Ontology Languages (COMP321)
Exercise 4

1. Let $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ be an interpretation defined by

   - $\Delta^{\mathcal{I}} = \{2, 3, 4\}$;
   - $A^{\mathcal{I}} = \{2\}$;
   - $B^{\mathcal{I}} = \{3, 4\}$;
   - $r^{\mathcal{I}} = \{(2, 2), (2, 3), (4, 2)\}$.

   Determine

   - $(A \sqcup B)^{\mathcal{I}}$;
   - $(A \sqcap B)^{\mathcal{I}}$;
   - $(\top \sqcap \neg(A \sqcup \neg B))^{\mathcal{I}}$;
   - $(\forall r.(A \sqcup B))^{\mathcal{I}}$;
   - $(\forall r.(A \sqcap B))^{\mathcal{I}}$;
   - $(\forall r.A \sqcap \exists r.B)^{\mathcal{I}}$.

   Which of the following statements are true:

   - $\mathcal{I} \models B \sqsubseteq \neg A$?
   - $\mathcal{I} \models \exists r.A \sqcap B \sqsubseteq \forall r.A$?
   - $\mathcal{I} \models \exists r.B \sqsubseteq A$?

2. Apply the $\mathcal{ALC}$-tableau algorithm to the following concepts and determine which are satisfiable and which are not. If a concept is satisfiable, give an interpretation satisfying it.

   - $A \sqcap \neg A$
   - $\exists r.\exists r.(A \sqcap \neg A)$
   - $\forall r.\forall r.(A \sqcap \neg A)$
   - $\exists r.A \sqcap \forall s.\neg A$
   - $\exists r.A \sqcap (\forall r.\neg A \sqcup \exists r.\neg A)$

3. Use the $\mathcal{ALC}$-tableau algorithm to determine whether $\emptyset \models \forall r.A \sqsubseteq \exists r.A$ (in words: determine whether the concept inclusion $\forall r.A \sqsubseteq \exists r.A$ follows from the empty TBox).