Deciding monodic fragments by temporal resolution*

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Abstract. In this paper we study the decidability of various fragments of monodic first-order temporal logic by temporal resolution. We focus on two resolution calculi, namely, monodic temporal resolution and fine-grained temporal resolution. For the first, we state a very general decidability result, which is independent of the particular decision procedure used to decide the first-order part of the logic. For the second, we introduce refinements using orderings and selection functions. This allows us to transfer existing results on decidability by resolution for first-order fragments to monodic first-order temporal logic and obtain new decision procedures. The latter is of immediate practical value, due to the availability of **TeMP**, an implementation of fine-grained temporal resolution.

1 Introduction

Temporal logics have long been recognised as introducing appropriate languages for specifying a wide range of important computational properties in computer science and artificial intelligence [6]. However, until recently, the practical use of temporal logics has largely been restricted to propositional temporal logics. First-order temporal logic has generally been avoided as no complete proof system can exist for this logic. However, recent work by Hodkinson, Wolter, and Zakharyaschev [11] shows that a specific fragment of first-order temporal logic, called the *monodic* fragment, or *monodic* first-order temporal logic, has the completeness property. This initial result was followed by an examination of the monodic fragment in terms of decidable subclasses, automated deduction, and applications.

Hodkinson et al. [10, 11, 23] show the decidability of the monadic, two-variable, fluted, and loosely guarded fragments of monodic first-order temporal logic without equality as well as the decidability of the monodic packed fragment of first-order temporal logic with equality. Kontchakov et al. [17] have developed a framework for devising tableau decision procedures for such decidable monodic first-order temporal logics. Using this framework they present tableau decision

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procedures for the one-variable fragment and for the fragment corresponding to the modal logic $\mathsf{S4}_u$ of monodic first-order temporal logic.

In parallel, Degtyarev, Dixon, Fisher, Hustadt, and Konev have investigated monodic first-order temporal logic in the context of resolution. Degtyarev et al. [5] present a temporal resolution calculus, called *monodic temporal resolution*, for this logic, which is then used in [4] to establish a general decidability result for temporal resolution. Decidability of all the classes from Hodkinson et al., as well as, the Gödel class and the dual Maslov class $\overline{\mathrm{K}}$ fragments of monodic first-order temporal logic are shown to be immediate consequences of this general result. Konev et al. [14, 15] devise the *fine-grained resolution calculus* as an alternative resolution calculus for monodic first-order temporal logic which is more amenable to mechanisation. This calculus forms the basis of the temporal monodic theorem prover **TeMP** presented in [12].

In this paper we focus on decidability results in the context of the fine-grained resolution calculus. To motivate why the general decidability result obtained in the context of monodic temporal resolution does not easily carry over to the fine-grained resolution calculus, we first provide a brief presentation of both calculi and some basic results about them, including the mentioned decidability result. A main contribution of this paper is the introduction of refinements of fined-grained resolution which incorporate advanced techniques developed in the context of first-order resolution (e.g. [1, 19]) into temporal resolution, namely orderings and selection functions. We prove completeness of this refined calculus by simulating derivations in the monodic temporal resolution calculus. The refined calculus, called *ordered fine-grained resolution with selection*, allows us to transfer decidability results and decision procedures obtained for fragments of first-order logic to the corresponding fragments of monodic first-order temporal logic.

2 First-order temporal logic

The language of First-Order Temporal Logic, FOTL, is an extension of classical first-order logic by temporal operators for a discrete linear model of time (isomorphic to $\mathbb{N}$, that is, the most commonly used model of time). The signature of FOTL (without equality and function symbols) consists of a countably infinite set of *variables* x_0, x_1, $\ldots$, a countably infinite set of *constants* c_0, c_1, $\ldots$, a non-empty set of *predicate symbols* P, P_0, $\ldots$, each with a fixed arity ≥ 0, the *propositional operators* $\top$, $\neg$, $\vee$, the *quantifiers* $\exists x_i$ and $\forall x_i$, and the *temporal operators* $\Box$ ('always in the future'), $\Diamond$ ('eventually in the future'), $\bigcirc$ ('at the next moment'), and U ('until'). The set of formulae of FOTL is defined as follows: $\top$ is a FOTL formula; if P is an n-ary predicate symbol and t_1, $\ldots$, t_n are variables or constants, then $P(t_1, \ldots, t_n)$ is an *atomic* FOTL formula; if φ and ψ are FOTL formulae, then so are $\neg\varphi$, $\varphi \vee \psi$, $\exists x\varphi$, $\forall x\varphi$, $\Box\varphi$, $\Diamond\varphi$, $\bigcirc\varphi$, and $\varphi \,\mathsf{U}\, \psi$. We also use $\bot$, $\wedge$, and $\Rightarrow$ as additional operators, defined using $\top$, $\neg$, and $\vee$. Free and bound variables of a formula are defined in the standard way, as well as the notions of open and closed formulae. Given a formula φ, we write

$\varphi(x_1, \ldots, x_n)$ to indicate that all the free variables of φ are among $x_1, \ldots, x_n$. As usual, a *literal* is either an atomic formula or its negation.

Formulae of this logic are interpreted over structures $\mathfrak{M} = (D_n, I_n)_{n \in \mathbb{N}}$ that associate with each element n of $\mathbb{N}$, representing a moment in time, a first-order structure $\mathfrak{M}_n = (D_n, I_n)$ with its own non-empty domain D_n and interpretation I_n. An *assignment* $\mathfrak{a}$ is a function from the set of variables to $\bigcup_{n \in \mathbb{N}} D_n$. The application of an assignment to terms is defined in the standard way, in particular, $\mathfrak{a}(c) = c$ for every constant c. The *truth relation* $\mathfrak{M}_n \models^{\mathfrak{a}} \varphi$ is defined (only for those a such that $\mathfrak{a}(x) \in D_n$ for every variable x) as follows:

