TRP++: A temporal resolution prover*

Ullrich Hustadt and Boris Konev**

Logic and Computation Group, Department of Computer Science
University of Liverpool, Liverpool L69 7ZP, UK
{U.Hustadt, B.Konev}@csc.liv.ac.uk

1 Introduction

Temporal logics are extensions of classical logic, with operators that deal with time. They have been used in a wide variety of areas within Computer Science and Artificial Intelligence, for example robotics [24], databases [26], hardware verification [13] and agent-based systems [21]. In particular, propositional temporal logics have been applied to:

- the specification and verification of reactive (e.g. distributed or concurrent) systems [19];
- the synthesis of programs from temporal specifications [18, 20];
- the semantics of executable temporal logic [8, 9];
- algorithmic verification via model-checking [3, 12]; and
- knowledge representation and reasoning [1, 6, 27].

In developing these techniques, temporal proof is often required, and we base our work on practical proof techniques for the clausal resolution method for temporal logic. The method is based on an intuitive clausal form, called SNF, comprising three main clause types and a small number of resolution rules [10]. While the approach has been shown to be competitive [14, 15] using a prototype implementation of the method, we now aim at an even more efficient implementation. This implementation, called TRP++, is the focus of this paper.

2 Basics of PLTL

Let $P$ be a set of propositional variables. The set of formulae of propositional linear time logic PLTL (over $P$) is inductively defined as follows: (i) $\top$ is a formula of PLTL, (ii) every propositional variable of $P$ is a formula of PLTL, (iii) if $\varphi$ and $\psi$ are formulae of PLTL, then $\neg \varphi$ and $(\varphi \lor \psi)$ are formulae of PLTL, and (iv) if $\varphi$ and $\psi$ are formulae of PLTL, then $\Box \varphi$ (in the next moment of time $\varphi$ is true), $\Diamond \varphi$ (sometimes in the future $\varphi$ is true), $\Box \varphi$ (always in the future $\varphi$ is true), $(\varphi U \psi)$ ($\varphi$ is true until $\psi$ is true), and $(\varphi W \psi)$ ($\varphi$ is true unless $\psi$ is true) are formulae of PLTL. Other Boolean connectives including $\bot$, $\land$, $\rightarrow$, and $\leftrightarrow$ are defined using $\top$, $\neg$, and $\lor$.

PLTL-formulae are interpreted over ordered pairs $I = \langle S, i \rangle$ where (i) $S$ is an infinite sequence of states $(s_i)_{i \in \mathbb{N}}$ and (ii) $i$ is an interpretation function assigning to each state a subset of $P$.

We define a binary relation $|=\!=$ between a PLTL-formula $\varphi$ and a pair consisting of a PLTL-interpretation $I = \langle S, i \rangle$ and a state $s_i \in S$ as follows.

$$
\begin{align*}
I, s_i &\models p \quad \text{iff} \quad p \in \iota(s_i) & I, s_i &\models \top \\
I, s_i &\models (\varphi \lor \psi) \quad \text{iff} \quad I, s_i \models \varphi \quad \text{or} \quad I, s_i \models \psi & I, s_i &\models \neg \varphi \quad \text{iff} \quad I, s_i \not\models \varphi \\
I, s_i &\models \Box \varphi \quad \text{iff} \quad I, s_{i+1} \models \varphi & I, s_i &\models \Diamond \varphi \quad \text{iff} \quad \text{for all} \quad j \in \mathbb{N}, j \geq i \quad \text{implies} \quad I, s_j \models \varphi \\
I, s_i &\models (\varphi U \psi) \quad \text{iff} \quad \text{there exists} \quad j \in \mathbb{N} \quad \text{such that} \quad j \geq i \quad \text{and} \quad I, s_j \models \varphi & I, s_i &\models (\varphi W \psi) \quad \text{iff} \quad \text{there exists} \quad j \in \mathbb{N} \quad \text{such that} \quad j \geq i, I, s_j \models \psi \quad \text{and} \quad \text{for all} \quad k \in \mathbb{N}, j > k \geq i \quad \text{implies} \quad I, s_k \models \varphi \\
I, s_i &\models (\varphi W \psi) \quad \text{iff} \quad I, s_i \models \varphi U \psi \quad \text{or} \quad I, s_i \models \neg \varphi & I, s_i &\models (\varphi W \psi) \quad \text{iff} \quad I, s_i \models \varphi U \psi \quad \text{or} \quad I, s_i \models \neg \varphi
\end{align*}
$$

