1. Let $S$ be the signature consisting of the unary predicate symbols `author`, `human_being`, and `book`, the binary predicate symbol `author_of`, and the individual constants `Rankin` and `TheFalls`. Translate the following sentences into first-order predicate logic sentences over $S$:

- Rankin is an author.
- “The Falls” is a book.
- Rankin is not a book.
- Rankin is the author of “The Falls”.
- Rankin is a human being.
- Human beings exist.
- Not everything is a human being.
- No book is a human being.
- Some human beings are not authors.
- Every author is a human being.
- Every book has an author.
- “The Falls” has an author.
- Every author is the author of something.
- Every author is the author of some book.
- Every author of something is a human being.

2. Let $S_{AR}$ be the signature for arithmetic defined on page 19 and $\mathcal{A}$ the standard structure for $S_{AR}$.

- Let $a(x) = 4$. Does $(\mathcal{A}, a) \models \text{smaller}(5, x)$ hold? Explain your answer.
- Let $a(x) = 12$. Does $(\mathcal{A}, a) \models \text{smaller}(5, x)$ hold? Explain your answer.
• Let \( a(x) = 3 \). Does \((\mathcal{A}, a) \models \exists y. \text{smaller}(y, x)\) hold? Explain your answer.

• Let \( a(x) = 5 \). Does \((\mathcal{A}, a) \models \exists y. \text{sum}(y, y, x)\) hold? Explain your answer.

• Let \( a(x) = 5 \). Does \((\mathcal{A}, a) \models \exists y_1. \exists y_2. \text{sum}(y_1, y_2, x)\) hold? Explain your answer.

• Does \( \mathcal{A} \models \forall x. \exists y. \text{smaller}(x, y)\) hold? Explain your answer.

• Does \( \mathcal{A} \models \forall x. \exists y. \text{smaller}(y, x)\) hold? Explain your answer.

3. Let \( S_{AR} \) be again the signature for arithmetic defined on page 19. Add a binary predicate symbol \texttt{equal} to \( S_{AR} \) and denote the resulting signature by \( S^=_{AR} \). We expand the standard interpretation \( \mathcal{A} \) by defining

\[
\text{equal}^\mathcal{A} = \{(n, m) \mid n = m\}
\]

Translate the following sentences into first-order predicate logic sentences over \( S^=_{AR} \):

• \( x \) is equal to 0.

• If the sum of two numbers is 0, then both numbers are 0.

• If \( y \) is the sum of \( x_1 \) and \( x_2 \) and \( x_2 \) is not equal to 0, then \( x_1 \) is smaller than \( y \).

• the product of any number with 0 is 0.

• the sum of any number and 0 is the number itself.

Solution for 1.

• Rankin is an author: \texttt{author(Rankin)}.

• “The Falls” is a book: \texttt{book(TheFalls)}.

• Rankin is not a book: \texttt{¬book(Rankin)}. 
- Rankin is the author of “The Falls”: author_of(Rankin, TheFalls).
- Rankin is a human being: human_being(Rankin).
- Human beings exist: \( \exists x. \text{human}_\text{being}(x) \).
- Not everything is a human being: \( \exists x. \neg \text{human}_\text{being}(x) \).
- No book is a human being: \( \forall x. (\text{book}(x) \rightarrow \neg \text{human}_\text{being}(x)) \).
- Some human beings are not authors:
  \[ \exists x. (\text{human}_\text{being}(x) \land \neg \text{author}(x)) \].
- Every author is a human being: \( \forall x. (\text{author}(x) \rightarrow \text{human}_\text{being}(x)) \).
- Every book has an author: \( \forall x. (\text{book}(x) \rightarrow \exists y. \text{author}_\text{of}(y, x)) \).
- “The Falls” has an author: \( \exists x. \text{author}_\text{of}(x, \text{TheFalls}) \).
- Every author is the author of something:
  \[ \forall x. (\text{author}(x) \rightarrow \exists y. \text{author}_\text{of}(x, y)) \].
- Every author is the author of some book:
  \[ \forall x. (\text{author}(x) \rightarrow \exists y. (\text{author}_\text{of}(x, y) \land \text{book}(y))) \]
- Every author of something is a human being:
  \[ \forall x. (\exists y. \text{author}_\text{of}(x, y)) \rightarrow \text{human}_\text{being}(x) \]

Solution for 2.
- Let \( a(x) = 4 \). \( \mathcal{A}, a \models \text{smaller}(5, x) \) does NOT hold because \( 5^A = 5 \) and \( 5 \not< 4 \).
- Let \( a(x) = 12 \). \( \mathcal{A}, a \models \text{smaller}(5, x) \) holds because \( 5^A = 5 \) and \( 5 < 12 \).
- Let \( a(x) = 3 \). \( \mathcal{A}, a \models \exists y. \text{smaller}(y, x) \) holds because there exist numbers smaller than 3, for example 2.
Let $a(x) = 5$. $(A, a) \models \exists y.\text{sum}(y, y, x)$ does NOT hold because 5 is not an even number. So there does not exist any number $x$ with $x + x = 5$.

Let $a(x) = 5$. $(A, a) \models \exists y_1.\exists y_2.\text{sum}(y_1, y_2, x)$ holds because 5 is the sum of two numbers, for example of 2 and 3.

$A \models \forall x.\exists y.\text{smaller}(x, y)$ holds because for every number there exists a larger number.

$A \models \forall x.\exists y.\text{smaller}(y, x)$ does not hold because there does NOT exist any $x$ with $x < 0$.

Solution for 3.

$x$ is equal to 0: $\text{equal}(x, 0)$.

If the sum of two numbers is 0, then both numbers are 0:

$\forall x_1.\forall x_2. (\text{sum}(x_1, x_2, 0) \rightarrow (\text{equal}(x_1, 0) \land \text{equal}(x_2, 0)))$

If $y$ is the sum of $x_1$ and $x_2$ and $x_2$ is not equal to 0, then $x_1$ is smaller than $y$:

$\forall x_1.\forall x_2.\forall y. ( (\text{sum}(x_1, x_2, y) \land \neg \text{equal}(x_2, 0)) \rightarrow \text{smaller}(x_1, y))$

the product of any number with 0 is 0: $\forall x.\text{prod}(x, 0, 0)$.

the sum of any number and 0 is the number itself: $\forall x.\text{sum}(x, 0, x)$. 