$$
\begin{array}{ll}
\mathfrak{M}_n \models^{\mathfrak{a}} \top & \\
\mathfrak{M}_n \models^{\mathfrak{a}} P(t_1, \ldots, t_n) & \text{iff } (I_n(\mathfrak{a}(t_1)), \ldots, I_n(\mathfrak{a}(t_n))) \in I_n(P) \\
\mathfrak{M}_n \models^{\mathfrak{a}} \neg\varphi & \text{iff not } \mathfrak{M}_n \models^{\mathfrak{a}} \varphi \\
\mathfrak{M}_n \models^{\mathfrak{a}} \varphi \vee \psi & \text{iff } \mathfrak{M}_n \models^{\mathfrak{a}} \varphi \text{ or } \mathfrak{M}_n \models^{\mathfrak{a}} \psi \\
\mathfrak{M}_n \models^{\mathfrak{a}} \exists x \varphi & \text{iff } \mathfrak{M}_n \models^{\mathfrak{b}} \varphi \text{ for some assignment } \mathfrak{b} \text{ that may differ} \\
& \quad \text{from } \mathfrak{a} \text{ only in } x \text{ and such that } \mathfrak{b}(x) \in D_n \\
\mathfrak{M}_n \models^{\mathfrak{a}} \forall x \varphi & \text{iff } \mathfrak{M}_n \models^{\mathfrak{b}} \varphi \text{ for every assignment } \mathfrak{b} \text{ that may differ} \\
& \quad \text{from } \mathfrak{a} \text{ only in } x \text{ and such that } \mathfrak{b}(x) \in D_n \\
\mathfrak{M}_n \models^{\mathfrak{a}} \bigcirc\varphi & \text{iff } \mathfrak{M}_{n+1} \models^{\mathfrak{a}} \varphi \\
\mathfrak{M}_n \models^{\mathfrak{a}} \Diamond\varphi & \text{iff there exists } m \geq n \text{ such that } \mathfrak{M}_m \models^{\mathfrak{a}} \varphi \\
\mathfrak{M}_n \models^{\mathfrak{a}} \Box\varphi & \text{iff for all } m \geq n,\ \mathfrak{M}_m \models^{\mathfrak{a}} \varphi \\
\mathfrak{M}_n \models^{\mathfrak{a}} \varphi \,\mathsf{U}\, \psi & \text{iff there exists } m \geq n \text{ such that } \mathfrak{M}_m \models^{\mathfrak{a}} \psi \text{ and} \\
& \quad \mathfrak{M}_i \models^{\mathfrak{a}} \varphi \text{ for every } i, n \leq i < m
\end{array}
$$

In this paper we make the *expanding domain assumption*, that is, $D_n \subseteq D_m$ if $n < m$, and we assume that the interpretation of constants is *rigid*, that is, $I_n(c) = I_m(c)$ for all $n, m \in \mathbb{N}$.

The set of valid formulae of this logic is not recursively enumerable. However, the set of valid *monodic* formulae is known to be finitely axiomatisable [23]. A formula φ of FOTL is called *monodic* if any subformula of φ of the form $\bigcirc\psi$, $\Box\psi$, $\Diamond\psi$, or $\psi_1 \,\mathsf{U}\, \psi_2$ contains at most one free variable. For example, the formulae $\forall x \Box \exists y P(x, y)$ and $\forall x \Box P(x, c)$ are monodic, while $\forall x \forall y (P(x, y) \Rightarrow \Box P(x, y))$ is not monodic.

Every monodic temporal formula can be transformed into an equi-satisfiable normal form, called *divided separated normal form (DSNF)* [14].

Definition 1. *A* monodic temporal problem P in divided separated normal form (DSNF) *is a quadruple* $\langle \mathcal{U}, \mathcal{I}, \mathcal{S}, \mathcal{E} \rangle$*, where*

1. *the universal part* $\mathcal{U}$ *and the initial part* $\mathcal{I}$ *are finite sets of first-order formulae;*
2. *the step part* $\mathcal{S}$ *is a finite set of clauses of the form* $p \Rightarrow \bigcirc q$*, where* p *and* q *are propositions, and* $P(x) \Rightarrow \bigcirc Q(x)$*, where* P *and* Q *are unary predicate symbols and* x *is a variable; and*
3. *the eventuality part* $\mathcal{E}$ *is a finite set of formulae of the form* $\Diamond L(x)$ *(a* non-ground *eventuality clause) and* $\Diamond l$ *(a* ground eventuality *clause), where* l *is a propositional literal and* $L(x)$ *is a unary non-ground literal with variable* x *as its only argument.*

With each monodic temporal problem $\langle \mathcal{U}, \mathcal{I}, \mathcal{S}, \mathcal{E} \rangle$ we associate the FOTL formula $\mathcal{I} \wedge \Box\mathcal{U} \wedge \Box\forall x\mathcal{S} \wedge \Box\forall x\mathcal{E}$. When we talk about particular properties of a temporal problem (e.g., satisfiability, validity, logical consequences, etc) we refer to properties of this associated formula.

The transformation to DSNF is based on using a renaming and unwinding technique which substitutes non-atomic subformulae and replaces temporal operators by their fixed point definitions as described, for example, in [8]. A step in this transformation which is of relevance for the results presented here is the following: We recursively rename each innermost open subformula $\xi(x)$, whose main connective is a temporal operator, by $P_\xi(x)$, where $P_{\xi(x)}$ is a new unary predicate, and rename each innermost closed subformula ζ, whose main connective is a temporal operator, by p_ζ, where p_ζ is a new propositional variable. In the terminology of [11] $P_\xi(x)$ and p_ζ are called the *surrogates* of $\xi(x)$ and ζ, respectively. Renaming introduces formulae defining $P_\xi(x)$ and p_ζ of the following form (since we are only interested in satisfiability, we use implications instead of equivalences for renaming positive occurrences of subformulae, see also [20]):

$$(a)\ \Box\forall x(P_\xi(x) \Rightarrow \xi(x)) \qquad \text{and} \qquad (b)\ \Box(p_\zeta \Rightarrow \zeta).$$

If the main connective of $\xi(x)$ or ζ is either $\Box$ or U, then the formula will be replaced by its fixed point definition. If the main connective of $\xi(x)$ or ζ is either the $\bigcirc$ or $\Diamond$ operator, the defining formula will be simplified further to obtain step or eventuality clauses.

Theorem 1 (see [4], Theorem 1). *Any monodic first-order temporal formula can be transformed into an equi-satisfiable monodic temporal problem in DSNF with at most a linear increase in the size of the problem.*

In the next section we briefly recall the temporal resolution calculus first developed in [5] and we present a general decidability result for this calculus.

3 Monodic temporal resolution

The monodic temporal resolution calculus does not directly operate on the formulae and clauses of a monodic temporal problem P, but, as described next, operates on merged derived step clauses and full merged step clauses computed from the constant flooded form of P. Let $\mathsf{P} = \langle \mathcal{U}, \mathcal{I}, \mathcal{S}, \mathcal{E} \rangle$ be a monodic temporal problem, then the temporal problem $\mathsf{P}^c = \langle \mathcal{U}, \mathcal{I}, \mathcal{S}, \mathcal{E}^c \rangle$ where $\mathcal{E}^c = \mathcal{E} \cup \{\Diamond L(c) \mid \Diamond L(x) \in \mathcal{E}, c \text{ is a constant in } \mathsf{P}\}$ is the *constant flooded form* of P. (Strictly speaking, P^c is not in DSNF: We have to rename ground eventualities by propositions.) Evidently, P^c is satisfiability equivalent to P. Let

$$P_{i_1}(x) \Rightarrow \bigcirc M_{i_1}(x), \ldots, P_{i_k}(x) \Rightarrow \bigcirc M_{i_k}(x) \tag{1}$$

be a subset of the set of step clauses of P^c. Then formulae of the form

$$P_{i_j}(c) \Rightarrow \bigcirc M_{i_j}(c) \qquad \text{and} \qquad \exists x \bigwedge_{j=1}^{k} P_{i_j}(x) \Rightarrow \bigcirc \exists x \bigwedge_{j=1}^{k} M_{i_j}(x), \tag{2}$$

where c is a constant in P^c and $j = 1, \ldots, k$, are called *derived* step clauses.[3] Note that formulae of the form (2) are logical consequences of (1). Let $\{\Phi_1 \Rightarrow$

[3] In [4] derived step clauses, are termed e-derived step clauses.

