COMP321 (Ontology Languages): Test 2

Lecturer: F. Wolter
Time: 50 minutes

This test makes up 10 percent of the final mark for this module. You can achieve 100 marks.

1. Consider the TBox $\mathcal{T}$ containing

   - $\mathsf{City} \sqcap \mathsf{Country} \sqsubseteq \bot$;
   - $\mathsf{Country} \sqsubseteq \exists \mathsf{capital\_of}^{-}.\mathsf{City}$;
   - $\mathsf{City} \sqsubseteq \forall \mathsf{capital\_of}.\mathsf{Country}$.

   Let $\mathcal{A}$ be the ABox containing

   - $\mathsf{City}(\mathsf{Paris}$;
   - $\mathsf{City}(\mathsf{Madrid})$;
   - $\mathsf{Country}(\mathsf{UK})$;
   - $\mathsf{capital\_of}(\mathsf{Paris}, \mathsf{France})$.

   Recall that the answers to Boolean queries given by database instances, and knowledge bases are "Yes", "No", "Don't know".

   Give the answers given by

   - the database instance $\mathcal{I}_{\mathcal{A}}$ corresponding to $\mathcal{A}$;
   - the knowledge base $(\mathcal{T}, \mathcal{A})$,

   to the following Boolean queries:

   - $\mathsf{City}(\mathsf{Paris})$;
   - $\mathsf{Country}(\mathsf{Paris})$;
   - $\mathsf{Country}(\mathsf{France})$;
   - $(\forall \mathsf{capital\_of}^{-}.\mathsf{City})(\mathsf{France})$;
   - $(\exists \mathsf{capital\_of}^{-}.\mathsf{City})(\mathsf{UK})$;
   - $(\exists \mathsf{capital\_of}^{-}.\mathsf{Country})(\mathsf{Madrid})$.

   Give a brief explanation for each answer.

   (36 marks)

2. Consider the following sentences:

   - Every organisation has at least 2 members.
   - If something is a member of an organisation, then it is a state or a human being.
   - The EU is an organisation whose members are not human beings.
   - All members of the EU are states.

   Translate the sentences into $\mathcal{SHOIQ}$ inclusions. Use $\mathsf{has\_member}$ as a role name (and no other role name) and state which other symbols are used as concept names and individual names in the translation.

   (**20 marks**)

3. Consider the TBox containing

$$\top \sqsubseteq \mathsf{red} \sqcup \mathsf{green}, \quad \mathsf{red} \sqcap \exists r.\mathsf{green} \sqsubseteq \mathsf{clash}, \quad \mathsf{green} \sqcap \exists r.\mathsf{red} \sqsubseteq \mathsf{clash}$$

   Let $\mathcal{A}$ be the ABox containing

$$r(0,1), r(1,2), r(2,3), r(3,0)$$

   What is the answer of $(\mathcal{T}, \mathcal{A})$ to the Boolean query $\exists x\, \mathsf{clash}(x)$? Explain your answer.

   (**14 marks**)

4. Why is data complexity regarded as a more informative measure of the complexity of query answering in relational databases than combined complexity?

   (**15 marks**)

5. Let $\mathcal{T}$ be the $\mathcal{EL}$ TBox containing

$$A \sqsubseteq \exists r.B, \quad B \sqsubseteq \exists r.B$$

   and let $\mathcal{A}$ contain

$$r(a,b), \quad A(b)$$

   Compute the interpretation $\mathcal{I}_{\mathcal{T},\mathcal{A}}$ from the lecture notes so that for all $\mathcal{EL}$-concepts C and $d \in \{a, b\}$

$$\mathcal{T}, \mathcal{A} \models C(d) \quad \Leftrightarrow \quad \mathcal{I}_{\mathcal{T},\mathcal{A}} \models C(d)$$

   (**15 marks**)

Solution for 1.

