1. Recall that a first-order sentence $G$ over a signature $S$ is satisfiable if, and only if, there exists an $S$-structure $F$ such that $F \models G$. Let $R$ be a binary predicate symbol, $Q$ a unary predicate symbol, and $c$ an individual constant.

(a) Show that the sentence $G_1 = \forall x.\forall y.(R(x, y) \rightarrow R(y, x))$ is satisfiable. In other words, define an $\{R\}$-structure $F_1$ such that $F_1 \models G_1$.

(b) Show that the sentence $G_2 = \neg \forall x.\forall y.(R(x, y) \rightarrow R(y, x))$ is satisfiable. In other words, define an $\{R\}$-structure $F_2$ such that $F_2 \models G_2$.

(c) Show that the sentence $G_3 = \forall x.R(x, x)$ is satisfiable. In other words, define an $\{R\}$-structure $F_3$ such that $F_3 \models G_3$.

(d) Show that the sentence $(\forall x.Q(x) \land \neg \forall x.c)$ is not satisfiable.

2. Show that $\forall x.(P(x) \land Q(x))$ and $(\forall x.P(x) \land \forall x.Q(x))$ are logically equivalent. In other words, show that

- $\{\forall x.(P(x) \land Q(x))\} \models (\forall x.P(x) \land \forall x.Q(x))$ and
- $\{\forall x.P(x) \land \forall x.Q(x)\} \models \forall x.(P(x) \land Q(x))$.

3. Show that $\{\forall x.(P(x) \lor Q(x))\} \not\models ((\forall x.P(x) \lor \forall x.Q(x))$. In other words, define a $\{P, Q\}$-structure $F$ such that $F \models \forall x.(P(x) \lor Q(x))$ and $F \not\models ((\forall x.P(x) \lor \forall x.Q(x))$.

4. Let $S = \{P, Q, c\}$, where $P$ and $Q$ are unary predicate symbols and $c$ is a individual constant. Show that

$$\{((Q(c) \land \exists x.(Q(x) \land P(x)))\} \not\models P(c)$$

In other words, define an $S$-structure $F$ such that

- $F \models Q(c) \land \exists x.(Q(x) \land P(x))$ and
5. Let \( P, Q, R \) be propositional formulas. Show that

\[
(P \rightarrow (Q \rightarrow R)) \equiv ((P \land Q) \rightarrow R)
\]