If $I, s_i \models \varphi$ then we say $\varphi$ is true, or holds, at $s_i$ in $I$. An interpretation $I$ satisfies a formula $\varphi$ iff $\varphi$ holds at $s_0$ in $I$ and it satisfies a set $N$ of formulae iff for every formula $\psi \in N$, $I$ satisfies $\psi$. In this case, $I$ is

---

* Work supported by EPSRC grant GR/L87491.
** On leave from Steklov Institute of Mathematics at St.Petersburg
a model for $\varphi$ and $N$, respectively, and we say $\varphi$ and $N$ are (PLTL-)satisfiable. The satisfiability problem of PLTL is known to be PSPACE-complete [25].

Arbitrary PLTL-formulae can be transformed into separated normal form (SNF) in a satisfiability equivalence preserving way using a renaming technique replacing non-atomic subformulae with new propositions and removing all occurrences of the $\forall$ and $\exists$ operator [7, 10].

The result is a set of SNF clauses of the following form (which differs slightly from [7, 10]).

$$\forall_{i=1}^{n} L_i \quad \Box(\forall_{j=1}^{m} K_j \lor \forall_{i=1}^{n} \circ L_i) \quad \Box(\forall_{j=1}^{m} K_j \lor \Diamond L)$$

(initial clause)                           (global clause)                           (eventuality clause)

Here, $K_j$, $L_i$, and $L$ (with $1 \leq j \leq m$, $1 \leq i \leq n$, and $1 \leq i \leq n$, $0 \leq n$) denote propositional literals.

In the following, we assume that SNF clauses are sets of (temporal) literals. Furthermore, if $C = L_1 \lor \ldots \lor L_n$ we use $\Box C$ to denote $\Box L_1 \lor \ldots \lor \Box L_n$.

### 3 Clausal temporal resolution

TRP++ is based on the resolution method for PLTL proposed by Fisher [7] (see also [4, 5, 10]) which involves the translation of PLTL-formulae to separated normal form, classical resolution within states (known as initial and step resolution) and temporal resolution over states between eventuality clauses containing a literal like $\Diamond \neg p$ and global clauses that together imply $\Box p$ (known as eventuality resolution). Figure 1 contains a list of all the inference rules. To simplify the presentation, we have represented the conclusion of the eventuality resolution rule as a PLTL-formula (which would have to be transformed into a set of SNF clauses). In our implementation, we directly produce the corresponding SNF clauses without reverting to the transformation procedure. It should be obvious that finding a set of SNF clauses which satisfies the side conditions of the eventuality resolution rule is a non-trivial problem.

These inference rules provide a sound and complete calculus for deciding the satisfiability of a set $N$ of SNF clauses. Furthermore, under the assumption that an inference step is performed only once for the same set of premises (or that we stop as soon as no new SNF clauses can be derived), any derivation from a set $N$ of SNF clauses will always terminate. Since any PLTL-formula can be transformed into a satisfiability equivalent set of SNF clauses, this means that the combination of this transformation process and the temporal resolution calculus provides a decision procedure for PLTL.

#### Initial resolution rules:

$$\frac{C_1 \lor L \quad \neg L \lor C_2}{C_1 \lor C_2} \quad \frac{C_1 \lor L \quad \Box(\neg L \lor D_2)}{C_1 \lor D_2}$$

where $C_1 \lor L$ and $\neg L \lor C_2$ are initial clauses, $\Box(\neg L \lor C_2)$ is a global clause and $\neg L \lor D_2$ is a propositional clause.

#### Step resolution rules:

$$\frac{\Box(C_1 \lor L)}{\Box(C_1 \lor C_2)} \quad \frac{\Box(\neg L \lor C_2)}{\Box(\neg L \lor C_2)} \quad \frac{\Box(C_1 \lor L) \quad \Box(\neg L \lor D_2)}{\Box(\circ L \lor D_2)} \quad \frac{\Box(D_1 \lor D_2)}{\Box(D_1 \lor D_2)}$$

where $\Box(C_1 \lor L)$ and $\Box(\neg L \lor C_2)$ are global clauses and $C_1 \lor L$ and $\neg L \lor C_2$ are propositional clauses, and $\Box(\circ L \lor D_2)$ and $\Box(D_1 \lor D_2)$ are global clauses. (The side conditions ensure that no clauses with nested occurrences of the $\circ$-operator can be derived.)

