo, Dory)
\end{align*}
\]

We query $\mathcal{D}_{Nemo}$ under closed world assumption (standard relational database semantics) and under open world assumption. Recall that under the closed world assumption we consider the interpretation $\mathcal{I} := \mathcal{I}_{\mathcal{D}_{Nemo}}$ defined as follows:

- $\Delta^\mathcal{I} = \{Nemo, Karl, Dory\}$;
- $\text{Clownfish}^\mathcal{I} = \{Nemo, Karl\}$;
- $\text{Surgeonfish}^\mathcal{I} = \{Dory\}$;
- $\text{has\_friend}^\mathcal{I} = \{(Nemo, Dory)\}$.

Consider the following Boolean queries (in description logic notation).

- $\text{Clownfish}(Karl)$
- $\text{Clownfish}(Dory)$
- $\text{Fish}(Nemo)$
- $\neg\text{Fish}(Nemo)$
- $(\exists\text{has\_friend}.\top)(Nemo)$
- $(\exists\text{has\_friend}.\text{Fish})(Nemo)$
- $(\text{Clownfish} \sqcap \neg\text{Surgeonfish})(Karl)$
- $\text{Fish}(Dory)$
- $(\text{Surgeonfish} \sqcap \neg\text{Fish})(Dory)$
(1) Write those Boolean queries in first-order predicate logic (FOPL) notation. (Note that for many queries there is no difference between description logic notation and FOPL notation).

Solution

- Clownfish(Karl) is already in FOPL
- Clownfish(Dory) is already in FOPL
- Fish(Nemo) is already in FOPL
- ¬Fish(Nemo) is already in FOPL
- (∃has_friend.Clownfish)(Nemo) is in FOPL: ∃y. has_friend(Nemo, y).
- (∃has_friend.Fish)(Nemo) is in FOPL:
  \[ \exists y. (\text{has\_friend}(\text{Nemo}, y) \land \text{Fish}(y)) \]

- (Clownfish \(\sqcap\) ¬Surgeonfish)(Karl) is in FOPL:
  \[ \text{Clownfish}(\text{Karl}) \land \neg\text{Surgeonfish}(\text{Karl}) \]

- Fish(Dory) is already in FOPL.
- (Surgeonfish \(\sqcap\) ¬Fish)(Dory) is in FOPL:
  \[ \text{Surgeonfish}(\text{Dory}) \land \neg\text{Fish}(\text{Dory}) \]

- (∃has_friend.Clownfish)(Karl) in in FOPL:
  \[ \exists y. (\text{has\_friend}(\text{Karl}, y) \land \text{Clownfish}(y)) \]

(2) Query answering under closed world assumption: check for each Boolean query \(F\) whether the answer to the query \(F\) given by \(\mathcal{D}_{\text{Nemo}}\) is “Yes” or “No”. In other words, check whether \(\mathcal{I} \models F\) or \(\mathcal{I} \models \neg F\).

Solution
• $\mathcal{I} \models \text{Clownfish}(\text{Karl})$? Yes
• $\mathcal{I} \models \text{Clownfish}(\text{Dory})$? No
• $\mathcal{I} \models \text{Fish}(\text{Nemo})$? No
• $\mathcal{I} \models \neg \text{Fish}(\text{Nemo})$? Yes
• $\mathcal{I} \models (\exists \text{has\_friend}. \top)(\text{Nemo})$? Yes
• $\mathcal{I} \models (\exists \text{has\_friend.}\text{Fish})(\text{Nemo})$? No
• $\mathcal{I} \models (\text{Clownfish} \sqcap \neg \text{Surgeonfish})(\text{Karl})$? Yes
• $\mathcal{I} \models \text{Fish}(\text{Dory})$? No
• $\mathcal{I} \models (\text{Surgeonfish} \sqcap \neg \text{Fish})(\text{Dory})$? Yes
• $\mathcal{I} \models (\exists \text{has\_friend.}\text{Clownfish})(\text{Karl})$? No

(3) Query answering under open world assumption: check for each Boolean query $F$ whether the certain answer to $F$ given by $\mathcal{D}_{\text{Nemo}}$ is “Yes”, “No”, or “Don’t know”. In other words, check whether $\mathcal{D} \models F$ or $\mathcal{D} \models \neg F$ or neither of these two hold.

Solution

• Clownfish(\text{Karl})? Yes
• Clownfish(\text{Dory})? Don’t know
• Fish(\text{Nemo})? Don’t know
• $\neg \text{Fish}(\text{Nemo})$? Don’t know
• $(\exists \text{has\_friend.}\top)(\text{Nemo})$? Yes
• $(\exists \text{has\_friend.}\text{Fish})(\text{Nemo})$? Don’t know
• $(\text{Clownfish} \sqcap \neg \text{Surgeonfish})(\text{Karl})$? Don’t know
• Fish(\text{Dory})? Don’t know
• $(\text{Surgeonfish} \sqcap \neg \text{Fish})(\text{Dory})$? Don’t know
• $(\exists \text{has\_friend.}\text{Clownfish})(\text{Karl})$? Don’t know
Consider the following non-Boolean queries $F_i$:

- $F_1(x) = \text{Clownfish}(x)$
- $F_2(x) = \neg \text{Surgeonfish}(x)$
- $F_3(x, y) = \text{has\_friend}(x, y)$
- $F_4(x) = \text{Clownfish}(x) \land \neg \text{has\_friend}(x, Dory)$

(4) For each query $F_i$, give $\text{answer}(F_i, \mathcal{I}_N)$.
Solution

- $\text{answer}(F_1, \mathcal{D}_\text{Nemo}) = \{\text{Nemo}, \text{Karl}\}$.
- $\text{answer}(F_2, \mathcal{D}_\text{Nemo}) = \{\text{Nemo}, \text{Karl}\}$.
- $\text{answer}(F_3, \mathcal{D}_\text{Nemo}) = \{(\text{Nemo}, \text{Dory})\}$.
- $\text{answer}(F_4, \mathcal{D}_\text{Nemo}) = \{\text{Karl}\}$.

(5) For each query $F_i$, give $\text{certanswer}(F_i, \mathcal{D}_\text{Nemo})$.
Solution

- $\text{certanswer}(F_1, \mathcal{D}_\text{Nemo}) = \{\text{Nemo}, \text{Karl}\}$.
- $\text{certanswer}(F_2, \mathcal{D}_\text{Nemo}) = \emptyset$.
- $\text{certanswer}(F_3, \mathcal{D}_\text{Nemo}) = \{(\text{Nemo}, \text{Dory})\}$.
- $\text{certanswer}(F_4, \mathcal{D}_\text{Nemo}) = \emptyset$. 