$\bigcirc\Psi_1, \ldots, \Phi_n \Rightarrow \bigcirc\Psi_n\}$ be a set of derived step clauses or *ground* step clauses in P^c. Then $(\bigwedge_{i=1}^n \Phi_i) \Rightarrow \bigcirc(\bigwedge_{i=1}^n \Psi_i)$ is called a *merged derived step clause.*

Let $\mathcal{A} \Rightarrow \bigcirc\mathcal{B}$ be a merged derived step clause, let $P_1(x) \Rightarrow \bigcirc M_1(x), \ldots, P_k(x) \Rightarrow \bigcirc M_k(x)$ be a subset of the original step clauses in P^c, and let $\mathcal{A}(x) \stackrel{\text{def}}{=} \mathcal{A} \wedge \bigwedge_{i=1}^k P_i(x)$, $\mathcal{B}(x) \stackrel{\text{def}}{=} \mathcal{B} \wedge \bigwedge_{i=1}^k M_i(x)$. Then $\forall x(\mathcal{A}(x) \Rightarrow \bigcirc\mathcal{B}(x))$ is called a *full merged step clause.*

In what follows, $\mathcal{A} \Rightarrow \bigcirc\mathcal{B}$ and $\mathcal{A}_i \Rightarrow \bigcirc\mathcal{B}_i$ denote merged derived step clauses, $\forall x(\mathcal{A}(x) \Rightarrow \bigcirc\mathcal{B}(x))$ and $\forall x(\mathcal{A}_i(x) \Rightarrow \bigcirc\mathcal{B}_i(x))$ denote full merged step clauses, and $\mathcal{U}$ denotes the (current) universal part of a monodic temporal problem P. We now define the temporal resolution calculus, $\mathfrak{I}_e$, for the expanding domain case. The inference rules of $\mathfrak{I}_e$ are the following.

– *Step resolution rule w.r.t.* $\mathcal{U}$:

$$\frac{\mathcal{A} \Rightarrow \bigcirc\mathcal{B}}{\neg\mathcal{A}}\ (\bigcirc^{\mathcal{U}}_{res}), \quad \text{if } \mathcal{U} \cup \{\mathcal{B}\} \models \bot.$$

– *Termination rule w.r.t.* $\mathcal{U}$ *and* $\mathcal{I}$*:*

$$\frac{}{\bot}\ (\bot^{\mathcal{U}}_{res}), \quad \text{if } \mathcal{U} \cup \mathcal{I} \models \bot.$$

– *Eventuality resolution rule w.r.t.* $\mathcal{U}$*:*

$$\frac{\forall x(\mathcal{A}_1(x) \Rightarrow \bigcirc\mathcal{B}_1(x)) \quad \ldots \quad \forall x(\mathcal{A}_n(x) \Rightarrow \bigcirc\mathcal{B}_n(x)) \qquad \Diamond L(x)}{\forall x \bigwedge_{i=1}^n \neg\mathcal{A}_i(x)}\ (\Diamond^{\mathcal{U}}_{res}),$$

where $\forall x(\mathcal{A}_i(x) \Rightarrow \bigcirc\mathcal{B}_i(x))$ are full merged step clauses such that for every i, $1 \leq i \leq n$, the *loop* side conditions $\forall x(\mathcal{U} \wedge \mathcal{B}_i(x) \Rightarrow \neg L(x))$ and $\forall x(\mathcal{U} \wedge \mathcal{B}_i(x) \Rightarrow \bigvee_{j=1}^n(\mathcal{A}_j(x)))$ are valid.[4]
The set of full merged step clauses, satisfying the loop side conditions, is called a *loop in* $\Diamond L(x)$ and the formula $\bigvee_{j=1}^n \mathcal{A}_j(x)$ is called a *loop formula.*

– *Ground eventuality resolution rule w.r.t.* $\mathcal{U}$*:*

$$\frac{\mathcal{A}_1 \Rightarrow \bigcirc\mathcal{B}_1 \quad \ldots \quad \mathcal{A}_n \Rightarrow \bigcirc\mathcal{B}_n \qquad \Diamond l}{\bigwedge_{i=1}^n \neg\mathcal{A}_i}\ (\Diamond^{\mathcal{U}}_{res}),$$

where $\mathcal{A}_i \Rightarrow \bigcirc\mathcal{B}_i$ are merged derived step clauses such that for every i, $1 \leq i \leq n$, the *loop* side conditions $\mathcal{U} \wedge \mathcal{B}_i \models \neg l$ and $\mathcal{U} \wedge \mathcal{B}_i \models \bigvee_{j=1}^n \mathcal{A}_j$ are valid. The notions of *ground loop* and *ground loop formula* are defined similarly to the case above.

Let P be a temporal problem. By TRes(P) we denote the set of all possible conclusions of the inference rules above applied to P^c.

Definition 2 (Derivation). *Let* $\mathsf{P} = \langle \mathcal{U}, \mathcal{I}, \mathcal{S}, \mathcal{E} \rangle$ *be a monodic temporal problem. A* derivation *from* P *is a sequence of universal parts,* $\mathcal{U} = \mathcal{U}_0 \subset \mathcal{U}_1 \subset \mathcal{U}_2 \subset \cdots$*, such that* $\mathcal{U}_{i+1}$ *is obtained from* $\mathcal{U}_i$ *by applying an inference rule to* $\langle \mathcal{U}_i, \mathcal{I}, \mathcal{S}, \mathcal{E}^c \rangle$ *and adding its conclusion to* $\mathcal{U}_i$*. The* $\mathcal{I}$*,* $\mathcal{S}$ *and* $\mathcal{E}^c$ *parts of the temporal problem are not changed during a derivation.*

[4] In the case $\mathcal{U} \models \forall x \neg L(x)$, the *degenerate clause*, $\top \Rightarrow \bigcirc\top$, can be considered as a premise of this rule; the conclusion of the rule is then $\neg\top$ and the derivation successfully terminates.