#### Eventuality resolution rule:

$$\frac{\Box(C_1 \lor \forall_{i=1}^{k_1} \circ D_{1,j,i}) \quad \ldots \quad \Box(C_n \lor \forall_{i=1}^{k_1} \circ D_{n,j,i})}{\Box(C \lor \Diamond L) \quad \Box(C \lor \Box L) \quad \Box(C \lor (\neg (\forall_{i=1}^{m} \lor_{j=1}^{n} C_{j,i}) \lor L))}$$

where for all $i$, $1 \leq i \leq n$, $(\forall_{i=1}^{m} \lor_{j=1}^{n} D_{j,i}) \rightarrow \neg L$ and $(\forall_{i=1}^{m} \lor_{j=1}^{n} D_{j,i}) \rightarrow (\forall_{i=1}^{m} \lor_{j=1}^{n} C_{j,i})$ are provable.

Fig. 1. The temporal resolution calculus
4 Implementation details

The main procedure of our implementation of the temporal resolution calculus of Section 3 consists of a loop where at each iteration (i) the set of SNF clauses is saturated under application of the initial and step resolution rules, and (ii) then for every eventuality clause in the SNF clause set, an attempt is made to find a set of premises for an application of the eventuality resolution rule. If we find such a set, the set of SNF clauses representing the conclusion of the application is added to the current set of SNF clauses. The main loop terminates if the empty clause is derived, indicating that the initial set of SNF clauses and the PLTL-formula it is stemming from are unsatisfiable, or if no new clauses have been derived during the last iteration of the main loop, which in the absence of the empty clause indicates that the initial set of SNF clauses and the PLTL-formula it is stemming from are satisfiable. Since the number of SNF clauses which can be formed over the finite set of propositional variables contained in the initial set of SNF clauses is itself finite, we can guarantee termination of the main procedure.

It is easy to check that under the natural arithmetic translation of initial and global clauses into first-order logic (an initial clause \((-q_1 \lor \ldots \lor (-q_n)\) is represented by the first-order clause \((-q_1(0) \lor \ldots \lor (-q_n(0))\) where 0 is a constant representing the natural number 0; a global clause \((-p_1 \lor \ldots \lor (-p_m) \lor (\forall \, ((-p_1(x) \lor \ldots \lor (-p_m(x)) \lor (-q_1(s(x)) \lor \ldots \lor (-q_n(s(x))\)\)) where s is representing the successor function on the natural numbers), initial and step resolution exactly correspond to usual first-order ordered resolution with respect to an atom ordering \(<\) where \(p(x) < q(s(x))\) for arbitrary predicate symbols \(p, q,\) and \(r\). The eventuality resolution rule (whose application is the computational most costly part of the method) can be implemented by a search algorithm which is again based on step resolution [5]. Hence, performance of the step resolution inference engine is critical for the system.

Using the arithmetic translation, any state-of-the-art first-order resolution system could perform step resolution (and initial resolution). However, our formulæ have a very restrictive nature, and TRP++ uses its own “near propositional” approach to deal with them.

Data representation. We represent SNF clauses as propositional clauses and supply each literal with an “attribute”—one of initial, global_now, and global_next with obvious meaning (eventuality clauses are kept and processed separately). In addition, we define a total ordering \(<\) on attributed literals which satisfies the constraint that for every initial literal \(K\), global_now literal \(L\), and global_next literal \(M\) we have \(K < L < M\).

The ordering is then used to restrict resolution inference steps to the maximal literals in a clause. This ensures, for example, that in a clause \(C \lor L\) where \(L\) is a global_now literal but \(C\) contains some global_next literals, a resolution step on \(L\) is impossible. (Note that this behaviour is in accordance with the inference rules of the temporal resolution calculus.)

To simulate the effect of first-order unification on the arithmetical translation of SNF clauses, unification of literals in our “near propositional” representation has to take their attributes into account. For example, it is impossible to unify an initial literal with a global_next literal (since it is impossible to unify a literal \((-p(0))\) with \((-p(s(x)))\)). However, it is possible to unify a global_now literal with a global_next literal and the unifying substitution will turn the global_now literal into a global_next literal (since it is possible to unify a literal \((-p(y))\) with \((-p(s(x)))\)). In this case we will also have to turn all other global_now literals in the clause in which the global_now literal occurs into global_next literals.

