COMP321 (Ontology Languages): Test 1

Lecturer: F. Wolter
Time: 50 minutes

This test makes up 10 percent of the final mark for this module. You can achieve 100 marks.

1. Consider the following $\mathcal{EL}$-TBox $\mathcal{T}$:

$$\begin{aligned}
\mathsf{Team} &\sqsubseteq \exists \mathsf{has_part}.\mathsf{Player} \\
\mathsf{Footballteam} &\sqsubseteq \mathsf{Team} \\
\mathsf{Player} &\sqsubseteq \mathsf{Human_being} \\
\exists \mathsf{has_part}.\mathsf{Human_being} &\sqsubseteq \mathsf{Organisation}
\end{aligned}$$

   - Apply the $\mathcal{EL}$-subsumption algorithm from the Comp321-Slides to compute $S(A)$ for every concept name A in $\mathcal{T}$ and $R(r)$ for every role name r of $\mathcal{T}$. In your answer, show how the rules (simpleR), (conjR), (rightR) and (leftR) are applied step-by-step in the computation of S and R.
   - Using S, determine whether $\mathsf{Footballteam} \sqsubseteq_{\mathcal{T}} \mathsf{Organisation}$ (i.e., whether Footballteam is subsumed by Organisation w.r.t. $\mathcal{T}$).
   - Using S, determine whether $\mathsf{Player} \sqsubseteq_{\mathcal{T}} \mathsf{Team}$ (i.e., whether Player is subsumed by Team w.r.t. $\mathcal{T}$). (**40 marks**)

2. Consider the interpretation $\mathcal{I}$ defined by

   - $\Delta^{\mathcal{I}} = \{a, b, c, d\}$;
   - $A^{\mathcal{I}} = \{a, b, c\}$;
   - $r^{\mathcal{I}} = \{(a, c), (b, c), (b, d)\}$;
   - $B^{\mathcal{I}} = \{c\}$.

   Determine the following set:

   - $(A \sqcap \exists r.B)^{\mathcal{I}}$;
   - $(\forall r.B)^{\mathcal{I}}$;
   - $(\exists r.B \sqcap \forall r.B)^{\mathcal{I}}$;
   - $(\neg A \sqcap \neg \exists r.B)^{\mathcal{I}}$.

(20 marks)

3. Consider the $\mathcal{ALC}$-concept

$$C = (\exists r.(A \sqcap \exists s.B)) \sqcap \forall r.(\neg A \sqcup E)$$

Apply the $\mathcal{ALC}$-tableau algorithm from the Comp321-Slides to the concept C to determine whether C is satisfiable or not. In your answer, show how the rules $\rightarrow_\sqcap$, $\rightarrow_\sqcup$, $\rightarrow_\exists$, and $\rightarrow_\forall$ are applied step-by-step to the constraint system $x : C$.

If C is satisfiable, construct an interpretation $\mathcal{I}$ satisfying C (i.e., with $C^\mathcal{I} \neq \emptyset$).

(40 marks)

Solution for 1.

The initial assignment is given by

$$\begin{aligned} S(\mathsf{Team}) &= \{\mathsf{Team}\} \\ S(\mathsf{Player}) &= \{\mathsf{Player}\} \\ S(\mathsf{Footballteam}) &= \{\mathsf{Footballteam}\} \\ S(\mathsf{Human\_being}) &= \{\mathsf{Human\_being}\} \\ S(\mathsf{Organisation}) &= \{\mathsf{Organisation}\} \\ R(\mathsf{has\_part}) &= \emptyset \end{aligned}$$

Now applications of (simpleR), (conjR), (rightR), (leftR) are step-by-step as follows:

- Update R using (rightR) for the first inclusion of $\mathcal{T}$:

$$R(\mathsf{has\_part}) = \{(\mathsf{Team}, \mathsf{Player})\}.$$

- Update S using (simpleR):

$$S(\mathsf{Footballteam}) = \{\mathsf{Footballteam}, \mathsf{Team}\}$$

- Update R using (rightR) for the first inclusion of $\mathcal{T}$:

$$R(\mathsf{has\_part}) = \{(\mathsf{Team}, \mathsf{Player}), (\mathsf{Footballteam}, \mathsf{Player})\}.$$

