COMP321 (Ontology Languages): Test 1

Lecturer: F. Wolter
Time: 50 minutes

This test makes up 10 percent of the final mark for this module. You can achieve 100 marks.

1. Consider the following $\mathcal{EL}$-TBox $\mathcal{T}$:

$$\begin{array}{rcl} \mathsf{SmallProject} & \sqsubseteq & \mathsf{Project} \\ \exists \mathsf{manages.Project} & \sqsubseteq & \mathsf{Manager} \\ \mathsf{JuniorManager} & \sqsubseteq & \exists \mathsf{manages.SmallProject} \\ \mathsf{JuniorManager} \sqcap \mathsf{Manager} & \sqsubseteq & \mathsf{Employee} \end{array}$$

   - Apply the $\mathcal{EL}$-subsumption algorithm from the Comp321-Slides to compute $S(A)$ for every concept name A in $\mathcal{T}$ and $R(r)$ for every role name r of $\mathcal{T}$. In your answer, show how the rules (simpleR), (conjR), (rightR) and (leftR) are applied step-by-step in the computation of S and R.
   - Using S, determine whether $\mathsf{Manager} \sqsubseteq_{\mathcal{T}} \mathsf{Employee}$ (i.e., whether Manager is subsumed by Employee w.r.t. $\mathcal{T}$).
   - Using S, determine whether $\mathsf{JuniorManager} \sqsubseteq_{\mathcal{T}} \mathsf{Manager}$ (i.e., whether JuniorManager is subsumed by Manager w.r.t. $\mathcal{T}$).
   - Using S, determine whether $\mathsf{JuniorManager} \sqsubseteq_{\mathcal{T}} \mathsf{Employee}$ (i.e., whether JuniorManager is subsumed by Employee w.r.t. $\mathcal{T}$). **(30 marks)**

2. Consider the interpretation $\mathcal{I}$ defined by

   - $\Delta^{\mathcal{I}} = \{a, b, c\}$;
   - $A^{\mathcal{I}} = \{a, b\}$, $B^{\mathcal{I}} = \{c\}$, $r^{\mathcal{I}} = \{(a, b), (a, c), (b, b)\}$.

   Determine the following sets:

   - $(\exists r.A)^{\mathcal{I}}$;
   - $(\forall r.A)^{\mathcal{I}}$;
   - $(\exists r.B)^{\mathcal{I}}$;
   - $(\forall r.B)^{\mathcal{I}}$. **(20 marks)**

3. Consider the $\mathcal{ALC}$-TBox $\mathcal{T}$:

$$\begin{aligned} \mathsf{Manager} &\sqsubseteq \exists \mathsf{works\_for}.\mathsf{Employer} \\ \mathsf{Manager} &\sqsubseteq \exists \mathsf{manages}.\mathsf{Unit} \\ \mathsf{Manager} &\sqsubseteq \forall \mathsf{manages}.\mathsf{Department} \\ \mathsf{Employer} &\sqsubseteq \exists \mathsf{registered\_at}.\mathsf{TaxOffice} \end{aligned}$$

- Translate $\mathcal{T}$ into natural language (English).
- Define a model $\mathcal{I}$ of $\mathcal{T}$ in which Manager is satisfied. (In other words, define an interpretation $\mathcal{I}$ such that $\mathcal{I} \models \mathcal{T}$ and $\mathsf{Manager}^{\mathcal{I}} \neq \emptyset$.)
- Add $\mathsf{Unit} \sqcap \mathsf{Department} \sqsubseteq \bot$ to $\mathcal{T}$. Is the resulting TBox satisfiable? Provide an (informal!) argument for your answer. **(20 marks)**

4. Consider the $\mathcal{ALC}$-concept

$$C = (\neg A) \sqcap (\exists r.\exists r.A) \sqcap (\forall r.\neg A)$$

Apply the $\mathcal{ALC}$-tableau algorithm from the Comp321-Slides to the concept C to determine whether C is satisfiable or not. In your answer, show how the rules $\rightarrow_{\sqcap}$, $\rightarrow_{\sqcup}$, $\rightarrow_{\exists}$, and $\rightarrow_{\forall}$ are applied step-by-step to the constraint system $x : C$.

If C is satisfiable, construct an interpretation $\mathcal{I}$ satisfying C (i.e., with $C^{\mathcal{I}} \neq \emptyset$). **(30 marks)**

Solution for 1.

The initial assignment is given by

$$\begin{aligned} S(\mathsf{SmallProject}) &= \{\mathsf{SmallProject}\} \\ S(\mathsf{Project}) &= \{\mathsf{Project}\} \\ S(\mathsf{Manager}) &= \{\mathsf{Manager}\} \\ S(\mathsf{Employee}) &= \{\mathsf{Employee}\} \\ S(\mathsf{JuniorManager}) &= \{\mathsf{JuniorManager}\} \\ R(\mathsf{manages}) &= \emptyset \end{aligned}$$

Now applications of (simpleR), (conjR), (rightR), (leftR) are step-by-step as follows:

- Update S using (simpleR):

$$S(\mathsf{SmallProject}) = \{\mathsf{Project}, \mathsf{Smallproject}\}$$

- Update R using (rightR) for the third inclusion of $\mathcal{T}$:

$$R(\mathsf{manages}) = \{(\mathsf{JuniorManager}, \mathsf{SmallProject})\}.$$

- Update S using (leftR) for the second inclusion of $\mathcal{T}$:

$$S(\mathsf{JuniorManager}) = \{\mathsf{JuniorManager}, \mathsf{Manager}\}$$

- Update S using (conjR) for the last inclusion:

$$S(\mathsf{JuniorManager}) = \{\mathsf{JuniorManager}, \mathsf{Manager}, \mathsf{Employee}\}.$$

The final assignment is

$$\begin{aligned} S(\mathsf{SmallProject}) &= \{\mathsf{SmallProject}, \mathsf{Project}\} \\ S(\mathsf{Project}) &= \{\mathsf{Project}\} \\ S(\mathsf{Manager}) &= \{\mathsf{Manager}\} \\ S(\mathsf{Employee}) &= \{\mathsf{Employee}\} \\ S(\mathsf{JuniorManager}) &= \{\mathsf{JuniorManager}, \mathsf{Manager}, \mathsf{Employee}\} \\ R(\mathsf{manages}) &= \{(\mathsf{JuniorManager}, \mathsf{SmallProject})\}. \end{aligned}$$

- $\mathsf{Manager} \sqsubseteq_{\mathcal{T}} \mathsf{Employee}$ does not hold since $\mathsf{Employee} \notin S(\mathsf{Manager})$.
- $\mathsf{JuniorManager} \sqsubseteq_{\mathcal{T}} \mathsf{Manager}$ holds since $\mathsf{Manager} \in S(\mathsf{JuniorManager})$.
- $\mathsf{JuniorManager} \sqsubseteq_{\mathcal{T}} \mathsf{Employee}$ holds since $\mathsf{Employee} \in S(\mathsf{JuniorManager})$.

Solution for 2.
Consider the interpretation $\mathcal{I}$ defined by

- $\Delta^{\mathcal{I}} = \{a, b, c\}$;
- $A^{\mathcal{I}} = \{a, b\}$;
- $B^{\mathcal{I}} = \{c\}$;
- $r^{\mathcal{I}} = \{(a, b), (a, c), (b, b)\}$.

We have:

- $(\exists r.A)^{\mathcal{I}} = \{a, b\}$;
- $(\forall r.A)^{\mathcal{I}} = \{b, c\}$;
- $(\exists r.B)^{\mathcal{I}} = \{a\}$;
- $(\forall r.B)^{\mathcal{I}} = \{c\}$.

Solution for 3.

- Every manager works for an employer;
- Every manager manages a unit.

- Every manager only manages departments;
- Every employer is registered at some tax office.

An interpretation $\mathcal{I}$ that is a model of $\mathcal{T}$ with $\mathsf{Manager}^{\mathcal{I}} \neq \emptyset$ is given by setting $\Delta^{\mathcal{I}} = \{a, b, c, d\}$ and

- $\mathsf{Manager}^{\mathcal{I}} = \{a\}$;
- $\mathsf{Employer}^{\mathcal{I}} = \{b\}$;
- $\mathsf{Unit}^{\mathcal{I}} = \{c\}$;
- $\mathsf{Department}^{\mathcal{I}} = \{c\}$;
- $\mathsf{TaxOffice}^{\mathcal{I}} = \{d\}$;
- $\mathsf{registered\_at}^{\mathcal{I}} = \{(b, d)\}$;
- $\mathsf{manages}^{\mathcal{I}} = \{(a, c)\}$.

The resulting TBox is satisfiable. For example, every $\mathcal{I}$ in which $\mathsf{Manager}^{\mathcal{I}} = \emptyset$ and $\mathsf{Employer}^{\mathcal{I}} = \emptyset$ is a model of the extended TBox. Note, however, that Manager is not satisfiable anymore because Unit and Department are disjoint (according to the new inclusion) but every manager manages a unit and any such unit is, by the value restriction in the third inclusion, a department.

Solution for 4.

Let

$$C = (\neg A) \sqcap (\exists r.\exists r.A) \sqcap (\forall r.\neg A)$$

As C is already in negation normal form, the tableaux algorithm starts with

$$S_0 = \{x : (\neg A) \sqcap (\exists r.\exists r.A) \sqcap (\forall r.\neg A)\}$$

An application of the rule $\rightarrow_\sqcap$ gives

$$S_1 = S_0 \cup \{x : \neg A, x : (\exists r.\exists r.A) \sqcap (\forall r.\neg A)\}$$

An application of the rule $\rightarrow_\sqcap$ gives

$$S_2 = S_1 \cup \{x : (\exists r.\exists r.A), x : (\forall r.\neg A)\}$$

An application of the rule $\rightarrow_\exists$ gives

$$S_3 = S_2 \cup \{(x, y) : r, y : \exists r.A\}$$

An application of the rule $\rightarrow_\exists$ gives

$$S_4 = S_3 \cup \{(y, z) : r, z : A\}$$

An application of the rule $\rightarrow_\forall$ gives

$$S_5 = S_4 \cup \{y : \neg A\}$$

No rule is applicable to S_5 and S_5 does not contain any clash. Thus, C is satisfiable.

An interpretation $\mathcal{I}$ satisfying C is given by

- $\Delta^{\mathcal{I}} = \{x, y, z\}$;
- $A^{\mathcal{I}} = \{z\}$;
- $r^{\mathcal{I}} = \{(x, y), (y, z)\}$.