Saturation by step resolution. We implement an OTTER-like saturation method where the set of all clauses is split into an active and a passive clause set, and all inferences are performed between a clause, selected from the passive clause set, and the active clause set (for a detailed description see e.g. [22]). Generated clauses are simplified by subsumption and forward subsumption resolution. As on a typical run of the saturation method, the given set of clauses is satisfiable (since the set of clauses are originating from the search algorithm needed for the eventuality resolution rule), we do not employ any special clause selection and clause preference technique. Instead, passive clauses are grouped according to their maximal literal.

Indexing. In order to speed-up resolution, we group active clauses according to their maximal literal. For (multi-literal) subsumption, we employ a trie-like data structure of the same kind that is used for string matching with wild-card characters [2]. For the current implementation, the subsumption algorithm does not distinguish literals with different attributes, thus providing us only with an imperfect filter whose result is re-checked afterwards.
Resolution engine performance. To evaluate the performance of the step resolution inference engine of TRP++, we compare TRP++ with theorem provers based on first-order resolution, SPASS 2.0\textsuperscript{1} and two versions of Vampire\textsuperscript{2}, namely Vampire 2.0-CASC (Vampire 2.0 for short) and Vampire 5.0. Vampire 5.0 has been the winner of CASC-18 in the MIX and FOF divisions.

For the first comparison, we have taken from the SATLIB benchmark problem library\textsuperscript{3} sets of both satisfiable (uf20-91, uf50-218, and flat30-60, each consisting of 100 problems) and unsatisfiable (uuf50-218, consisting of 100 problems) randomly generated propositional formulae in CNF form and five instances of the Pigeon-Hole principle, hole6–hole10. The tests have been performed on a PC with a 1.3GHz AMD Athlon processor, 512MB main memory, and 1GB virtual memory running RedHat Linux 7.1. For each individual satisfiability test a time-limit of 10000 CPU seconds was used. All theorem provers were used in ‘auto mode’, except for SPASS 2.0 where we have disabled splitting. Otherwise, SPASS 2.0 behaves quite well with Vampire 2.0 and SPASS 2.0 on the whole (as indicated by the total CPU time). One of the reasons why the other provers beat TRP++ on uf50-218 is because of their clause selection and preference techniques that speed up proof search. However, it is surprising that TRP++ performs much better than the other provers on hole8–hole10.

For the second comparison, we again used the problems in uf20-91, flat30-60, uf50-218, and uuf50-218, but this time considering the clauses as global clauses. To be able to use the first-order theorem provers on these problems, we apply the arithmetic translation to them. These test should provide some indication whether the particular data representation we have used for SNF clauses has an advantage over a ‘first-order’ representation. Two additional tests were conducted on the arithmetic translation of two problems, ring3 and ring5, describing the behaviour of an algorithm\textsuperscript{11} that orients rings with 3 and 5 nodes, respectively. In order to describe the non-deterministic behaviour of the original algorithm.

---

\textsuperscript{1}http://spass.mpi-sb.mpg.de/
\textsuperscript{2}http://www.math.miami.edu/~tptp/CASC/
\textsuperscript{3}http://www.intellektik.informatik.tu-darmstadt.de/SATLIB/benchm.html
in PLTL, eventuality clauses would be needed to which the arithmetic translation described above cannot be applied. Therefore, we used a reformulation of the two problems which avoids these clauses. Both problems are quite large (ring3 contains 24 variables and 268 clauses, ring5 contains 40 variables and 449 clauses) and unsatisfiable. Figure 3 summarises the results of this second comparison. As we can see, the performance of TRP++ is not affected by the change from propositional to global clauses while for all other theorem provers we see a negative impact. On ring3 and ring5, TRP++ can compete with Vampire 5.0 while the other provers fail.

Overall the results of these experiments indicate that there is still room for improvements of our implementation.

5 Comparison with other temporal provers

For comparison with other PLTL decision procedures, we selected the following systems: TRP++, a tableau-based procedure developed by McGuire et al. [17] which is incorporated in STeP, two tableau-based procedures included in the Logic Workbench 1.1, one developed by Janssen [16], the other by Schwendimann [23], and TRP, our previous prototype implementation of temporal resolution in SICStus Prolog.

