# Ontology Languages (COMP321)

## Exercise 2

1. Let $\mathcal{T} = \{A \sqsubseteq \exists r.B, E \sqsubseteq A\}$. Show that $\mathcal{T} \not\models A \sqsubseteq E$ by giving an interpretation $\mathcal{I}$ such that $\mathcal{I} \models \mathcal{T}$ and $\mathcal{I} \not\models A \sqsubseteq E$.

2. Let $\mathcal{T}$ be an $\mathcal{EL}$-TBox consisting of the following concept inclusions:

$$\begin{aligned} \mathsf{Bird} &\equiv \mathsf{Vertebrate} \sqcap \exists \mathsf{has}.\mathsf{Wing} \\ \mathsf{Reptile} &\sqsubseteq \mathsf{Vertebrate} \sqcap \exists \mathsf{lays}.\mathsf{Egg} \end{aligned}$$

   (a) Is $\mathcal{T}$ in normal form? Explain.

   (b) Given $\mathcal{T}$, compute an $\mathcal{EL}$-TBox $\mathcal{T}'$ in normal form using the pre-processing algorithm from the lecture.

   (c) Apply the algorithm from the lecture notes deciding whether $A \sqsubseteq_{\mathcal{T}'} B$ (equivalently, $\mathcal{T}' \models A \sqsubseteq B$), where $A, B$ are concept names. Use the normalised TBox $\mathcal{T}'$ as input and explain step-by-step which rules are applied.

   (d) Using the output of the algorithm, decide whether

   - $\mathsf{Reptile} \sqsubseteq_{\mathcal{T}'} \mathsf{Vertebrate}$
   - $\mathsf{Vertebrate} \sqsubseteq_{\mathcal{T}'} \mathsf{Bird}$

3. Let $\mathcal{T}$ be an $\mathcal{EL}$-TBox containing the following concept inclusions:

$$A \sqsubseteq X \tag{1}$$
$$A \sqsubseteq Y \tag{2}$$
$$B \sqsubseteq B' \tag{3}$$
$$X \sqcap Y \sqsubseteq Z \tag{4}$$
$$Z \sqsubseteq \exists r.B \tag{5}$$
$$\exists r.B' \sqsubseteq A' \tag{6}$$

(a) Is $\mathcal{T}$ in normal form?

(b) Apply the algorithm from the lecture notes deciding $E \sqsubseteq_{\mathcal{T}} F$ (equivalently, $\mathcal{T} \models E \sqsubseteq F$), where $E, F$ are concept names. Use $\mathcal{T}$ as input and explain how the rules of the algorithm are applied step-by-step.

(c) Using the output of the algorithm, decide whether

- $A \sqsubseteq_{\mathcal{T}} Z$
- $B \sqsubseteq_{\mathcal{T}} Z$
- $X \sqsubseteq_{\mathcal{T}} Y$
- $A \sqsubseteq_{\mathcal{T}} A'$
- $B \sqsubseteq_{\mathcal{T}} B'$