Exercise 7:
Consider the following instance of a single commodity Wardrop game \((G, r, c)\) where a flow of 1 has to be sent from \(s\) to \(t\):

\[
\begin{align*}
2x + 2 & \quad \text{from } s \to u \\
3x + 1 & \quad \text{from } u \to t \\
x^2 + 4 & \quad \text{on path } s \to v \to t \\
1 & \quad \text{on edge } s \to v \\
4 & \quad \text{on edge } v \to t
\end{align*}
\]

(a) Explain why in a Wardrop equilibrium no flow will be sent along the edge \((u, v)\).

(b) Determine the Wardrop equilibrium for \((G, r, c)\).

(c) Construct a new instance \((G, r, c^*)\) by replacing each cost function \(c_e\) with its marginal cost function \(c^*_e\).

(d) Determine an optimal flow for the original instance \((G, r, c)\). Compute the price of anarchy \(\rho(G, r, c)\).