We have compared the systems on two classes of randomly generated PLTL-formulae introduced in [15]. These classes, called $C_{ran}^1$ and $C_{ran}^2$, are intended to show the relative strength and weaknesses of tableau-based and resolution-based decision procedures for PLTL. In particular, we expect that resolution-based decision procedures like TRP and TRP++ can outperform tableau-based procedures on $C_{ran}^1$, while the opposite is true for $C_{ran}^2$ (more precisely, for satisfiable PLTL-formulae in $C_{ran}^2$). In general, formulae in these classes are characterised by three parameters, $n$, $k$, and $p$, where $n$ determines the number of propositional variables in a formula, $k$ the number of disjunctions in a SNF clause and $p$ the probability with which an atom occurs positively in a SNF clause. Here we will focus on just one choice of these parameters, namely $n = 12$, $k = 3$, and $p = 0.5$. For a detailed description we refer the reader to [15].

Again, the tests have been performed on a PC with a 1.3GHz AMD Athlon processor, 512MB main memory, and 1GB virtual memory running RedHat Linux 7.1. For each individual satisfiability test of a set of SNF clauses a time-limit of 1000 CPU seconds was used.

The left-hand side of Figure 4 depicts behaviour of the systems on $C_{ran}^1$. On the x-axis of all graphs we see the ratio of randomly generated SNF clauses in these sets versus the parameter $n$, ranging from 0 to 8. For each ratio, 100 sets of SNF clauses have been generated and tested. The upper part of the figure shows the percentage of satisfiable sets of SNF clauses for a given ratio. The vertical line divides the figure at the point where the number of satisfiable sets of SNF clauses equals the number of unsatisfiable ones. The right-hand side of the figure gives the same information for $C_{ran}^2$. For each ratio we have measured the CPU time each system has required to solve each of the 100 SNF clause sets for that ratio and computed the median. The middle part of the figure shows the resulting graphs for the median CPU time consumption of each of the systems, while the lower part of the figure shows the graphs for the maximal CPU time consumption. Note that in all performance graphs, a point for a system above the 1000 CPU seconds mark indicates that the median or maximal CPU time required by the system has exceeded our time-limit.

Important points to note are that TRP and TRP++ perform as expected compared to the other systems and that TRP++ performs considerably better than TRP on both classes indicating that the improved data representation and indexing techniques used in TRP++ compared to TRP have paid off. An interesting observation is that on $C_{ran}^2$ the behaviour of TRP and TRP++ differs in a way that is not easily explained by these improvements alone. While the median CPU time consumption of TRP on $C_{ran}^2$ grows steadily as the number of clauses in the clause sets under consideration increases, the median CPU time consumption of TRP++ remains constants and shows even a significant drop at the point where the majority of clauses turns unsatisfiable. Figure 5 depicts graphs showing the median number of clauses derived by TRP and TRP++ on $C_{ran}^1$ (left-hand side) and $C_{ran}^2$ (right-hand side). We see a good correlation to the median CPU time consumption of the two systems. We can also see that on $C_{ran}^1$, both system derive roughly the same number of clauses. Thus, the difference in performance of both systems on $C_{ran}^1$ is mainly due to the implementational improvements discussed before. However, on $C_{ran}^2$ we see that the differing behaviour of TRP versus TRP++ is reflected in differing numbers of derived clauses. It turns out that this is due to different orderings used by two systems. TRP uses an ordering based on the lexicographical ordering on the names of propositional variables while TRP++,
Fig. 4. Performance of the systems on $C_{ran}^1$ and $C_{ran}^2$

by default, uses an ordering based on the order in which the propositional variables occur in the clause set. On unsatisfiable formulae, which dominate the behaviour for ratios greater than 4.875 on $C_{ran}^2$, a smaller number of applications of the eventuality rule occurs than on satisfiable formulae. Applications of this rule again lead to step resolution inferences, the number of which will depend on the ordering used. For TRP++, this reduction in the number of applications of the eventuality rule leads to an observable reduction in the number of derived clauses, while for TRP the effect is not sufficiently significant.

References

Fig. 5. Number of derived clauses for TRP and TRP++ on $C_{ran}^1$ and $C_{ran}^2$.