A derivation *terminates* if, and only if, either a contradiction is derived, in which case we say that the derivation *terminates successfully*, or if no new formulae can be derived by further inference steps. Any derivation can be continued yielding a terminating derivation. Note that since there exist only finitely many different full merged step clauses, the number of different conclusions of the inference rules of monodic temporal resolution is finite. Therefore, every derivation is finite.

A derivation $\mathcal{U} = \mathcal{U}_0 \subset \mathcal{U}_1 \subset \mathcal{U}_2 \subset \cdots \subset \mathcal{U}_n$ from $\langle \mathcal{U}, \mathcal{I}, \mathcal{S}, \mathcal{E} \rangle$ is called *fair* (we adopt terminology from [1]) if for any $i \geq 0$ and formula $\varphi \in \mathrm{TRes}(\langle \mathcal{U}_i, \mathcal{I}, \mathcal{S}, \mathcal{E}^c \rangle)$, there exists $j \geq i$ such that $\varphi \in \mathcal{U}_j$.

It is important to note that all the inference rules have side conditions which are first-order problems. For example, consider a temporal problem $\mathsf{P} = \langle \mathcal{U}, \mathcal{I}, \mathcal{S}, \mathcal{E} \rangle$, where only $\mathcal{I}$ is non-empty, that is, P is simply a first-order problem. Then the only inference rule applicable is the termination rule. If the rule can be applied, then a single application of the rule would derive a contradiction indicating that P is unsatisfiable. If the rule cannot be applied, because $\mathcal{I}$ is not contradictory, then the derivation terminates without a single inference step being performed, indicating that P is satisfiable. This also illustrates why all derivations can be finite although the satisfiability problem of monodic FOTL is only semi-decidable.

So, in general, the side conditions of our inference rules are only semi-decidable and in the case a side condition is false, it may happen that the test of this side condition does not terminate. To ensure fairness we must make sure that each such test cannot indefinitely block the investigation of alternative applications of inference rules in a derivation.

Theorem 2 (see [4, Theorem 10]). *The rules of $\Im_e$ preserve satisfiability over expanding domains. A monodic temporal problem P is unsatisfiable over expanding domains iff any* fair *derivation in $\Im_e$ from P^c terminates successfully.*

4 Decidability by monodic temporal resolution

Monodic temporal resolution provides a decision procedure for a class of monodic FOTL formulae provided that there exists a first-order decision procedure for the side conditions of all inference rules. Examination of the side conditions shows that we are interested in the satisfiability of (i) the conjunction of the (current) universal part and the initial part, and (ii) the conjunction of the (current) universal part and sets of monadic formulae built from predicate symbols which occur in the step and eventuality part of a temporal problem. At the same time, in each step of the derivation the universal part is extended by monadic formulae from the conclusion of the inference rule applied in the inference step. So, after imposing restrictions on the form of the universal and initial parts of a class of temporal problems, we can guarantee decidability of this class.

However, formalising which fragments of monodic FOTL are decidable by monodic temporal resolution is slightly more complex, since we have to take our "rename and unwind" transformation to divided separated normal form into account, as the following example illustrates.

Example 1. Let $\varphi(x, y, z, u)$ be the first-order formula $Q_1(x, y, z) \vee Q_2(y, z) \vee Q(x, y, z, u)$. Then the formula $\exists x \forall y \forall z \exists u \varphi(x, y, z, u)$ belongs to the dual of Maslov's class K which is decidable. In contrast, consider the temporal formula $\exists x \Box \Diamond \forall y \forall z \exists u \varphi(x, y, z, u)$ with the same φ. Once transformed to an equisatisfiable temporal problem $\mathsf{P} = \langle \mathcal{U}, \mathcal{I}, \mathcal{S}, \mathcal{E} \rangle$ in DSNF, the universal part $\mathcal{U}$ contains the formula $\forall x(P_\varphi(x) \Rightarrow \forall y \forall z \exists u \varphi(x, y, z, u))$ which does not belong to Maslov's class K. (It belongs to the undecidable Surányi class $\forall^3\exists$ [2].)

To solve this problem, we define decidable fragments in terms of surrogates.

Definition 3 (Temporalisation by Renaming). *Let $\mathfrak{C}$ be a class of first-order formulae. Let φ be a monodic temporal formula in negation normal form (that is, the only Boolean connectives are conjunction, disjunction and negation, and negations are only applied to atoms). Let $\overline{\varphi}$ denote the formula that results from φ by replacing all of its subformulae whose main connective is a temporal operator and which is not within a scope of another temporal operator with their surrogates.*

We say that φ belongs to the class $\mathcal{T}_{ren}\mathfrak{C}$ if

1. *$\overline{\varphi}$ belongs to $\mathfrak{C}$ and*
2. *for every subformula of the form $\mathcal{T}\psi$, where $\mathcal{T}$ is a temporal operator (or of the form $\psi_1 \mathcal{T} \psi_2$ if $\mathcal{T}$ is binary), either $\overline{\psi}$ is a closed formula belonging to $\mathfrak{C}$ or the formula $\forall x(P(x) \Rightarrow \overline{\psi})$, where P is a new unary predicate symbol, belongs to $\mathfrak{C}$ (analogous conditions for ψ_1, ψ_2).*

Note that the formulae indicated in the first and second items of the definition exactly match the shape of the formulae contributing to $\mathcal{U}$ when we reduce a temporal formula to the normal form by renaming the complex expressions and replacing temporal operators by their fixed point definitions.

Theorem 3 (Decidability by Temporal Resolution). *Let $\mathfrak{C}$ be a decidable class of first-order formulae which does not contain equality and functional symbols, but possibly contains constants, such that*

- *$\mathfrak{C}$ is closed under conjunction;*
- *$\mathfrak{C}$ contains universal monadic formulae.*

Then $\mathcal{T}_{ren}\mathfrak{C}$ is decidable.

Proof [See also [4, Theorem 8.3]] After reduction to DSNF, all formulae from $\mathcal{U}$ belong to $\mathfrak{C}$. The (monadic) formulae from side conditions and the (monadic) formulae generated by temporal resolution rules belong to $\mathfrak{C}$. Therefore, testing the applicability of one of the temporal resolution rules becomes decidable. Given that all derivations are finite, due to the finiteness of the set of merged derived step clauses and full merged step clauses, decidability follows. ❑

A consequence of Theorem 3 is the decidability of a wide range of temporal monodic classes. These include the monadic, two-variable, fluted, guarded, and loosely guarded fragments of monodic first-order temporal logic which have also been shown to be decidable in [11, 23]. In addition, decidability also follows for other classes, for example, the class $\mathcal{T}_{ren}\exists^*\forall^2\exists^*$ and the class $\mathcal{T}_{ren}\overline{\mathrm{K}}$ where $\overline{\mathrm{K}}$ is the dual of Maslov's class K [18]. Moreover, combining the constructions from [10] and the saturation-based decision procedure for the guarded fragment with equality [9], it is possible to build a temporal resolution decision procedure for the monodic guarded and loosely guarded fragments with equality [16].

