Exercise 8:
In this exercise you are completing the proof of Theorem 3.6.
Let \( c(x) = a \cdot x + b \) be a linear function with \( a \geq 0, b \geq 0 \). Let \( 0 \leq x_1 \leq x_2 \).

(a) Use the definition of \( c \) to express \( x_1 \cdot (c(x_2) - c(x_1)) \) as a quadratic function in \( x_1 \).

(b) Use calculus to show that for any fixed \( x_2 \) the term \( x_1 \cdot (c(x_2) - c(x_1)) \) is maximised for \( x_1 = \frac{x_2}{2} \).

(c) Use (b) to show that
\[
x_1 \cdot (c(x_2) - c(x_1)) \leq \frac{1}{4} \cdot x_2 \cdot c(x_2).
\]

Exercise 9:
Consider the following refinement of Theorem 3.7:

- Let \( f \) be a Wardrop equilibrium for \((G, r, c)\) and \( f^\star \) a feasible flow for \((G, (1 + \beta)r, c)\) for some \( \beta > 0 \).
  Show an upper bound on \( C(f) \) with respect to \( C(f^\star) \) and \( \beta \).

Hint: Theorem 3.7 gives such a bound for \( \beta = 1 \)