Ontology Languages (COMP321)

Exercise 1

1. Recall the syntax of the Description Logic $\mathcal{EL}$. Assume that A and B are concept names and r and s are role names. For each of the following expressions, state whether

   - it is a $\mathcal{EL}$ concept;
   - a $\mathcal{EL}$ concept definition;
   - a primitive $\mathcal{EL}$ concept definition;
   - $\mathcal{EL}$ concept inclusion;
   - none of the above.

   (a) $A \sqcap B$
   (b) $(A \sqcap B) \sqcup A$
   (c) $\neg B$
   (d) $A \sqsubseteq B$
   (e) $\exists r.(A \sqcap B)$
   (f) $A \sqcap B \sqsubseteq B$
   (g) $A \equiv A \sqcap B$
   (h) $\exists A.B$
   (i) $r \sqsubseteq s$
   (j) $A \equiv \exists s.B$
   (k) $\bot \sqsubseteq \top$

2. Create an $\mathcal{EL}$ TBox $\mathcal{T}$ that models the following facts:

   (a) Mammals are animals.
   (b) Cats are mammals that are carnivores.
   (c) Elephants are mammals that are herbivores.
   (d) Carnivores eat meat.

(e) A vertebrate is any animal that has, amongst other things, a backbone.

Is the following $\mathcal{EL}$-TBox an $\mathcal{EL}$-terminology? Explain your answer. Express each concept inclusion in natural language:

$$
\begin{aligned}
\mathsf{Fish} &\sqsubseteq \mathsf{Animal} \sqcap \exists \mathsf{lives\_in}.\mathsf{Water} \\
\exists \mathsf{eat}.\mathsf{Meat} &\sqsubseteq \mathsf{Carnivore} \\
\mathsf{Bird} &\equiv \mathsf{Vertebrate} \sqcap \exists \mathsf{has\_part}.\mathsf{Wing} \\
& \quad \sqcap \exists \mathsf{has\_part}.\mathsf{Leg} \sqcap \exists \mathsf{lays}.\mathsf{Egg} \\
\mathsf{Reptile} &\sqsubseteq \mathsf{Vertebrate} \sqcap \exists \mathsf{lays}.\mathsf{Egg}
\end{aligned}
$$

3. Let $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ be an interpretation, where

- $\Delta^{\mathcal{I}} = \{a, b, c, d, e, f\}$
- $A^{\mathcal{I}} = \{a, b\}$
- $B^{\mathcal{I}} = \{c, d, e, f\}$
- $r^{\mathcal{I}} = \{(a, c), (a, e), (b, f)\}$

Determine the extension $C^{\mathcal{I}}$ of the following $\mathcal{EL}$-concepts C under $\mathcal{I}$:

- $A \sqcap B$
- $\exists r.B$
- $\exists r.(A \sqcap B)$
- $\top$
- $A \sqcap \exists r.B$

Which of the following are true?

- $\mathcal{I} \models A \equiv \exists r.B$
- $\mathcal{I} \models A \sqcap B \sqsubseteq \top$
- $\mathcal{I} \models \exists r.A \sqsubseteq A \sqcap B$
- $\mathcal{I} \models \top \sqsubseteq B$
- $\mathcal{I} \models B \sqsubseteq \exists r.A$