5 Monodic fine-grained temporal resolution

The main drawback of monodic temporal resolution is that the notion of merged derived step clauses and full merged step clauses is quite involved and that the search for merged step clauses to which one of the deduction rules can successfully be applied is computationally hard, in general it is only semi-decidable. The idea underlying the *monodic fine-grained temporal resolution calculus*, fine-grained resolution for short, is to refine the deduction rules of $\mathfrak{I}_e$ in such a way that they perform much smaller steps, but with decidable side conditions for their applicability. Of course, the price that has to be paid is that derivations are no longer guaranteed to be finite.

In more detail, fine-grained resolution differs from the calculus $\mathfrak{I}_e$ in two aspects. First, instead of the step resolution and the termination rule of $\mathfrak{I}_e$, we use a set of deduction rules operating on clausified problems. Second, we use a particular algorithm, called FG-BFS, to determine the loops to which we apply the ground and non-ground eventuality resolution rule of $\mathfrak{I}_e$.

Definition 4. *Let* $\mathsf{P} = \langle \mathcal{U}, \mathcal{I}, \mathcal{S}, \mathcal{E} \rangle$ *be a monodic temporal problem. The* clausification $\mathrm{Cls}(\mathsf{P})$ of P *is a quadruple* $\langle \mathcal{U}', \mathcal{I}', \mathcal{S}', \mathcal{E} \rangle$ *such that (i)* $\mathcal{U}'$ *is a set of clauses, called* universal clauses*, obtained by clausification of* $\mathcal{U}$*; (ii)* $\mathcal{I}'$ *is a set of clauses, called* initial clauses*, obtained by clausification of* $\mathcal{I}$*; (iii)* $\mathcal{S}'$ *is the smallest set of step clauses such that all step clauses from* $\mathcal{S}$ *are in* $\mathcal{S}'$ *and for every non-ground step clause* $P(x) \Rightarrow \bigcirc L(x)$ *in* $\mathcal{S}$ *and every constant* c *occurring* P*, the clause* $P(c) \Rightarrow \bigcirc L(c)$ *is in* $\mathcal{S}'$*.*

Example 2. Let $\mathsf{P} = \langle \mathcal{U}, \mathcal{I}, \mathcal{S}, \mathcal{E} \rangle$ where $\mathcal{U} = \{\exists x Q(x)\}$, $\mathcal{I} = \{P(c)\}$, $\mathcal{S} = \{P(x) \Rightarrow \bigcirc Q(x)\}$, and $\mathcal{E} = \emptyset$. Then $\mathrm{Cls}(\mathsf{P}) = \langle \mathcal{U}', \mathcal{I}', \mathcal{S}', \mathcal{E} \rangle$ where $\mathcal{U}' = \{Q(d)\}$ with d a Skolem constant, $\mathcal{I}' = \{P(c)\}$, and $\mathcal{S}' = \{P(x) \Rightarrow \bigcirc Q(x), P(c) \Rightarrow \bigcirc Q(c)\}$.

During a derivation more general *step* clauses can be derived, which are of the form $C \Rightarrow \bigcirc D$, where C is a *conjunction* of propositions, atoms of the form $P(x)$ and ground formulae of the form $P(c)$, where P is a unary predicate symbol and c is a constant such that c occurs in the input formula, and D is a *disjunction* of arbitrary literals.

Let us first define the deduction rules of fine-grained step resolution which replace the step resolution and the termination rule of $\mathfrak{I}_e$. In the following, we assume that different premises and conclusions of the deduction rules have no variables in common; variables may be renamed if necessary.

(1) *First-order resolution between two universal clauses.* Defined as standard first-order resolution between two clauses. The result is a universal clause.
(2) *First-order factoring on a universal clause.* Again, defined as standard first-order factoring on a clause. The result is a universal clause.
(3) *First-order resolution between an initial and a universal clause, between two initial clauses, and factoring on an initial clause.* Defined in analogy to the two deduction rules above only that the result is an initial clause.
(4) *Fine-grained step resolution.*

$$\frac{C_1 \Rightarrow \bigcirc(D_1 \vee L) \quad C_2 \Rightarrow \bigcirc(D_2 \vee \neg M)}{(C_1 \wedge C_2)\sigma \Rightarrow \bigcirc(D_1 \vee D_2)\sigma} \ ,$$

where $C_1 \Rightarrow \bigcirc(D_1 \vee L)$ and $C_2 \Rightarrow \bigcirc(D_2 \vee \neg M)$ are step clauses and σ is a most general unifier of the literals L and M *such that σ does not map variables from C_1 or C_2 into a constant or a functional term.*[5]

$$\frac{C_1 \Rightarrow \bigcirc(D_1 \vee L) \quad D_2 \vee \neg M}{C_1\sigma \Rightarrow \bigcirc(D_1 \vee D_2)\sigma} \ ,$$

where $C_1 \Rightarrow \bigcirc(D_1 \vee L)$ is a step clause, $D_2 \vee \neg M$ is a universal clause, and σ is a most general unifier of the literals L and M *such that σ does not map variables from C_1 into a constant or a functional term.*
(5) *Fine-grained step factoring.*

$$\frac{C \Rightarrow \bigcirc(D \vee L \vee M)}{C\sigma \Rightarrow \bigcirc(D \vee L)\sigma} \ ,$$

where σ is a most general unifier of the literals L and M *such that σ does not map variables from C into a constant or a functional term.*
(6) *Clause conversion.* A step clause of the form $C \Rightarrow \bigcirc\bot$ is rewritten to the *universal clause*[6] $\neg C$.

Besides the rules above we still need the eventuality resolution rule and the ground eventuality resolution rule of $\mathfrak{I}_e$. However, we use a particular algorithm, called FG-BFS (for fine-grained breadth-first search), to find loop formulae, that is, to find a disjunction of the left-hand sides of full merged step clauses that together with an eventuality literal forms the premises for the ground and non-ground eventuality resolution rules. This algorithm internally uses the deduction rules above with the exception of the clause conversion rule.

Let *fine-grained resolution* be the calculus consisting of the rules (1) to (6) above, together with the ground and non-ground eventuality resolution rules, restricted to loops found by the FG-BFS algorithm. We denote this calculus by $\mathfrak{I}_{FG}$. The calculus can be extended by first-order redundancy elimination rules, e.g. tautology and subsumption deletion, as well as analogous rules for step clauses.

[5] This restriction justifies skolemisation of the universal part: Skolem constants from one moment of time do not propagate to the previous moment.

