1. Let

• $p_1$ denote the proposition: Tom’s house is red;
• $p_2$ denote the proposition: Jim’s house is red;
• $p_3$ denote the proposition: Mary’s house is red.

Translate into propositional logic:

• Tom’s house is red or Jim’s house is red;
• If Jim’s house is red, then Tom’s house is red and Mary’s house is red;
• Tom’s house is not red;
• Jim’s house is red if, and only if, Tom’s house is not red;
• Neither Jim’s nor Mary’s house is red.

Let $P_1, P_2, \ldots, P_5$ be the translations into propositional logic of the five sentences above. For each $P_i$, $1 \leq i \leq 5$:

• give an interpretation $I$ such that $I(P_i) = 1$ and
• give an interpretation $J$ such that $J(P_i) = 0$.

2. Give a definition of satisfiable formulas.

3. Which of the following formulas are satisfiable? Check this

(a) using truth tables and
(b) using the tableau algorithm.

• $\neg\neg p \land \neg(p \lor q)$;
• $\neg(\neg p \lor q) \land \neg r$;
• $((p \rightarrow q) \land \neg(q \rightarrow p))$;
• $((p \rightarrow q) \land \neg(p \rightarrow q))$;
• \(((\neg (p \land q) \land p) \land q)\).

(Hint: We do not have tableau rules for \(\to\). Thus, \(\to\) has to be replaced by its definition before the tableau rules can be applied.)

4. Call a propositional formula \(P\) positive if it does not contain the symbol \(\neg\). Give an inductive definition of positive formulas. Show that every positive formula is satisfiable.