- Update S using (simpleR) for the third inclusion of $\mathcal{T}$:

$$S(\mathsf{Player}) = \{(\mathsf{Player}, \mathsf{Human\_being})\}.$$

- Update S using (leftR) for the last inclusion of $\mathcal{T}$:

$$S(\mathsf{Team}) = \{\mathsf{Team}, \mathsf{Organisation}\}$$

- Update S using (leftR) for the last inclusion of $\mathcal{T}$:

$$S(\mathsf{Footballteam}) = \{\mathsf{Footballteam}, \mathsf{Team}, \mathsf{Organisation}\}$$

The final assignment is

$$\begin{array}{rcl} S(\mathsf{Team}) & = & \{\mathsf{Team}, \mathsf{Organisation}\} \\ S(\mathsf{Player}) & = & \{\mathsf{Player}, \mathsf{Human\_being}\} \\ S(\mathsf{Footballteam}) & = & \{\mathsf{Footballteam}, \mathsf{Team}, \mathsf{Organisation}\} \\ S(\mathsf{Human\_being}) & = & \{\mathsf{Human\_being}\} \\ S(\mathsf{Organisation}) & = & \{\mathsf{Organisation}\} \\ R(\mathsf{has\_part}) & = & \{(\mathsf{Team}, \mathsf{Player}), (\mathsf{Footballteam}, \mathsf{Player})\} \end{array}$$

- $\mathsf{Footballteam} \sqsubseteq_{\mathcal{T}} \mathsf{Organisation}$ holds since $\mathsf{Organisation} \in S(\mathsf{Footballteam})$.
- $\mathsf{Player} \sqsubseteq_{\mathcal{T}} \mathsf{Team}$ does not hold since $\mathsf{Team} \notin S(\mathsf{Player})$.

Solution for 2.
We have

- $(A \sqcap \exists r.B)^{\mathcal{I}} = \{a, b\}$;
- $(\forall r.B)^{\mathcal{I}} = \{a, c, d\}$;
- $(\exists r.B \sqcap \forall r.B)^{\mathcal{I}} = \{a\}$;
- $(\neg A \sqcap \neg\exists r.B)^{\mathcal{I}} = \{d\}$.

Solution for 3.
Let

$$C = (\exists r.(A \sqcap \exists s.B)) \sqcap \forall r.(\neg A \sqcup E)$$

As C is already in negation normal form, the tableaux algorithm starts with

$$S_0 = \{x : (\exists r.(A \sqcap \exists s.B)) \sqcap \forall r.(\neg A \sqcup E)\}$$

An application of the rule $\rightarrow_{\sqcap}$ gives

$$S_1 = S_0 \cup \{x : \exists r.(A \sqcap \exists s.B), x : \forall r.(\neg A \sqcup E)\}$$

An application of the rule $\rightarrow_{\exists}$ gives

$$S_2 = S_1 \cup \{(x, y) : r, y : A \sqcap \exists s.B\}$$

An application of the rule $\rightarrow_{\exists}$ gives

$$S_3 = S_2 \cup \{(y, z) : z : B\},$$

An application of the rule $\rightarrow_\forall$ gives

$$S_4 = S_3 \cup \{y : (E \sqcup \neg A)\}$$

An application of the branching rule $\rightarrow_\sqcup$ gives

$$S_5 = S_4 \cup \{y : \neg A\}$$

S_5 contains the clash $\{y : A, y : \neg A\}$. Thus, we have to try the second option: The other application of the branching rule $\rightarrow_\sqcup$ to S_4 gives

$$S_6 = S_4 \cup \{y : E\}$$

No rule is applicable to S_6 and S_6 does not contain any clash. Thus, C is satisfiable.

An interpretation $\mathcal{I}$ satisfying C is given by

- $\Delta^\mathcal{I} = \{x, y, z\}$;
- $A^\mathcal{I} = \{y\}$;
- $B^\mathcal{I} = \{z\}$;
- $E^\mathcal{I} = \{y\}$
- $r^\mathcal{I} = \{(x, y)\}$;
- $s^\mathcal{I} = \{(y, z)\}$.