[6] Here, and further, $\neg(L_1(x) \wedge \cdots \wedge L_k(x))$ abbreviates $(\neg L_1(x) \vee \cdots \vee \neg L_k(x))$.

A (linear) *derivation* in $\mathfrak{I}_{FG}$ from the clausification $\mathrm{Cls}(\mathsf{P}^c)$ of a constant flooded monodic temporal problem P^c is a sequence of clauses $C_1, \ldots$ such that each clause C_i is either an element of $\mathrm{Cls}(\mathsf{P}^c)$ or else the conclusion by a deduction rule applied to clauses from premises $C_1, \ldots, C_{i-1}$. A derivation $C_1, \ldots, C_m$ is also called a *proof of* C_m. A proof of the empty clause is called a *refutation*. A derivation $C_1, \ldots, C_m$ *terminates* iff for any derivation $C_1, \ldots, C_m, C_{m+1}$, the clause C_{m+1} is a variant of a clause in $\mathrm{Cls}(\mathsf{P}^c) \cup \{C_1, \ldots, C_m\}$.

Theorem 4 ([15] Theorems 5 and 9). *Fine-grained resolution is sound and complete for constant flooded monodic temporal problems over expanding domains.*

For the class of problems where all the literals in a problem are propositional or ground, fine-grained resolution is a decision procedure, as the inference steps performed by it are exactly those performed by the clausal temporal calculus [8] for propositional linear-time temporal logic, which is an exponential time decision procedure for the satisfiability problem of that logic. However, for all the classes mentioned at the end of Section 4, termination of fine-grained resolution cannot be guaranteed. So, in analogy to the approach taken to obtain resolution decision procedure for decidable fragments of first-order logic, we develop sound and complete *refinements* of fine-grained resolution to ensure termination of derivations. We assume that we are given an *atom ordering* $\succ$, that is, a total and well-founded ordering on ground first-order atoms which is stable under substitution, and a *selection function* S which maps any first-order clause C to a (possibly empty) subset of its negative literals. An atom ordering $\succ$ is extended to literals by $(\neg)A \succ (\neg)B$ if $A \succ B$ and $\neg A \succ A$. A literal L is called (strictly) maximal w.r.t. a clause C iff there exists a ground substitution σ such that for all $L' \in C$: $L\sigma \succeq L'\sigma$ ($L\sigma \succ L'\sigma$). A literal L is *eligible* in a clause $L \vee C$ if either it is selected in $L \vee C$, or no literal is selected in C and L is maximal w.r.t. C.

The atom ordering $\succ$ and the selection function S are used to restrict the applicability of the deduction rules of fine-grained resolution as follows.

(1) *First-order ordered resolution with selection between two universal clauses*

$$\frac{C_1 \vee A \quad \neg B \vee C_2}{(C_1 \vee C_2)\sigma} \ ,$$

if σ is the most general unifier of A and B, $A\sigma$ *is eligible in* $(C_1 \vee A)\sigma$, *and* $\neg B\sigma$ *is eligible in* $(\neg B \vee C_2)\sigma$.

(2) *First-order ordered positive factoring with selection*

$$\frac{C_1 \vee A \vee B}{(C_1 \vee A)\sigma} \ ,$$

if σ is the most general unifier of A and B, and $A\sigma$ *is eligible in* $(C_1 \vee A \vee B)\sigma$.

(3) *First-order ordered resolution with selection between an initial and a universal clause, between two initial clauses, and ordered positive factoring with selection on an initial clause.* These are defined in analogy to the two deduction rules above with the only difference that the result is an initial clause.

(4) *Ordered fine-grained step resolution with selection.*

$$\frac{C_1 \Rightarrow \bigcirc(D_1 \vee A) \quad C_2 \Rightarrow \bigcirc(D_2 \vee \neg B)}{(C_1 \wedge C_2)\sigma \Rightarrow \bigcirc(D_1 \vee D_2)\sigma},$$

where $C_1 \Rightarrow \bigcirc(D_1 \vee L)$ and $C_2 \Rightarrow \bigcirc(D_2 \vee \neg M)$ are step clauses, σ is a most general unifier of the literals L and M such that σ does not map variables from C_1 or C_2 into a constant or a functional term, *$A\sigma$ is eligible in $(D_1 \vee A)\sigma$, and $\neg B\sigma$ is eligible in $(D_2 \vee \neg B)\sigma$.*

$$\frac{C_1 \Rightarrow \bigcirc(D_1 \vee L) \quad D_2 \vee \neg M}{C_1\sigma \Rightarrow \bigcirc(D_1 \vee D_2)\sigma},$$

where $C_1 \Rightarrow \bigcirc(D_1 \vee L)$ is a step clause, $D_2 \vee \neg M$ is a universal clause, and σ is a most general unifier of the literals L and M such that σ does not map variables from C_1 into a constant or a functional term, *$N\sigma$ is eligible in $(D_2 \vee \neg N)\sigma$, and $L\sigma$ is eligible in $(D_1 \vee L)\sigma$.*

(5) *Ordered fine-grained positive step factoring with selection.*

$$\frac{C \Rightarrow \bigcirc(D \vee A \vee B)}{C\sigma \Rightarrow \bigcirc(D \vee A)\sigma},$$

where σ is a most general unifier of the atoms A and B such that σ does not map variables from C into a constant or a functional term, and *$A\sigma$ is eligible in $(D \vee A \vee B)\sigma$.*

(6) *Clause conversion.* A step clause of the form $C \Rightarrow \bigcirc\bot$ is rewritten to the universal clause $\neg C$.

Let *ordered fine-grained resolution with selection* be the calculus consisting of the rules (1) to (6) above, together with the ground and non-ground eventuality resolution rules, restricted to loops found by the FG-BFS algorithm which now uses the rules (1) to (5) above instead of their unrefined variants. We denote this calculus by $\mathfrak{I}^{S,\succ}_{FG}$. Again, the calculus can be extended by first-order redundancy elimination rules as well as analogous rules for step clauses.

Note that for ordered fine-grained step resolution with selection, the ordering and selection function only influence which literals on the right-hand side of a step clause are eligible, literals on the left-hand side are not taken into account.

Theorem 5. *Ordered fine-grained resolution with selection is sound and complete for constant flooded monodic temporal problems over expanding domains.*

Proof [Sketch] Soundness of $\mathfrak{I}^{S,\succ}_{FG}$ is straightforward as any derivation in $\mathfrak{I}^{S,\succ}_{FG}$ is also a derivation in $\mathfrak{I}_{FG}$, which is sound according to Theorem 4.