- $\mathsf{City}(\mathsf{Paris})$: the answer is always YES because $\mathsf{City}(\mathsf{Paris})$ is in the ABox.
- $\mathsf{Country}(\mathsf{Paris})$: the answer given by $\mathcal{I}_\mathcal{A}$ is NO because UK is the only country in the database. The answer given by $(\mathcal{T}, \mathcal{A})$ is NO because Paris is a city and the classes city and country are disjoint according to the TBox.
- $\mathsf{Country}(\mathsf{France})$: the answer given by $\mathcal{I}_\mathcal{A}$ is NO because UK is the only country in the database. The answer given by $(\mathcal{T}, \mathcal{A})$ is YES because Paris is a city and is capital of France. By the third inclusion, anything a city is a capital of is a country.
- $(\forall \mathsf{capital\_of}^{-}.\mathsf{City})(\mathsf{France})$: The answer given by $\mathcal{I}_\mathcal{A}$ is YES because France has one capital and that's Paris which is a city. The answer given by $(\mathcal{T}, \mathcal{A})$ is DON'T KNOW because it is not stated in the TBox that countries have only one capital and it is not stated that capitals of countries are cities.
- $(\exists \mathsf{capital\_of}^{-}.\mathsf{City})(\mathsf{UK})$. The answer given by $\mathcal{I}_\mathcal{A}$ is NO because UK does not occur in the table for $\mathsf{capital\_of}$. The answer given by $(\mathcal{T}, \mathcal{A})$ is YES because the UK is a country and every country has a capital that is a city by the second inclusion.
- $\exists \mathsf{capital\_of}^{-}.\mathsf{Country})(\mathsf{Madrid})$. The answer given by $\mathcal{I}_\mathcal{A}$ is NO because Madrid does not occur in the table for $\mathsf{capital\_of}$. The answer given by $(\mathcal{T}, \mathcal{A})$ is DON'T KNOW because the TBox does not state properties of the relation "has capital" that allow us to deduce that this is not the case. Note that the query asks whether Madrid has a capital that is a country. Clearly this does not follows from the TBox and the ABox. So the answer is definitely not "yes". It is not "No" because one can easily construct a model of the TBox and ABox in which the query is true.

Solution for 2.

- Every organisation has at least 2 members.

$$\mathsf{Organisation} \sqsubseteq (\geq 2 \mathsf{has\_member}.\top)$$

- If something is a member of an organisation, then it is a state or a human being.

$$\exists \mathsf{has\_member}^{-}.\mathsf{Organisation} \sqsubseteq \mathsf{State} \sqcup \mathsf{Human\_being}$$

- The EU is an organisation whose members are not human beings.

$$\{\mathsf{EU}\} \sqsubseteq \mathsf{Organisation} \sqcap \neg \exists \mathsf{has\_member}.\mathsf{Human\_being}$$

- All members of the EU are states.

$$\exists \mathsf{has\_member}^{-}.\{\mathsf{EU}\} \sqsubseteq \mathsf{State}$$

Solution for 3.

The answer is "Don't Know". Clearly the answer is not "No" because there is a model of $(\mathcal{T}, \mathcal{A})$ in which clash is non-empty. The answer is not "Yes" because the interpretation $\mathcal{I}$ with

- $\Delta^{\mathcal{I}} = \{0, 1, 2, 3\}$,
- $r^{\mathcal{I}} = \{(0, 1), (1, 2), (2, 3), (3, 0)\}$,
- $\mathsf{green}^{\mathcal{I}} = \{0, 1, 2, 3\}$,
- $\mathsf{red}^{\mathcal{I}} = \emptyset$
- $\mathsf{clash}^{\mathcal{I}} = \emptyset$

is a model of $\mathcal{T}$ and $\mathcal{A}$.

Solution for 4.

The input to the query answering problem are data D and a query q. In data complexity, when measuring the complexity of answering q over D, the query is regarded as fixed and only D grows. In combined complexity, both q and D grow. Data complexity is more meaningful because queries are typically very small compared to the data.

Solution for 5. The interpretation $\mathcal{I} = \mathcal{I}_{\mathcal{T},\mathcal{A}}$ is given by setting

- $\Delta^{\mathcal{I}} = \{a, b, d_A, d_B\}$;
- $r^{\mathcal{I}} = \{(a, b), (b, d_B), (d_B, d_B), (d_A, d_B)\}$;
- $A^{\mathcal{I}} = \{b, d_A\}$;
- $B^{\mathcal{I}} = \{d_B\}$.