Logic in Computer Science (COMP118)
Tutorial Problems 1

1. Let

   - p_1 denote the proposition: Tom's house is red;
   - p_2 denote the proposition: Jim's house is red;
   - p_3 denote the proposition: Mary's house is red.

   Translate into propositional logic:

   - Tom's house is red or Jim's house is red;
   - If Jim's house is red, then Tom's house is red and Mary's house is red;
   - Tom's house is not red;
   - Jim's house is red if, and only if, Tom's house is not red;
   - Neither Jim's nor Mary's house is red.

   Let $P_1, P_2, \ldots, P_5$ be the translations into propositional logic of the five sentences above. For each P_i, $1 \leq i \leq 5$:

   - give an interpretation I such that $I(P_i) = 1$ and
   - give an interpretation J such that $J(P_i) = 0$.

2. Give a definition of satisfiable formulas.

3. Which of the following formulas are satisfiable? Check this

   (a) using truth tables and

   (b) using the tableau algorithm.

   - $(\neg\neg p \land \neg(p \lor q))$;
   - $(\neg(\neg p \lor q) \land \neg r)$;
   - $((p \rightarrow q) \land \neg(q \rightarrow p))$;
   - $((p \rightarrow q) \land \neg(p \rightarrow q))$;

- $((\neg(p \wedge q) \wedge p) \wedge q)$.

(Hint: We do not have tableau rules for $\rightarrow$. Thus, $\rightarrow$ has to be replaced by its definition before the tableau rules can be applied.)

4. Call a propositional formula P *positive* if it does not contain the symbol $\neg$. Give an inductive definition of positive formulas. Show that every positive formula is satisfiable.