The proof of completeness proceeds along the lines of the completeness proof of $\mathfrak{I}_{FG}$ presented in [14]. Assume that $\mathsf{P}^c = \langle \mathcal{U}_0, \mathcal{I}, \mathcal{S}, \mathcal{E} \rangle$ is a constant flooded monodic temporal problem and $\Delta = \mathcal{U}_0, \ldots, \mathcal{U}_n$ is a derivation from P^c in $\mathfrak{I}_e$ such that $\mathcal{U}_n$ contains $\bot$, that is, $\mathfrak{I}_e$ is able to derive a contradiction from P^c. By induction on the length of the derivation we show that this derivation can be simulated by $\mathfrak{I}^{S,\succ}_{FG}$. We construct a refutation $\Delta' = C^1_0, \ldots, C^{n_0}_0, \ldots, C^1_n, \ldots, C^{n_k}_n$

of the clausification Cls(P^c) of P^c where each step in Δ will correspond to one or more steps in Δ'. At the start Δ just consists of $\mathcal{U}_0$ and the corresponding derivation Δ' consists of all the clauses $C_0^1, \ldots, C_0^{n_0}$ in Cls(P^c). Let $U(\Delta')$ and $I(\Delta')$ denote the set of all universal and initial clauses in Δ', respectively. By the fact that clausification preserves satisfiability, $\mathcal{U}_0$ is satisfiable iff $U(\Delta')$ is satisfiable and $\mathcal{U}_0 \cup \mathcal{I}$ is satisfiable iff $U(\Delta') \cup I(\Delta')$ is satisfiable. Furthermore, if $\mathcal{U}_0$ would contain $\bot$, then Δ' would contain the empty clause.

Now, in each step of Δ a first-order formula u_i, $1 \leq i \leq n$, is added to $\mathcal{U}_{i-1}$ to obtain $\mathcal{U}_i$, where u_i is the conclusion of one the deduction rules of $\mathfrak{I}_e$ applied to $\langle \mathcal{U}_{i-1}, \mathcal{I}, \mathcal{S}, \mathcal{E} \rangle$. We show that using $\mathfrak{I}_{FG}^{S,\succ}$ we can derive a clause $C_i^{n_i}$ from the clauses in the derivation Δ' constructed so far such that the universal closure of $C_i^{n_i}$ implies u_i. This also implies that $\mathcal{U}_i$ is satisfiable iff $U(\Delta') \cup \{C_i^{n_i}\}$ is satisfiable and $\mathcal{U}_i \cup \mathcal{I}$ is satisfiable iff $U(\Delta') \cup I(\Delta') \cup \{C_i^{n_i}\}$ is satisfiable. We then add $C_i^{n_i}$ and all intermediate clauses $C_i^1, \ldots, C_i^{n_i - 1}$ used in its derivation to Δ'. To show the existence and derivability of $C_i^{n_i}$ we consider which deduction rule of $\mathfrak{I}_e$ has been used to derive u_i.

Suppose u_i has been derived by an application of the termination rule (which implies that u_i is $\bot$). Then the set $\mathcal{U}_{i-1} \cup \mathcal{I}$ of first-order formulae is unsatisfiable, which, by induction hypothesis, implies that $U(\Delta') \cup I(\Delta')$ is unsatisfiable. By completeness of first-order ordered resolution with selection (see, e.g. [1]), we will be able to derive the empty clause from the clauses in $U(\Delta') \cup I(\Delta')$ using the resolution and factoring rules of $\mathfrak{I}_{FG}^{S,\succ}$ for universal and initial clauses, that is, rules (1) to (3), and extend Δ' accordingly.

Suppose u_i has been derived by an application of the step resolution rule. Then there is a merged derived step clause $\mathcal{A} \Rightarrow \bigcirc \mathcal{B}$ such that the formula $\mathcal{U}_{i-1} \cup \{\mathcal{B}\}$ is unsatisfiable. The merged derived step clause $\mathcal{A} \Rightarrow \bigcirc \mathcal{B}$ is constructed from some step clauses $p_j \Rightarrow \bigcirc q_j$, $1 \leq j \leq m_1$, $P_k(c_k) \Rightarrow \bigcirc Q_k(c_k)$, $1 \leq k \leq m_2$, and $P_l(x_l) \Rightarrow \bigcirc Q_l(x_l)$, $1 \leq l \leq m_3$, in $\mathcal{S}$ which are also present in Δ'. Define a set $L(\mathcal{B})$ of literals as $\{q_j \mid 1 \leq j \leq m_1\} \cup \{Q_k(c_k) \mid 1 \leq k \leq m_2\} \cup \{Q_l(x_l) \mid 1 \leq l \leq m_3\}$. Again, due to the completeness of first-order ordered resolution with selection, a derivation of the empty clause from $L(\mathcal{B}) \cup U(\Delta')$ exists. Then inspection of the rules (4) and (5) for ordered fine-grained step resolution with selection and for ordered right positive factoring with selection, respectively, shows that we can also construct a derivation from $\{p_j \Rightarrow \bigcirc q_j \mid 1 \leq j \leq m_1\} \cup \{P_k(c_k) \Rightarrow \bigcirc Q_k(c_k) \mid 1 \leq k \leq m_2\} \cup \{P_l(x_l) \Rightarrow \bigcirc Q_l(x_l) \mid 1 \leq l \leq m_3\} \cup U(\Delta')$. This will not be a derivation of the empty clause, but of a step clause $\mathcal{P} \Rightarrow \bigcirc \bot$ where $\mathcal{P}$ is a conjunction of literals in $\{P_j \mid 1 \leq j \leq m_1\} \cup \{P_k(c_k) \mid 1 \leq k \leq m_2\} \cup \{P_l(x_l) \mid 1 \leq l \leq m_3\}$, though not necessarily all of them. An application of rule (6) for clause conversion allows us to derive the universal clause $\neg \mathcal{P}$. We can show that the universal closure of $\neg \mathcal{P}$ implies u_i. We add all the clauses in the derivation of $\mathcal{P} \Rightarrow \bigcirc \bot$ to Δ' as $C_i^1, \ldots, C_i^{n_i - 1}$ for some n_i, and also add $\neg \mathcal{P}$ as $C_i^{n_i}$.

Finally, concerning the ground and non-ground eventuality resolution rules and the use of the FG-BFS algorithm to compute loops, we simply observe that using rules (1) to (5) in the algorithm will not change the loops the algorithm will compute. This follows from the considerations above. ❑

6 Decidability by ordered fine-grained resolution with selection

Ordered fine-grained resolution with selection allows us to transfer decidability results and decision procedures obtained for fragments of first-order logic to the corresponding fragments of monodic first-order temporal logic.

