Exercise 12:
Prove the upper bound in Corollary 3.17 (a): Show that $(1 + \frac{\Phi}{2}, \frac{1}{2\Phi}) \in A(C)$, that is, for all reals $y > 0$, $z \geq 0$ and all latency functions of the form $c(x) = a_1 \cdot x + a_0$ where $a_1, a_0 \geq 0$ we have

$$y \cdot c(z + y) \leq (1 + \frac{\Phi}{2}) \cdot y \cdot c(y) + \frac{1}{2\Phi} \cdot z \cdot c(z).$$

Recall that $\Phi = \frac{1 + \sqrt{5}}{2}$ is the golden ratio, which has the property $\Phi^2 = 1 + \Phi$.
Use this property and Prop. 3.15 to show the upper bound on the price of anarchy.

Exercise 13:
Consider the unweighted network congestion game $G$ with 4 players and linear latency functions, which is given by the following graph:

(a) Show that $\text{PoA}(G) \geq \frac{5}{2}$.

(b) Modify the weights of the players so that the price of anarchy of the resulting weighted network congestion game is exactly $\frac{3 + \sqrt{5}}{2} \approx 2.618$. 