We present two examples. First, we consider the temporalisation $\mathcal{T}_{ren}GF$ of the guarded fragment by renaming according to Definition 3 and we show how a decision procedure can be constructed from the procedure for the guarded fragment developed in [9]. Second, using the same approach we derive a decision procedure for $\mathcal{T}_{ren}\overline{\mathrm{K}}$ and $\mathcal{T}_{ren}\overline{\mathrm{DK}}$ based on the procedure for $\overline{\mathrm{K}}$ and $\overline{\mathrm{DK}}$ developed in [13] ($\overline{\mathrm{DK}}$ is the class containing all conjunctions of formulae of the class $\overline{\mathrm{K}}$).

Ganzinger and de Nivelle [9] use the following ordering $\succ_{GF}$ and selection function S_{GF} to decide the guarded fragment: $\succ_{GF}$ is an arbitrary lexicographic path ordering on terms and atoms based on a precedence $\succ$ on function and predicate symbols such that $f \succ c \succ p$ for any non-constant function symbol f, constant c, and predicate symbol p. The selection function S_{GF} selects one of the guards in any clause that is non-functional[7] and contains at least one guard; it selects one of the functional negative literals in a clause containing such literals; and it does not select any literal in a clause containing a positive functional literal but no negative functional literal. On guarded clauses, that is, the class of clauses which contains the clause normal form of any guarded formula, the selection function S_{GF} is well-defined. In addition, the decision procedure in [9] requires that in the computation of the clausification of guarded formulae *structural transformation* [7, 20] is used to introduce surrogates for universally quantified subformulae. Let $\mathrm{ST}_{\mathrm{GF}}$ denote this transformation.

We can use exactly the same ordering and selection function to obtain a decision procedure for $\mathcal{T}_{ren}GF$.

Theorem 6. *Let $\succ_{GF}$ and S_{GF} be the ordering and selection function defined above. Then $\mathfrak{I}_{FG}^{S_{GF},\succ_{GF}}$ decides the satisfiability problem of $\mathcal{T}_{ren}GF$.*

Proof By Theorem 5, $\mathfrak{I}_{FG}^{S_{GF},\succ_{GF}}$ is sound and complete. It remains to show termination. Let φ be a formula in $\mathcal{T}_{ren}GF$ and P^c be the corresponding constant flooded temporal problem. In analogy to [9], we use the structural transformation $\mathrm{ST}_{\mathrm{GF}}$ in the computation of the clausification of P^c. Let P^c_{Cls} denote $\mathrm{Cls}(\mathrm{ST}_{\mathrm{GF}}(\mathsf{P}^c))$. First, we give a syntactical characterisation of the clauses in P^c_{Cls} and of the clauses we might have derived from it. To do so, we extend the notion of a guarded clause to step clauses as follows. A step clause $C \Rightarrow \bigcirc D$ is guarded iff the first-order clause $\neg C \vee D$ is guarded and C is monadic. Then all the universal, initial, and step clauses in P^c_{Cls} are guarded. We can also show that all inference steps possible by $\mathfrak{I}_{FG}^{S_{GF},\succ_{GF}}$ on guarded (step) clauses will result in a guarded (step) clause. Second, the number of guarded (step) clauses (up to variable renaming) over the signature Σ of P^c_{Cls} is finitely bounded, more

[7] An expression is *functional* if it contains a constant or a function symbol, and *non-functional* otherwise.

precisely, there is a double exponential upper bound in the size of Σ on their number. Consequently, any derivation from P^c_{Cls} will either eventually produce the empty clause or no new clauses can be added to the derivation. ❑

Our second example is a decision procedure for $\mathcal{T}_{ren}\overline{\mathrm{K}}$ and $\mathcal{T}_{ren}\overline{\mathrm{DK}}$ based on the resolution decision procedure for $\overline{\mathrm{K}}$ and $\overline{\mathrm{DK}}$ by Hustadt and Schmidt [13]. The procedure uses an atom ordering $\succ_K$ which is a recursive path ordering based on a total precedence $\succ$ on function and predicate symbols which basically gives precedence to symbols of greater arity. The selection function S_K maps any clause to the empty set. The decision procedure also uses an additional inference rule, namely *splitting*, to perform case analysis on clauses consisting of variable-disjoint subclauses. While it is possible to extend the calculus $\mathfrak{I}^{S,\succ}_{FG}$ by a splitting inference rule, it is easier to use splitting through new predicate symbols instead [3, 21]. Here, whenever we have a clause $C \vee D$ such that C and D are variable-disjoint, we replace it by two clauses $C \vee p$ and $\neg p \vee D$, where p is a new predicate symbol of arity 0 smaller than any other predicate symbol. Finally, the procedure requires the use of structural transformation in the computation of the clausification of formulae in $\overline{\mathrm{K}}$ and $\overline{\mathrm{DK}}$. Here, certain occurrences of one-variable literals with constant or duplicate variable arguments have to replaced by surrogates (see [13] for details). Let $\mathrm{ST}_{\overline{\mathrm{K}}}$ denote this transformation.

Theorem 7. *Let $\succ_K$ and S_K be the ordering and selection function defined above. Then $\mathfrak{I}^{S_K,\succ_K}_{K}$ decides the satisfiability problem of $\mathcal{T}_{ren}\overline{\mathrm{K}}$ and $\mathcal{T}_{ren}\overline{\mathrm{DK}}$.*

Proof Along the lines of the proof of Theorem 6. Let φ be a formula in $\mathcal{T}_{ren}\overline{\mathrm{K}}$ or $\mathcal{T}_{ren}\overline{\mathrm{DK}}$, let P^c be the corresponding constant flooded temporal problem, and let P^c_{Cls} be $\mathrm{Cls}(\mathrm{ST}_{\overline{\mathrm{K}}}(\mathsf{P}^c))$. The characterisation of clauses in P^c_{Cls} and of the clauses we derive from it is based on the notions of (strongly) k-regular and (strongly) CDV-free clauses introduced in [13]. Again, we need to extend these notions to step clauses. We can then show that all universal, initial, and step clauses in P^c_{Cls} are strongly CDV-free and k-regular, or strongly k-regular if φ belongs to $\mathcal{T}_{ren}\overline{\mathrm{DK}}$. Inference steps restricted by $\succ_K$ and S_K will also only derive clauses with these properties. There is a double exponential upper bound on the number of (strongly) k-regular, (strongly) CDV-free clauses in the size of the signature of P^c_{Cls}. This shows termination of any derivation in $\mathfrak{I}^{S_K,\succ_K}_{K}$. ❑

7 Future work

One motivation for our interest in classes decidable by ordered fine-grained resolution with selection is that with the theorem prover **TeMP** [12] we have an implementation of fine-grained resolution. **TeMP** takes advantage of the arithmetic translation of temporal problems which allows us to use a first-order theorem prover, in our case Vampire, to implement the inference rules of fine-grained resolution. Consequently, we will have to transfer the restrictions imposed by ordered fine-grained resolution with selection to the level of the first-order theorem prover employed by **TeMP** to realise the decision procedures presented in this paper.

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